Survey of Control for Groups of Robots and Evolutionary Dynamics of Behavior in Social Networks

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Introduction

Schedule Plan of My Research

4	5	6	7	8	9
	FL Seminar				Make Research Plan
	Survey	Survey	Survey		

Theme

- Vision-based Control
- Cooperative Control of Robotic Networks on SE(3)
- Social or Power Network with Evolutionary Dynamics

Outline

- Introduction
- Survey : Control for Groups of Robots[1-3]
- Survey : Evolutionary Dynamics of Behavior in Social Networks[4,5]

Background

Cooperative Control

multi agents achieve specified tasks or behaviors

ex. formation flight, search and rescue, mapping of unknown environment

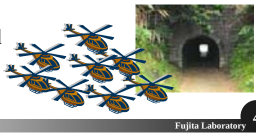
Advantages

- robustness to individual failures
- the possibility to cover wide regions

Situation[1,2,3]

required to move as a team from an initial to a final region

ex. moving many robots through a tunnel while staying grouped



➡ not exceed a certain value

Overview of [1-3]

Central Concept

develop a low-dimensional abstraction of the large teams of robots the position, orientation, and shape of the team

Advantages

- independent of the number of agents and the ordering of the robots
- the pose and shape control laws are decoupled

⟶ good scaling properties and robust to individual failures

Disadvantage

- requires a global observer

[1]C. Belta and V. Kumar, "Abstraction and control for groups of robots," *IEEE Trans. Robot.*, vol. 20, no. 5, pp. 865--875, Oct. 2004.
[2]N. Michael and V. Kumar, "Control of Ensembles of Aerial Robots," in *Proc. IEEE*, vol.99, No. 9, pp. 1587--1602, Sept. 2011.
[3]N. Michael, C. Belta, and V. Kumar, "Controlling three dimensional swarms of robots." in Proc. IEEE Int. Conf. Robot. Autom., Orlando, FL, May 2006, pp. 964--969.

Problem Settings

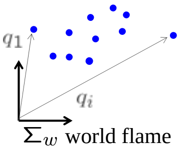
State Space Q

N-point Robots

$q = [q_1^T, \cdots, q_i^T, \cdots, q_N^T]^T \quad q_i \in Q_i = \mathcal{R}^2$

with respect to world frame

$Q = Q_1 \times Q_2 \times \cdots \times Q_N \in \mathcal{R}^{2N}$



2N-dimentional Control System

$\dot{q} = u \qquad u = [u_1^T, \cdots, u_i^T, \cdots, u_N^T]^T \in U = \mathcal{R}^{2N}$

Abstract Space (Swarm) A

$\phi : Q \to A \quad \phi(q) = a$

$p_i = [x_i, y_i]^T = R^T(q_i - \mu)$

Pose (Lie group)

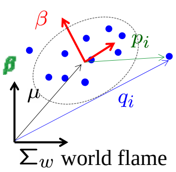
$g = (R, \mu) \in SE(2)$

Centroid of the Group

$\mu = \frac{1}{N}\sum_{i=1}^{N} q_i \in \mathcal{R}^2$

Orientation of the Flame β

$R \in SO(2) : \sum_{i=1}^{N} x_i y_i = 0$



Shape (semimajor and seminimor axes)

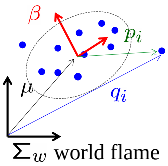
$s = (s_1, s_2) \quad s_1 = \frac{1}{N-1}\sum_{i=1}^{N} x_i^2, \; s_2 = \frac{1}{N-1}\sum_{i=1}^{N} y_i^2$

Abstract Space

State Space (N-point robots)

$\dot{q} = u \qquad q = [q_1, \cdots, q_i, \cdots, q_N] \in \mathcal{R}^{2N}$

$\phi : Q \to A$



Abstract Space(Swarm)

$a = (g, s)$

Pose (Lie group) $g = (R, \mu) \in SE(2)$

Shape $s = (s_1, s_2)$

Control System

$\dot{g} = g\hat{u}_g$

$\dot{s} = u_s$

$u_i = u_i(q_i, a)$

Decoupled: u_g does not affect shape s, u_s does not affect the group g

→ can control the group and the shape independently

Goal

$a = [g, s] \to a^{\mathrm{des}} = [g^{\mathrm{des}}, s^{\mathrm{des}}]$

Control Laws

$u_s = \dot{s}^{\mathrm{des}} + K_s(s^{\mathrm{des}} - s)$

$u_g = ((g^{\mathrm{des}})^{-1}\dot{g}^{\mathrm{des}})^{\vee} + K_g(\log(g^{-1}g^{\mathrm{des}}))^{\vee}$

