Game Theoretic Optimal Power Allocation:
Distributed Welfare Game and its Simulation

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Background*

National Strategy "S+3E"
Propose a new energy best mix in the light of
- (i) Energy Security
- (ii) Economic Efficiency
- (iii) Environment
- + (iv) Safety (after 3.11)

Realization of Optimal Supply Structure			Realization of Energy-Saving Type Demand Structure
Fossil Fuel	Nuclear Power	Renewable Energy	Energy/Carbon
Development of New Energy System (Stable Supply, Competitive, Economic Efficiency)			
Development of New Energy Technology			
International Strategy			
Securing Resources	Contribution for Global Warming Prob.	International Cooperation	

One of the Solution
Distributed Cooperative Energy Management System (DCEMS) [*] 経済産業省 "エネルギー政策見直しの基本的視点," 2011/6
➡ Robustness to event uncertainty

Problems
Renewables Volatility
➡ Demand and Supply Balance
Frequency Stabilization

Solutions Optimization or Game Theory Distributed Cooperative EMS

Objective*

Target
Unbundling-based Power Network (Development of New Energy System)
(Unbundling: Supply Part is divided into Generation and Transmission)
➡ Electricity Liberalization

Supply Part
Demand Part

Problem
Optimal Power Allocation Problem
(Many Power Sources (PV), Many Consumers
Each consumer chooses needed power sources)

Approach
Game Theoretic Control[1,2] (Development of New Energy Technology)
(Utility Design: Welfare Game (Potential Game)[3]
Learning Design: Payoff-based Learning[4,5])

Merits: Scalability, Adaptability in real time
Solve the more complex situations than other approaches easily

Objective
Propose a new method of energy best mix

Outline

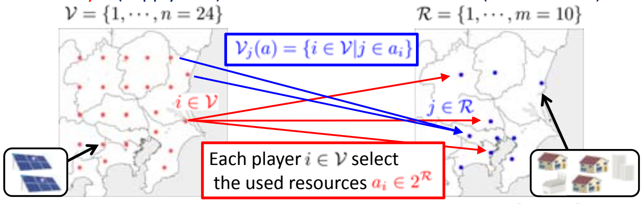
This Seminar

Objective: Design the Distributed Welfare Game
Show the result of its simulation

Problem Setting: Player and Resource

Player (Supply Side) $\mathcal{V}=\{1,\cdots,n=24\}$

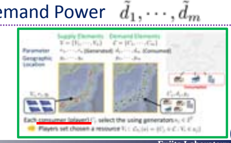
Resource (Demand Side) $\mathcal{R}=\{1,\cdots,m=10\}$



Generated Power $r_1,\cdots,r_n$
Location $p_1,\cdots,p_n$

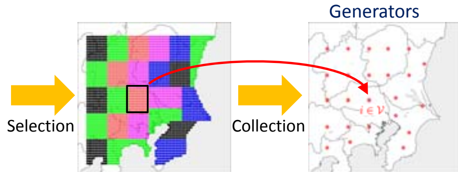
Consumed Power $d_1,\cdots,d_m$
Location $q_1,\cdots,q_m$
Population $s_1,\cdots,s_m$
Predicted Demand Power $\tilde{d}_1,\cdots,\tilde{d}_m$

Last Seminar(5/14, FL12-4-1)
(Supply Side) Resource ⟷ Reverse ⟷ (Demand Side) Player
(∵ Cannot compose a distributed welfare)



Problem Setting: Data

Data (MTSAT-2, quasi-real time[*])
Visualization[#]



Spec. (Sampling Time: Every hour (from 3h to 19h JST)
Value: Global Solar Radiation per second [W/m^2]
Term: 2010/7/23-current)

Generated Power r_i (Sum of the value at the measured points)

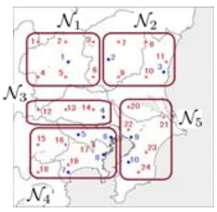
Features Panel Size S_i at $i\in\mathcal{V}$ ⟷ (Correspondence) Sum of the value at the points

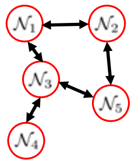
(If the conversion efficiency is the common value and the numerical scaling is ignored, this setting is a realistic situation.)

