Proof of Passivity based Human-robot Team Cooperative Control

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5th Nov., 2013

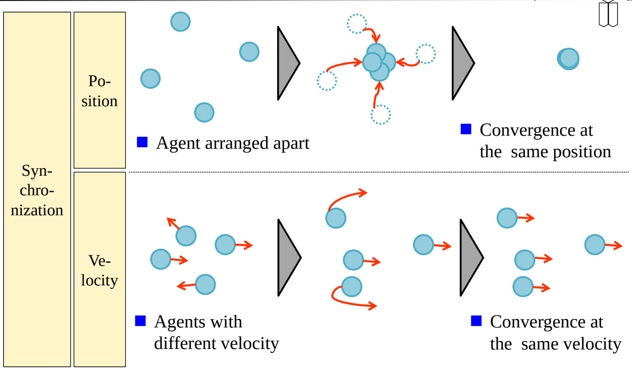
Today's Topic

Introduction
- What is Synchronization ? (which is my research's basis.)
  - Position Synchronization
  - Velocity Synchronization
- What is Human interaction ?

Settings Theorem
- Problem Settings
- Assumption
- Control Object
- Theorem

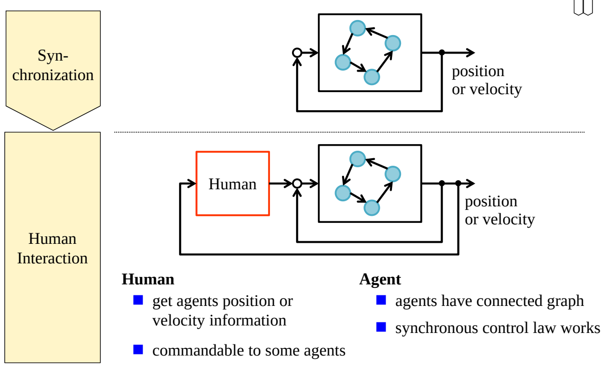
Proof | Proof
- Proof
  - Position Synchronization
  - Velocity Synchronization

What is Synchronization ?



It expresses motion of swarm.

Human Interaction

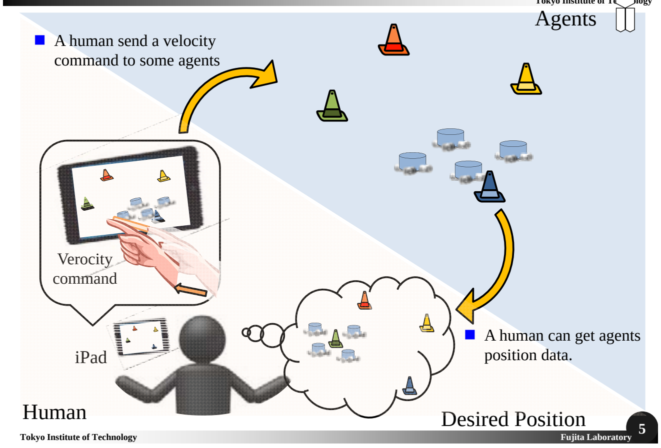


Human
- get agents position or velocity information
- commandable to some agents

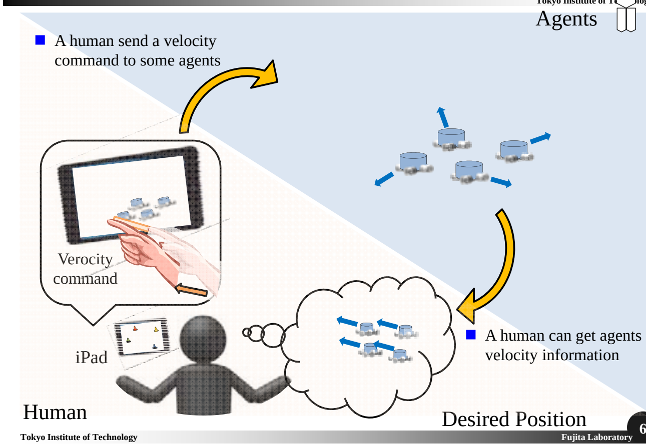
Agent
- agents have connected graph
- synchronous control law works

Human interaction enables the control using agent information.

Situation : Position



Situation : Velocity



Settings

Notation

n agents : $\mathcal{A}_1, \cdots, \mathcal{A}_n$

position : $q = [q_1 \ \cdots \ q_n]^T$

$q_i \in \mathcal{R}^{1\times 2}$

desired value in human brain: $r_p \in \mathcal{R}^{n\times 2}$ (position) or $r_v \in \mathcal{R}^{n\times 2}$ (velocity)

command vector : $E = [e_1 \ \cdots \ e_n]$

$e_i = \begin{cases} 1 & \text{(commandable)} \\ 0 & \text{(Not commandable)} \end{cases}$

command from human : $v_d \in \mathcal{R}^{1\times 2}$

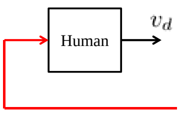
m : number of commandable agents

		Position Control	Velocity Control
Problem Setting	Agent	$\dot{v} = -Lq$, $\dot{q} = -Lq + Lv + Ev_d$	
	Human	Desired position r_p	Desired velocity r_v
		Velocity command: v_d	
Control Objective		$\lim_{t\to\infty}(q - r_p)$	$\lim_{t\to\infty}(\dot{q} - r_v)$

