Network Structure of Replicator-mutator Dynamics Model

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Background

Multi-agent System

system composed of multiple interacting intelligent agent

be used to solve problem that are difficult or impossible for an agent

Example

- formation flight
- cooperative control of UAVs
- sensor network
- energy network

there are signal exchanges via local interaction

Communication Network

agents within a limited communication range can exchange information or interact directly

⇓

graph based interaction model

Background

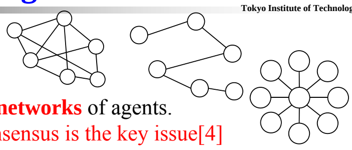
Approach

Consensus [1-7]

Objective

to reach an agreement in **networks** of agents.

the speed of reaching a consensus is the key issue[4]



Application

formation control, flocking, sensor network, attitude alignment

Evolutionary Dynamics in social networks[8-12]

Objective

to forecast and control the behaviors in social **networks**

Application

analysis of social phenomena, crowd simulation

Orientation

based on Evolutionary Dynamics

New consider the speed of convergence

Consensus in Networked Systems[1-7]

Network Topology

$G = (V, E, A)$ $V = \{v_1, \cdots v_n\}$ v_i : agent

connected undirected graph $E = \{(b_i, b_j) : a_{ij} > 0\}$

$A = [a_{ij}]$: adjacency matrix

Consensus Dynamics

$\dot{\mathbf{x}}(t) = -L\mathbf{x}(t)$ $L = D - A$ D: degree matrix

solves a consensus problem

achieve a average consensus

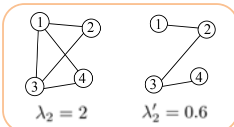
$\mathbf{x}(\mathbf{0}) = (z_1, \cdots z_n)$: initial state ➡ $\mathbf{x}^* = (\alpha, \cdots, \alpha)$ $(\alpha = 1/n \sum_i z_i)$

Algebraic Connectivity

the eigenvalues of Laplacian

$0 = \lambda_1 \leq \lambda_2 \leq \cdots \lambda_n$

λ_2 : measure of speed of convergence



- maximize the second smallest eigenvalues[2]
- small world network gives considerably larger λ_2[5]

Convergence Speed*

Reaching Agreement

$U = [\mathbf{u_1}\ \mathbf{u_2}\ \cdots\ \mathbf{u_n}]$: matrix consisting of eigenvectors of L

$$\dot{\mathbf{x}}(t) = -L\mathbf{x}(t)$$

$$\longrightarrow \mathbf{x}(\mathbf{t}) = e^{-Lt}\mathbf{x_0}$$

$$\longrightarrow \mathbf{x}(\mathbf{t}) = e^{-\lambda_1 t}(\mathbf{u}_1^T\mathbf{x_0})\mathbf{u_1} + e^{-\lambda_2 t}(\mathbf{u}_2^T\mathbf{x_0})\mathbf{u_2} + \cdots + e^{-\lambda_n t}(\mathbf{u}_n^T\mathbf{x_0})\mathbf{u_n}$$

$$\mathbf{x}(t) \to (\mathbf{u}_1^T\mathbf{x_0})\mathbf{u_1} = \frac{\mathbf{1}^T\mathbf{x_0}}{n}\mathbf{1} \qquad (\because \lambda_1 = 0)$$

λ_2 is the smallest positive eigenvalue of the Laplacian

➡ λ_2 is the slowest mode of convergence

λ_2 is the measure of *speed of convergence* of the consensus algorithm

Evolutionary Dynamics in Social Networks[8-12]

Behavior Network Topology

$G = (V, E, A)$ $V = \{b_1, \cdots b_n\}$ b_i : behavior (action)

connected undirected graph $E = \{(b_i, b_j) : a_{ij} > 0\}$

$A = [a_{ij}]$: adjacency matrix (assump. : $a_{ii} = 1, \forall i$)

Replicator Mutator Dynamics (RMD)

$\dot{\mathbf{x}} = Q^T F\mathbf{x} - \phi\mathbf{x}$ $F = \mathrm{diag}(\mathbf{f})$ x_i : fraction