K_s, K_g : 2×2 positive definite matrix

the errors in shape and pose goes exponentially to zero

Output

$q = [q_1, \cdots, q_i, \cdots, q_N] \longrightarrow a = (g, s)$

Main Results

Abstract Space(Swarm)

$\dot{g} = g\hat{u}_g \quad u_s, u_g$

$\dot{s} = u_s$

$u_s = \dot{s}^{\mathrm{des}} + K_s(s^{\mathrm{des}} - s)$

$u_g = ((g^{\mathrm{des}})^{-1}\dot{g}^{\mathrm{des}})^{\vee} + K_g(\log(g^{-1}g^{\mathrm{des}}))^{\vee}$

State Space (i-th point robot)

$\dot{q}_i = u_i \quad u_i$

Individual Control Law

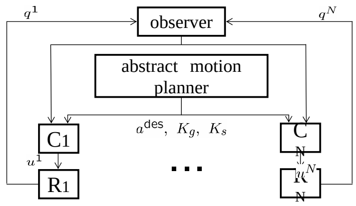
$$u_i = \dot{\mu} + \frac{s_1 - s_2}{s_1 + s_2} T(q_i - \mu)\omega + \sum_{k=1}^{2} \frac{\dot{s}_k}{2s_k} H_k(q_i - \mu)$$

Proposition 1

If a is bounded, then so are q_i.

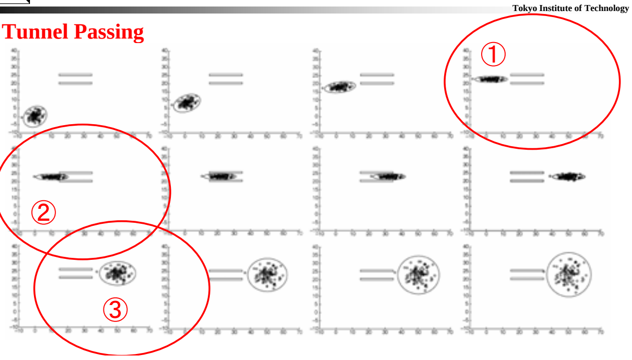
Proposition 2

For any a^{des}, the closed-loop system globally asymptotically converges to the equilibrium manifold $a = a^{des}$.



Simulation Results

Tunnel Passing



①gather the robots in front of the tunnel in such a shape that they can pass through

Outline

- Introduction
- Survey : Control for Groups of Robots[1-3]
- Survey : Evolutionary Dynamics of Behavior in Social Networks[4,5]
  - Evolutionary Dynamic Model[4]
  - Individual-based Evolutionary Dynamic Model[5]
  - Discussion

Example of Behavior Network

Main Objective[4,5]

developing and analyzing an evolutionary dynamics model for networked social behaviors

Example : Companies / Products

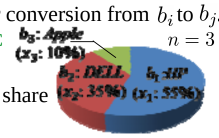
Situation ***n* companies try to increase their market share.**

behavior $b_i \to$ **a customer buying a product from company *i*.**

fraction $x_i \to$ **market share of company *i*.** $\sum_{i=1}^{n} x_i$

reward $a_{ij} \to$ reward gained by an agent for conversion from b_i to b_j.

ex) Companies: HP, DELL, Apple / Products: PC $\quad n = 3$



Dominant Behavior

a (few) company(s) having the largest market share

ex) 'iPod' by *Apple* and 'Walkman' by *sony*

How dominant social norms emerge?

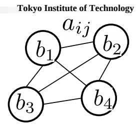
→ **shed light on "how to control the social interactions to achieve a desired dominant emergent behavior." [4]**

Evolutionary Dynamic Model

Notation of Behavior Network

weighted graph $\quad G = (V, E)$

$V = \{b_1, \cdots b_n\} \quad E = \{(b_i, b_j) : a_{ij} > 0\}$



b_i : behavior of a group of people in a social network

a_{ij} : interaction coefficient **(assumption : $a_{ii} = 1, \forall i$.)**

$w_{ij} = a_{ij} / (\sum_j a_{ij})$: normalized weight

evolutionary dynamics

x_i : fraction of the population with behavior b_i $\;(\sum_{i=1}^{n} x_i = 1)$

$f_i = \sum_j a_{ij} x_j$: fitness of behavior b_i

q_{ij} : mutation rate, rate of conversion from $b_i \quad b_j \quad (\sum_j q_{ij} = 1)$