[*] CEReS 4VL http://www.cr.chiba-u.jp/~4vl/wiki/wiki.cgi [#] http://atmos.cr.chiba-u.ac.jp/takenaka/

Problem Setting: Region and Transmission

Region / **Transmission Route**





[Assume Realistic Model] — Lines over 500kV In Tokyo Electric Power Co.

Constraint Condition 1

Receive the trans. energy from Own and Neighboring Regions

Ex. $\mathcal{N}_1$, $\mathcal{N}_2$, $\mathcal{N}_5$ (2, 3); $\mathcal{N}_1$, $\mathcal{N}_2$, $\mathcal{N}_3$

Constraint Condition 2

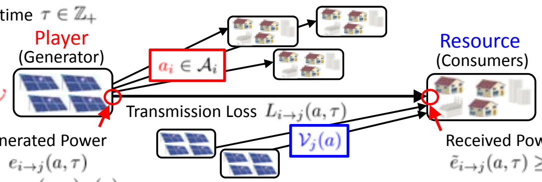
Each generator sends the energy to least 1 resource $\mathcal{A}_i \subseteq 2^{\mathcal{R}} \backslash \emptyset, \ \forall i \in \mathcal{V}$

Each resource receives the energy from least 1 generator $\mathcal{V}_j(a) \subseteq 2^{\mathcal{V}} \backslash \emptyset, \ \forall j \in \mathcal{R}$

Problem Setting: Energy Transmission

Energy Transmission

At time $\tau \in \mathbb{Z}_+$



Generated Power $e_{i \to j}(a, \tau) = w_{ij}(a, \tau) r_i(\tau)$

Received Power $\tilde{e}_{i \to j}(a, \tau) \geq 0$

$$w_{ij}(a, \tau) = \frac{\tilde{d}_j(\tau)}{\sum_{k \in a_i} \tilde{d}_k(\tau)}$$

Depending on demand power

Sum of Received Powers except Loss $E_j(a, \tau) = \sum_{i \in \mathcal{V}_j(a)} e_{i \to j}(a, \tau)$

Sum of Received Powers $\tilde{E}_j(a, \tau) = \sum_{i \in \mathcal{V}_j(a)} \tilde{e}_{i \to j}(a, \tau)$

Simulation Model

$L_{i \to j}(a, \tau) = k \| p_i - q_j \|_2 e^2_{i \to j}(a, \tau)$ $k \in \mathbb{R}_+$: loss param.

$\tilde{e}_{i \to j}(a, \tau) = e_{i \to j}(a, \tau) - L_{i \to j}(a, \tau) + F_j(a, \tau)$ $F_j(a, \tau)$: Penalty func.

Problem Setting: Limitation

Transmission Limitation $\mathcal{N}_i \xrightarrow{v_{ij}(a,\tau)} \mathcal{N}_j$, $\mathcal{N}_j \xrightarrow{v_{ji}(a,\tau)} \mathcal{N}_i$

v_{ij} v_{ji} Sum of Trans. Energy from Red to Blue

$\mathcal{N}_i$ $0 \leq v_{ij} - v_{ji} \leq M_{ij}$ $\mathcal{N}_j$

Over Trans. $\mathcal{N}_i$ $M_{ji} \geq v_{ji} - v_{ij} \geq 0$ $\mathcal{N}_j$ Over Trans.