$\mathbf{f} = A\mathbf{x}$: Payoff

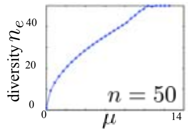
$Q = I - \mu L$: social choice model $\phi = \mathbf{f}^T\mathbf{x}$: average payoff

$L = I - D^{-1}A$: fixed $\mu > 0$: mutation parameter (constant)

Diversity

how many species exist in steady state

$n_e = 1/\sum x_i^2$



- roll of the structure of network and μ on diversity[10]
- small-world network improve the diversity on homogeneous networks[11]

Diversity*

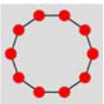
Phases for Evolution[8]

$n_e = 1/\sum x_i^2 \qquad 1 \le n_e \le n$

Name	n_e	μ	$\mathbf{x}$
① Behavioral Flocking	1	0	$x_{i^*}=1, x_j=0, \forall j \neq i^*$
② Cohesion	$1 < n_e \ll n$	↕	
③ Collapse	$1 \ll n_e < n$		
④ Complete Collapse	n	大	$x_i = 1/n, \forall i$

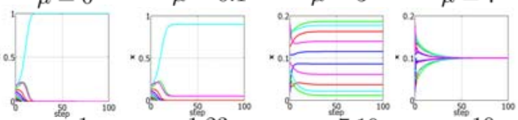
Example

node : 10



circle graph

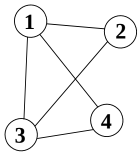
① Behavioral Flocking $\mu = 0$ ② Cohesion $\mu = 0.1$ ③ Collapse $\mu = 3$ ④ Complete Collapse $\mu = 7$



$n_e = 1$ $n_e = 1.23$ $n_e = 7.19$ $n_e = 10$

Comparison of Two Approaches

	Consensus Dynamics	Evolutionary dynamics
Network	graph theory (node, edge)	
Graph Laplacian	$L = D - A$	$L = I - D^{-1}A$ [*]
Dynamics	$\dot{\mathbf{x}} = -L\mathbf{x}$	$\dot{\mathbf{x}} = Q^T F \mathbf{x} - \phi \mathbf{x}$ $(Q = I - \mu L)$
Features	convergence speed	diversity



$A \quad \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad (a_{ii} = 1)$

$D \quad \mathrm{diag}(3,2,3,2) \qquad \mathrm{diag}(4,3,4,3)$

$L \quad \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0.75 & -0.25 & -0.25 & -0.25 \\ -0.33 & 0.67 & -0.33 & 0 \\ -0.25 & -0.25 & 0.75 & -0.25 \\ -0.33 & 0 & -0.33 & 0.67 \end{bmatrix}$

[*] J. A. Fax, R. M. Murray, "Information Flow and Cooperative Control of Vehicle Formations, "

Purpose of Study

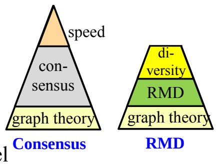
Orientation

- use the RMD model

→ deal with not only consensus but also population of a certain behavior

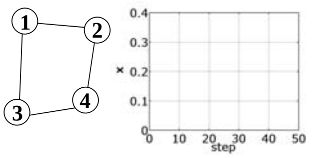
- consider the convergence speed based on second smallest eigenvalue in RMD model (New)

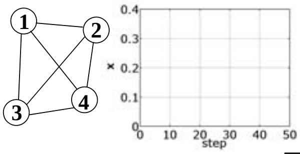
→ some kind of adjustment for Laplacian of RMD model is nesessary



Simulation Example

Settings : $\mathbf{x}_0 = [0.1\ 0.2\ 0.3\ 0.4]^T \quad \mu = 1 \quad a = 0.5$





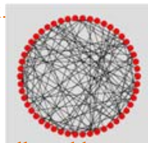
Future Works

RMD Model Analysis

simulation for various network structure

relation between
- network structure and diversity
- mutation parameter and diversity
- initial value and diversity

ex.



small-world network

Convergence Speed in RMD Model

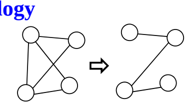
relation between network structure and convergence speed

connection between the second smallest eigenvalue and RMD model

Time-varying Parameter or Switching Topology

in case μ is not constant

in case network structure is time-varying



Concrete Situation Settings

consider the application

ex) formation control, power network, decision making control

スケジュール

Reference

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