Replicator Mutator Dynamics

$$\dot{x}_i = \sum_{j=1}^{n} x_j f_j q_{ji} - \phi x_i, \quad i = 1, \cdots, n$$

cf. no mutation

$$\dot{x}_i = x_i(f_i - \phi)$$

$\phi = \sum_i x_i f_i$: average fitness

Emergence of Social Behavior

$A = [a_{ij}]$: rewards matrix $Q = [q_{ij}]$: mutation matrix

$W = [w_{ij}]$: weight matrix $D = \mathrm{diag}(d_1, \cdots, d_n)$: degree matrix

L : graph Laplacian $L = I - D^{-1}A = I - W$

Social Choice Model[4]

$Q = 1 - \mu L = (1-\mu)I + \mu D^{-1}A$ $\mu \geq 0$: mutation parameter

$q_{ij} = \mu w_{ij},\ q_{ii} = 1 - \mu(1 - w_{ii})$

proportional to relative rewards w_{ij} and controlled by μ

Diversity

$n_e(x) = \frac{1}{\sum_i x_i^2} = 1/\|x\|^2$ the effective number of species $(1 \leq n_e \leq n)$

Three Phase of Evolution

behavioral flocking
all agents adopt to a single behavior, $n_e = 1$

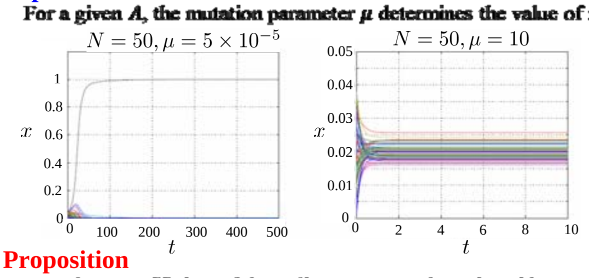
cohesion phase **few dominant behaviors emerge, $1 \leq n_e \ll n$**

collapse phase **has no dominant behaviors, $n_e \leq n$**

Main Results

Experimental Results

For a given A, the mutation parameter μ determines the value of n_e.



Proposition

Let $\bar{x}$ be an equilibrium of the replicator-mutator dynamics with $Q = (1-\mu)I + \mu W$. Then, the following statements hold:

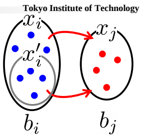
1) An absolutely dominant behavior results from $\mu = 0$.

2) For large behavior networks $n \gg 1$, a single relatively dominant behavior can only emerge from evolution with a relatively small μ.

Individual-based Evolutionary Dynamic Model

Assumption and Concerns of [4]

(1) society is homogeneous → single reward matrix A
The reward of switching from one behavior to another is the same across the population.



(2) mutation rate is homogeneous
Different members of society do have different mutation rate.

➡ focus on individual diversity in an individual-based evolutionary dynamic model.

Individual-based Evolutionary Dynamic Model

Notation

N individuals, n behaviors

x_j^k : **inclination of individual k toward b_j** *Cf.* x_j **: fraction with b_j**

a_{ij}^k : **subjective reward assigned by individual k to switch from b_i to b_j.**

$f^k = A^k x^k$: **how individual k perceives fitness of different behaviors.**

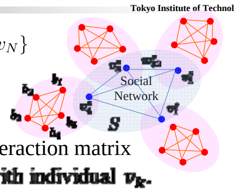
$Q^k = I - \mu^k L^k$: **mutation matrix associated with individual k.**

μ^k : how easily the individual tends to change its inclination

Effect of Social Interaction

Social Interaction Model

weighted graph $S = (V, E)$ $V = \{v_1, \cdots v_N\}$ $E = \{e_{ij}\}$



v_i^s : **individual i**

$A^s = [a_{ij}^s]$: interaction matrix

$W^s = [w_{ij}^s] = a_{ij}^s / (\sum_{j=1}^N a_{ij}^s)$: weighted interaction matrix

f_k^s : **Social fitness parameter associated with individual v_k.**

individual replicator-mutator dynamic model with the effect of social interaction

$$\dot{x}^k = (Q^k)^T F^k x^k - \phi^k x^k + \sum_{j=1}^n f_j^s w_{kj}^s x^j - \phi_k^s x^k$$