Assumption

Assumption I	Graph Structure : balanced and strongly connected	⇒	$\mathbb{1}^T L = 0$
Assumption II	Desired values : same values with all agents	⇒	$r_p = c_p \mathbb{1}$, $r_v = c_v \mathbb{1}$
Assumption III	Human : passive from input to output	⇒	$\dot{S} \le \tau^T y$
			Miyazawa's presentation
Assumption IV	(Only position control) Human : when agents stay desired value, human don't command to them . when human don't command to agents, they stay desired value.	⇒	$v_d = 0 \Leftrightarrow q - r_p = 0$
Assumption V	(Only velocity control) Human : when agents don't move at desired velocity, human change his command	⇒	$q_r \neq r_v \Rightarrow \dot{v}_d \neq 0$

Control Law and Theorem



what to feedback ?

	Position Control	Velocity Control
Dynamics	$\dot{v} = -Lq$, $\dot{q} = -Lq + Lv + Ev_d$ (1)	
Feedback to human	$-\frac{1}{m}E^T(q - r_p)$ (2) The center of gravity of position error of commandable agent	$-\frac{1}{m}E(\dot{q} - r_v)^T$ (3) The average of velocity error of commandable agent
Theorem	Consider the dynamical system described by (1) with the feedback to human (2) or (3) . Then under assumption I~V, the system is globally stable and the agents output synchronize at the desired value.	

Outline of Proof : Position Synchronization

Dynamics

$\dot{v} = -Lq$, $\dot{q} = -Lq + Lv + Ev_d$ ⇒ $\dot{v} = -Lq_r$, $\dot{q}_r = -Lq_r + Lv + Ev_d$

Energy func. Passivity

$\dot{V}_{all} = \dot{V} + \dot{S} \le 0.$

$V = \frac{1}{2}q_r^T q_r + \frac{1}{2}v^T v$ (Agents' energy)

$\dot{S} \le \tau^T y$ (Human's energy)

Lasalle

$\dot{V}_{all} = 0 \Rightarrow q_r = \dot{q}_r = 0$

Step 1

$L(Lv + Ev_d) = 0 \Rightarrow Lv + Ev_d = 0$

- Assumption III : $\dot{S} \le \tau^T y$

Step 2

$Lv + Ev_d = 0 \Rightarrow q_r = 0$

- Contraposition : $q_r \neq 0 \Rightarrow Lv + Ev_d \neq 0$
- Assumption IV : $v_d = 0 \Leftrightarrow q - r_p = 0$

$U = \{(q_r, \dot{q}_r) \mid \dot{V}_{all} = 0\}$. $M = \{(q_r, \dot{q}_r) \mid q_r = \dot{q}_r = 0\}$.

- Lasalle invariance principle

Proof (1): Position Synchronization

In the system (1), replace q with $q_r = q - r_p$

$\dot{v} = -Lq$, $\dot{q} = -Lq + Lv + Ev_d$ ⇒ $\dot{v} = -Lq_r$, $\dot{q}_r = -Lq_r + Lv + Ev_d$

Define a positive definite Lyapunov function for the system.

$$V = \tfrac{1}{2}q_r^T q_r + \tfrac{1}{2}v^T v$$

The derivative of this function along trajectories of the system is given by

$$\dot{V} = -q_r^T Lq_r + q_r^T Lv + q_r^T Ev_d - v^T Lq_r = -q_r^T Lq_r + q_r^T Ev_d$$

Consider the human system that have S satisfy the condition $\dot{S} \le \tau^T y$ (Assumption III) and whose input τ is $-E^T q_r$.

The derivative of the total energy including the human system energy is

$$\dot{V}_{all} = \dot{V} + \dot{S} = -q_r^T Lq_r \le 0.$$

Thus the system is globally stable and all signals are bounded.

Consider the set $U = \{(q_r, \dot{q}_r) \mid \dot{V}_{all} = 0\}$.

The maximum invariant set in U is $M = \{(q_r, \dot{q}_r) \mid q_r = \dot{q}_r = 0\}$.

Proof (1): Position Synchronization

Consider the set $U = \{(q_r, \dot{q}_r) \mid \dot{V}_{all} = 0\}$.

The maximum invariant set in U is $M = \{(q_r, \dot{q}_r) \mid q_r = \dot{q}_r = 0\}$.

When $\dot{V}_{all} = 0$

$Lq_r = 0 \Rightarrow L\dot{q}_r = 0$

Then $L\dot{q}_r = L(Lv + Ev_d) = 0$

Assume that $\dot{q}_r = c\mathbb{1}$, then $\frac{d}{dt}(v_d^T q_r) = v_d^T \dot{q}_r = \frac{m}{n}\|v_d\|^2 \ge 0$

where m is the number of commandable agents, n is the number of agents.

