Weighted Coverage Control with Voronoi Partition

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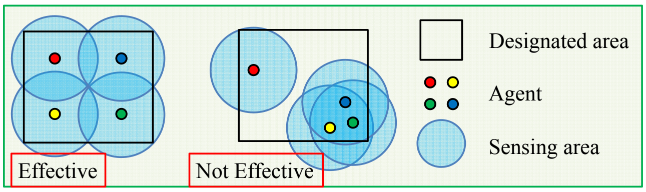
Coverage Control

Control position of each agent to cover a designated area effectively.



- Importance of the area is uniform.
- Performance of each agent is the same.

Voronoi diagram: An approach to this control

Voronoi Diagram

$$V(p_i) = \{p \mid d(p,p_i) \leqq d(p,p_j), j \neq i\}$$

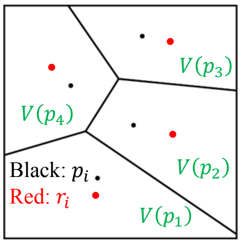
p_i: Position of agent i

$V(p_i)$: Voronoi region of agent i

$d(p,p_i)$: Distance between p and p_i

r_i: Reference position of agent i (Centroid of $V(p_i)$)

Procedure on coverage control ($^\forall i$)
1. Get p_i
2. Calculate $V(p_i)$
3. Calculate r_i
4. Move agent i to r_i
5. Repeat until $p_i = r_i$



Advantages of Variable Weight

Past system: Uniform weight

$r_i (p_i = r_i, ^\forall i)$ is fixed.

If $p_i = r_i$ ($^\forall i$), each agent does not move anymore.

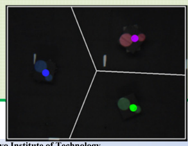
Current system: Variable weight

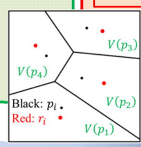
$r_i (p_i = r_i, ^\forall i)$ is not fixed.

Each agent can go to various point.

If we change weight, each agent starts to move.

Each agent can keep moving.





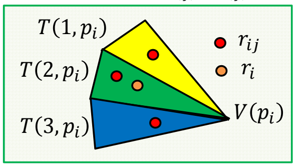
Derivation of r_i (Centroid of $V(p_i)$)

$$r_{i,j} = \left(\frac{\sum_k \rho x_{i,j,k}}{\sum_k \rho}, \frac{\sum_k \rho y_{i,j,k}}{\sum_k \rho}\right) \quad r_i = \left(\frac{\sum_j \sum_k \rho x_{i,j,k}}{\sum_j \sum_k \rho}, \frac{\sum_j \sum_k \rho y_{i,j,k}}{\sum_j \sum_k \rho}\right)$$

$T(j,p_i)$: A triangle constituting $V(p_i)$ $\quad$ $r_{i,j}$: Centroid of $T(j,p_i)$

$(x_{i,j,k}, y_{i,j,k})$: kth point in $T(j,p_i)$ $\quad$ ρ: Weight at $(x_{i,j,k}, y_{i,j,k})$

Procedure on derivation of r_i
1. Divide $V(p_i)$ into $T(j,p_i)$
2. Derive $r_{i,j}$
3. Derive r_i



If designated area is convex, $V(p_i)$ is also convex.

⇒ $V(p_i)$ can be divided into some triangles easily.

⇒ We have to consider only triangles for any $V(p_i)$.

Derivation of $r_{i,j}$ (on C++)

Express $T(j,p_i)$ as a set of square.

$$r_{i,j} = \left(\frac{\sum_k \rho x_{i,j,k}\, ds}{\sum_k \rho\, ds}, \frac{\sum_k \rho y_{i,j,k} ds}{\sum_k \rho\, ds}\right) = \left(\frac{\sum_k \rho x_{i,j,k}}{\sum_k \rho}, \frac{\sum_k \rho y_{i,j,k}}{\sum_k \rho}\right)$$

$(x_{i,j,k}, y_{i,j,k})$: kth red point in $T(j,p_i)$ $\quad$ ρ: Weight $\quad$ ds: Area of a square

$dm = \rho(x_{i,j,k}, y_{i,j,k})ds$

- Shape of the square is fixed.
- Weight at the red point represents weight at whole points in the square.