own valuation of different behaviors b_j

influence of all other individuals' inclinations x^j in the social network

Theorem

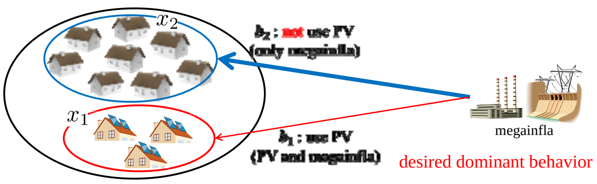
Let $x_\star^k$ be an equilibrium of the modified individual-based replicator-mutator social dynamic model, then an absolutely dominant equilibrium behavior results if and only if $x_\star^k = e_i$ for some i and $\mu^k = 0$ for all $k = 1, \ldots, n$.

Discussion

Sunaga's Scenario ➡ Evolutionary Dynamic Model[4]

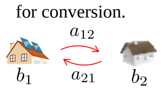
Objective

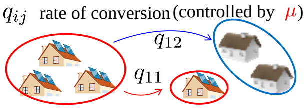
to achieve desired dominant emergent behavior



a_{ij} reward gained by an agent for conversion.

q_{ij} rate of conversion (controlled by μ)

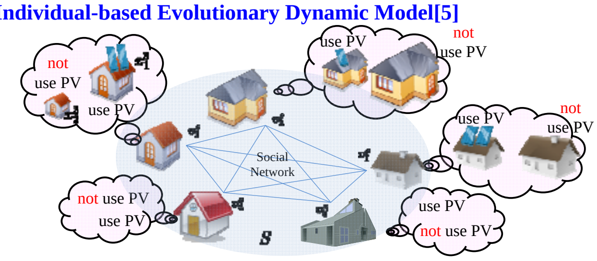




f_i fitness

Discussion

Individual-based Evolutionary Dynamic Model[5]



x_1^k **inclination of individual k toward using PV**

x_2^k **inclination of individual k toward not using PV**

We can analyze how to set each parameter to make all houses using PV.

References

[1]C. Belta and V. Kumar, “Abstraction and control for groups of robots,” *IEEE Trans. Robot*., vol. 20, no. 5, pp. 865--875, Oct. 2004.

[2]N. Michael and V. Kumar, “Control of Ensembles of Aerial Robots,” in *Proc. IEEE*, vol.99, No. 9, pp. 1587--1602, Sept. 2011.

[3]N. Michael, C. Belta, and V. Kumar, “Controlling three dimensional swarms of robots,” in *Proc*. IEEE Int. Conf. Robot. Autom., Orlando, FL, May 2006, pp. 964-969.

[4]R. Olfati-Saber, “Evolutionary Dynamics of Behavior in Social Networks,” in *Proc. of the 46th IEEE Conference on Decision and Control*, Dec. 2007, pp. 4051-4056.

[5]Islam I. Hussein, “An Individual-Based Evolutionary Dynamics Model for Networked Social Behaviors,” *Proc. of the* 2012 *American Control Conference*, 2012.

Appendix

Controlling three dimensional swarms of robots

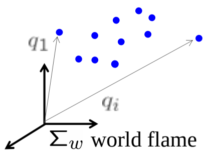
State Space Q

N-point Robots

$q = [q_1^T, \cdots, q_i^T, \cdots, q_N^T]^T \quad q_i \in Q_i = \mathcal{R}^3$

with respect to world frame

$Q = Q_1 \times Q_2 \times \cdots \times Q_N \in \mathcal{R}^{3N}$



3N-dimentional Control System

$\dot{q} = u \qquad u = [u_1^T, \cdots, u_i^T, \cdots, u_N^T]^T \in U = \mathcal{R}^{3N}$

Abstract Space (Swarm) A

$\phi : Q \to A \quad \phi(q) = a$

$p_i = [x_i, y_i, z_i]^T = R^T(q_i - \mu)$

Pose (Lie group)

$g = (R, \mu) \in SE(3)$

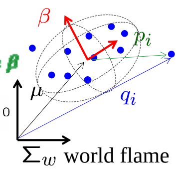
Centroid of the Group

$\mu = \frac{1}{N}\sum_{i=1}^{N} q_i \in \mathcal{R}^3$

Orientation of the Flame β

$R \in SO(3)$

$: \sum_{i=1}^{N} x_i y_i = \sum_{i=1}^{N} y_i z_i = \sum_{i=1}^{N} z_i x_i = 0$



Shape

$s = (s_1, s_2, s_3) \quad s_1 = \frac{1}{N-1}\sum_{i=1}^{N} x_i^2, \; s_2 = \frac{1}{N-1}\sum_{i=1}^{N} y_i^2$

Proof[1]

Proposition 1

If α is bounded, then so are q_i.