Ex. 1-6 $\xrightarrow{v_{12}}$ 2,3; 1 $\xrightarrow{v_{21}}$ 7-11 ($\mathcal{N}_1$, $\mathcal{N}_2$)

$$v_{12}(a, \tau) = \sum_{\substack{j \in a_i \\ i \in \mathcal{N}_1, j \in \mathcal{N}_2}} e_{i \to j}(a, \tau)$$

$$v_{21}(a, \tau) = \sum_{\substack{j \in a_i \\ i \in \mathcal{N}_2, j \in \mathcal{N}_1}} e_{i \to j}(a, \tau)$$

($F_j(a, \tau)$: Penalty func.
Each player shoulder the over/short value depending on demand in the region when "over trans." mode happens)

Action Limitation (Restricted Action Set)

$\mathcal{R}_i(a_i) = \{a_i' \in \mathcal{A}_i | \max\{|a_i' \backslash a_i|, |a_i \backslash a_i'|\} \in \{0, 1\}\}$

Each player can add/remove at most 1 resource at 1 step

Ex. Player 1

1 → {1, 2, 3, 4} — Current: {2, 3} ⇒ Next Available Actions: {2, 3}, {2}, {3}, {1, 2, 3}, {2, 3, 4}

Welfare, Utility and Potential Game 1

Resource Side At each step

Transmission Loss

$$v_j^1(a) = -\sum_{\tau \in [T_0, T_0 + \Delta T]} \left(\left| \frac{E_j(a, \tau) - \tilde{E}_j(a, \tau)}{E_j(a, \tau)} \right| + \left| \frac{F_j(a, \tau)}{d_j(\tau)} \right| \right)$$

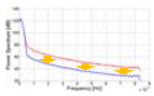
Trans. Loss Rate   Penalty Rate   known?

Smooth Effect (psd: power spectrum density)

Translating Data $\bar{E}_j(a, \tau) = \dfrac{\tilde{E}_j(a, \tau)}{\sum_{\tau' \in [T_0, T_0 + \Delta T]} \tilde{E}_j(a, \tau')}, \ \tau \in [T_0, T_0 + \Delta T]$

Translated psd Data $P_j(a; \omega)$

$v_j^2(a) = -\int_{\underline{\omega}}^{\bar{\omega}} \gamma(\omega) P_j(a; \omega) d\omega$ Weight func. $\gamma(\omega) = \dfrac{\omega}{\underline{\omega}}$



Player Side

$W(a) = w_1 \sum_{j \in \mathcal{R}} v_j^1(a) + w_2 \sum_{j \in \mathcal{R}} v_j^2(a)$ w_1, w_2 : weight param.

$U_i(a) = W(a) = \phi(a), \ \forall i \in \mathcal{V}$ ⇒ Compose PG (not Distributed Welfare Game)

Other Translating Data* $\bar{E}_j'(a, \tau) = \dfrac{\tilde{E}_j(a, \tau) - d_j(\tau)}{d_j(\tau)}$

Welfare, Utility and Potential Game 2

Welfare Func. $W(a) = \sum_{j \in \mathcal{R}} W_j(\mathcal{V}_j(a))$ $W_j(\mathcal{V}_j(a)) = w_1 v_j^1(a) + w_2 v_j^2(a)$

Utility Func. $U_i(a) = \sum_{j \in a_i} U_i^j(a)$ [If $|\mathcal{V}_j(a)| = 0$ then $W_j(\mathcal{V}_j(a)) = 0$; $\mathcal{V}_j(a) \subseteq 2^{\mathcal{V}} \backslash \emptyset$ ⇒ Not be used]

Form 1 ESU $U_i^j(a) = W_j(\mathcal{V}_j(a)) / |\mathcal{V}_j(a)|$

Form 2 WLU $U_i^j(a) = W_j(\mathcal{V}_j(a)) - W_j(\mathcal{V}_j(a) \backslash \{i\})$ How to calculate?

$$v_j^1(a) = -\sum_{\tau \in [T_0, T_0 + \Delta T]} \left(\left| \frac{E_j(a, \tau) - \tilde{E}_j(a, \tau)}{E_j(a, \tau)} \right| + \left| \frac{F_j(a, \tau)}{d_j(\tau)} \right| \right)$$

Is it possible to separate $\tilde{e}_{i \to j}$ only and calculate a changed Penalty Func.?
(in sense of actual system)   ($F_j(a \backslash \{i\}, \tau)$: unknown)

If $|\mathcal{V}_j(a)| = 1$, then how is the psd value $v_j^2(a \backslash \{i\})$?