$v_d^T \dot{q}_r \ge 0$ is contrary to the assumption III, $v_d^T \dot{q}_r \le 0$ (a human is passive).

So $\dot{q}_r \neq c\mathbb{1}$. Hence $L(Lv + Ev_d) = 0 \Rightarrow Lv + Ev_d = 0$.

Next we show that $Lv + Ev_d = 0 \Rightarrow q_r = 0$.

To prove above equation, we show a contraposition $q_r \neq 0 \Rightarrow Lv + Ev_d \neq 0$.

Here, assume $Lv + Ev_d = 0$ and multiply $\mathbb{1}^T$ from left side when $q_r \neq 0$.

$\mathbb{1}^T Lv + \mathbb{1}^T Ev_d = 0 \quad \therefore \ \mathbb{1}^T Ev_d = 0 \quad \therefore mv_d = 0$

Then, $q_r = 0$ from assumption IV, but this is contradictory to $q_r \neq 0$.

Therefore the assumption $Lv + Ev_d = 0$ is wrong.

Hence $q_r \neq 0 \Rightarrow Lv + Ev_d \neq 0$. $\therefore$ $Lv + Ev_d = 0 \Rightarrow q_r = 0$

From the above, the maximum invariant set in U is $M = \{(q_r, \dot{q}_r) \mid q_r = \dot{q}_r = 0\}$.

Using Lasalle's Invariance Principle, the agents output synchronize at desired position.

Outline of Proof : Velocity Synchronization

Energy func. Passivity

$\dot{V}_{all} = \dot{V} + \dot{S} \leq 0.$

$V = \frac{1}{2}(\dot{q} - r_v)^T(\dot{q} - r_v) + \frac{1}{2}q^T L^2 q$ (Agents' energy)

$\dot{S} \leq \tau^T y$ (Human's energy)

Lasalle

$\dot{V}_{all} = 0 \Rightarrow q_r - r_v = 0$

Step 1

$E\dot{v}_d = 0$

- Achievement of velocity synchronization : $L\dot{q} = 0$

Step 2

$\dot{q} = r_v$

- Assumption V : $q_r \neq r_v \Rightarrow \dot{v}_d \neq 0$
- Contradiction: $E\dot{v}_d = 0$

$U = \{(q, \dot{q}) \mid \dot{V}_{all} = 0\}$. $M = \{(q, \dot{q}) \mid \dot{q} - r_v = 0\}$.

- Lasalle invariance principle

Proof (1): Velocity Synchronization

Define a positive definite Lyapunov function for the system.

$$V = \frac{1}{2}(\dot{q} - r_v)^T(\dot{q} - r_v) + \frac{1}{2}q^T L^2 q$$

The derivative of this function along trajectories of the system is given by

$$\begin{aligned}\dot{V} &= (\dot{q} - r_v)^T(-L\dot{q} + L\dot{v} + E\dot{v}_d) + q^T L^2 u \\ &= (\dot{q} - r_v)^T E\dot{v}_d - (\dot{q} - r_v)^T(-L\dot{q} - L^2 q) + q^T L^2 \dot{q} \\ &= (\dot{q} - r_v)^T E\dot{v}_d - \dot{q}^T L\dot{q}\end{aligned}$$

Consider the human system that have *S* satisfy the condition $\dot{S} \leq \tau^T y$ (Assumption I) and whose input τ is $-E^T(\dot{q} - r_v)$

The derivative of total energy including the human system energy is

$$\dot{V}_{all} = \dot{V} + \dot{S} = -\dot{q}^T L\dot{q} \leq 0.$$

When $\dot{q}^T L\dot{q} = 0$, $L\dot{q} = 0$. This means that the velocity of each agent synchronize.

Consider the set $U = \{(q, \dot{q}) \mid \dot{V}_{all} = 0\}$.

The maximum invariant set in *U* is $M = \{(q, \dot{q}) \mid \dot{q} - r_v = 0\}$.

Proof (2): Velocity Synchronization

Consider the set $U = \{(q, \dot{q}) \mid \dot{V}_{all} = 0\}$.

The maximum invariant set in *U* is $M = \{(q, \dot{q}) \mid \dot{q} - r_v = 0\}$.

From $L\dot{q} = 0$, relative position of each agent don't change. So,

$$Lq = \text{const}, \quad Ev_d = \text{const}.$$

Thus we get $E\dot{v}_d = 0$.

Assume $\dot{q} - r_v \neq 0$, $\dot{v}_d \neq 0$ from Assumption V. That is $E\dot{v}_d \neq 0$.

However, this contradict the above equation $E\dot{v}_d = 0$.

Under Assumption V, $\dot{q} = r_v$.

From the above, the maximum invariant set in *U* is $M = \{(q, \dot{q}) \mid \dot{q} - r_v = 0\}$.

Using Lasalle's Invariance Principle, the agents output synchronize at desired velocity.