⇒ WLU composes PG (Distributed Welfare Game)

(Exact) Restricted Potential Game

A game $\Gamma = (\mathcal{V}, \mathcal{A}, \{U_i\}_{i \in \mathcal{V}}, \{\mathcal{R}_i\}_{i \in \mathcal{V}}, \phi)$ satisfying

$\exists \phi$ s.t. $\forall i \in \mathcal{V}, \forall a_i \in \mathcal{A}_i, \forall a_i' \in \mathcal{R}_i(a_i), \forall a_{-i} \in \mathcal{A}_{-i}$

$U_i(a_i', a_{-i}) - U_i(a_i, a_{-i}) = \phi(a_i', a_{-i}) - \phi(a_i, a_{-i})$

Welfare, Utility and Potential Game 2*

Form 2 WLU $U_i^j(a) = W_j(\mathcal{V}_j(a)) - W_j(\mathcal{V}_j(a) \backslash \{i\})$ How to calculate?

$$v_j^1(a) = -\sum_{\tau \in [T_0, T_0 + \Delta T]} \left(\left| \frac{E_j(a, \tau) - \tilde{E}_j(a, \tau)}{E_j(a, \tau)} \right| + \left| \frac{F_j(a, \tau)}{d_j(\tau)} \right| \right)$$

Is it possible to separate $\tilde{e}_{i \to j}$ only and calculate a changed Penalty Func.?
(in sense of actual system)   ($F_j(a \backslash \{i\}, \tau)$: unknown)

Solution Method (estimation) $\tilde{e}_{i \to j}$ Suppose it takes the rate of $\tilde{E}_j(a, \tau)$ depending on the rate of Received Powers except Loss

$$\tilde{e}_{i \to j}(\tau) = \frac{e_{i \to j}(a, \tau)}{\sum_{k \in \mathcal{V}_j(a)} e_{k \to j}(a, \tau)} \tilde{E}_j(a, \tau)$$

$F_j(a \backslash \{i\}, \tau)$ Equal with $F_j(a, \tau)$ (ignore this element)

(In simulation, suppose $\tilde{e}_{i \to j}$ is known)

If $|\mathcal{V}_j(a)| = 1$, then how is the psd value $v_j^2(a \backslash \{i\}), \ j \in a_i$?

Problem $\tilde{E}_j(a \backslash \{i\}, \tau) \equiv 0$ ⇒ $P_j(a \backslash \{i\}; \omega) \equiv 0, \ \forall \omega \neq 0$
⇒ $v_j^2(a \backslash \{i\}) = 0 > v_j^2(a)$

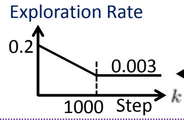
Solution Method $v_j^2(a \backslash \{i\}) \equiv v_j^2(a)$

Simulation: Setting

Initial State
Access within the own region

Learning Algorithm
Payoff-based Inhomogeneous Partially Irrational Play (PIPIP)[4]
Irrational Parameter $\kappa = 0.2$
Exploration Rate

if $\exists j \in \mathcal{R}$ s.t. $\mathcal{V}_j(a(k)) = \emptyset$
Select 1 player $i \in \mathcal{V}$ satisfying
$j \in a_i(k-1)$, and change $a_i(k) = a_i(k-1)$



Prob. that at least 1 player acts an exploration
99.6% → 7%

Simulation Model
Consumed Power $d_j(\tau) = w_j S(\tau)$ $w_j = \frac{s_j}{\sum_{k\in\mathcal{R}} s_k}$ $S(\tau) = \sum_{i\in\mathcal{V}} r_i(\tau)$

Predicted Demand Power $\tilde{d}_j(\tau) = \frac{S'_j(\tau)}{T+1}$ $S'_j(\tau) = \sum_{t\in[t_0,t_0+T]} d_j(t)$ $t_0, T \in \mathbb{Z}_+$: param.

Simulation 1: Setting

Situation Region Limitation
Transmission Limitation

Data/Cycle 60 data (cycle) Time/Step Δt 60 times

Function
Transmission Loss

$$v_j^1(a) = -\sum_{\tau\in[T_0,T_0+\Delta T]} \left(\left| \frac{E_j(a,\tau) - \tilde{E}_j(a,\tau)}{E_j(a,\tau)} \right| + \left| \frac{F_j(a,\tau)}{d_j(\tau)} \right| \right)$$

Trans. Loss Rate

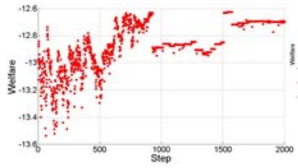
Smooth Effect

$$\bar{E}_j(a,\tau) = \frac{\tilde{E}_j(a,\tau)}{\sum_{\tau'\in[T_0,T_0+\Delta T]} \tilde{E}_j(a,\tau')}, \ \tau \in [T_0, T_0+\Delta T]$$

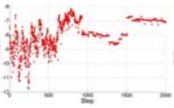
Game 1st model (not DWG) $U_i(a) = W(a) = \phi(a), \ \forall i \in \mathcal{V}$
Weight $w_1 =$ 0.1 $w_2 =$ 5000 $M =$ 10000 $\underline{\omega} =$ 0.0001
$k =$ 1e-5 (miss setting $F_s = 1/60$)

Simulation 1: Result

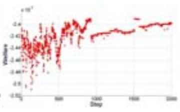
Welfare



Welfare 1 (Loss)



Welfare 2 (PSD)



PSD $9 \in \mathcal{R}$
[dB]

Time/Demand Response $9 \in \mathcal{R}$

Connection
↓
Movie

Some resources are worse
The reason: Gap between Demand and Available Supply is too large

Simulation 2: Setting

Situation Distributed Welfare Game
No Region Limitation

Data/Cycle 60 data (cycle) Time/Step Δt 60 times

Function
Transmission Loss

$$v_j^1(a) = -\sum_{\tau\in[T_0,T_0+\Delta T]} \left(\left| \frac{E_j(a,\tau) - \tilde{E}_j(a,\tau)}{E_j(a,\tau)} \right| + \left| \frac{F_j(a,\tau)}{d_j(\tau)} \right| \right)$$

Trans. Loss Rate Penalty Rate

Smooth Effect

$$\bar{E}'_j(a,\tau) = \frac{\tilde{E}_j(a,\tau) - d_j(\tau)}{d_j(\tau)} \quad \tau \in [T_0, T_0+\Delta T]$$

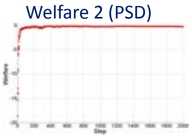
Game 2nd model (WLU, DWG)
Weight $w_1 =$ 0.1 $w_2 =$ 5000 $k =$ 1e-5 $\underline{\omega} =$ 0.005
(setting $F_s = 1$)

Simulation 2: Result

Welfare

Welfare 1 (Loss)

Welfare 2 (PSD)

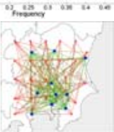


PSD $6 \in \mathcal{R}$
[dB]

Time/Demand Response $6 \in \mathcal{R}$

A few S/D gap exists

Connection
↓
Movie



Reference

[1] J. R. Marden, G. Arslan and J. S. Shamma, "Cooperative Control and Potential Games," *IEEE Transactions on Systems, Man and Cybernetics*, Vol. 39, No. 6, pp. 1393-1407, 2009.
[2] R. Gopalakrishnan, J. R. Marden and A. Wierman, "An Architectural View of Game Theoretic Control," *ACM SIGMETRICS Performance Evaluation Review*, Vol. 38, No. 3, pp. 31-36, 2011.
[3] J. R. Marden and A. Wierman, "Distributed Welfare Games," *Operations Research*, to appear, 2012.
[4] T. Goto, T. Hatanaka and M. Fujita, "Payoff-based Inhomogeneous Partially Irrational Play for Potential Game Theoretic Cooperative Control: Convergence Analysis," Proc. of the 2012 American Control Conference, pp. 2380-2387, 2012.
[5] J. R. Marden, G. Arslan and J. S. Shamma, "Joint strategy fictitious play with inertia for potential games," *IEEE Transactions on Automatic Control*, Vol. 54, No. 2, pp. 208-220, 2009.