Proof :

Assumption

$\|\mu - \mu^d\| \le M_\mu \quad |s_1 - s_1^d| \le M_{s_1} \quad |s_2 - s_2^d| \le M_{s_2}$

$$s_1 + s_2 = \frac{1}{N-1}\sum_{i=1}^{N}(q_i - \mu)^T(q_i - \mu)$$

From $|s_1 - s_1^d| \le M_{s_1}, \quad |s_2 - s_2^d| \le M_{s_2}$

$$\|q_i - \mu^d\| \le \sqrt{N(s_1 + s_2)}$$
$$\le \sqrt{(N-1)(M_{s_1} + M_{s_2}) + s_1^d + s_2^d}$$

Using $\|\mu - \mu^d\| \le M_\mu$

$$\|q_i - \mu^d\| \le \|q_i - \mu\| + \|\mu - \mu^d\|$$
$$\le \sqrt{(N-1)(M_{s_1} + M_{s_2} + s_1^d + s_2^d)} + M_\mu$$

Proof[1]

Proposition 2

For any α^{des},
the closed-loop system globally asymptotically converges to the equilibrium manifold $\alpha = \alpha^{des}$.

Lyapunov function

$V(q) = \frac{1}{2}\|\mu^d - \mu\|^2 + \frac{1}{2}(\theta^d - \theta)^2 + \frac{1}{2}(s_1^d - s_1)^2 + \frac{1}{2}(s_2^d - s_2)^2$

$\dot{V}(q) = -K_\mu\|\mu^d - \mu\|^2 - k_\theta(\theta^d - \theta)^2 - k_{s_1}(s_1^d - s_1)^2 - k_{s_2}(s_2^d - s_2)^2$

$\therefore \dot{V}(q) \le 0$

According to the global invariant set theorem(LaSalle),
we only have to prove that $V(q) \to \infty$ as $\| q \| \to \infty$

We prove this by contradiction.

Perron Matrix

Properties of Perron Matrix ($Q = I - \mu L$)

Let G be a digraph with n nodes and maximum degree $\Delta = max_i(\sum_{j \neq i} a_{ij})$. $\qquad \epsilon \in (0, \frac{1}{\Delta}]$

(1)a row stochastic nonnegative matrix with a trivial eigenvalues of 1

(2)all eigenvalues are in a unit circle

(3)If G is a balanced graph, then Q is a doubly stochastic matrix

(4)If G is strongly connected and $0 < \epsilon < 1/\Delta$, then Q is a primitive matrix

R. Olfati-Saber, J. A. Fax, and R. M. Murray,
“Consensus and cooperation in networked multi-agent systems,”
Proc. of the IEEE, 95, Jan. 2007.

Diversity

Properties

$n_e(x) = \frac{1}{\sum_i x_i^2} = 1/\|x\|^2$ **varies between 1 and n**

Minimum diversity

a single species has a frequency of $x_{i\star} = 1$

other species are extinct, $x_j = 0, \forall j \neq i\star$

$\longrightarrow$ $n_e = 1$ $b_{i\star}$: dominant behavior

Maximum diversity

all x_i are equal, then $x_i = 1/n, \forall i,$ and $n_e = n$

Balance Condition Lemma[4]

Lemma

Define the following set of functions

$$\eta_i(x) = \frac{(\phi - f_i)x_i}{x_i f_i (w_{ii} - 1) + \sum_{j \neq i} x_i f_i w_{ij}}$$

Let $\bar{x}$ be an equilibrium of the evolutionary system.

Then,

$$\mu = \eta_1(\bar{x}) = \eta_1(\bar{x}) = \cdots = \eta_n(\bar{x})$$

Proof[4]

Proposition

Let $\bar{x}$ be an equilibrium of the replicator-mutator dynamics with $Q = (1-\mu)I + \mu W$. Then, the following statements hold:

1) An absolutely dominant behavior results from $\mu = 0$.

2) For large behavior networks $n \gg 1$, a single relatively dominant behavior can only emerge from evolution with a relatively small μ.

Proof :

(1)If $\bar{x}$ has a element $\bar{x}_i = 1$ and $\bar{x}_j = 0, \forall j \neq i$ corresponding to behavioral flocking, then $\mu = 0$.

(2)Ignore the terms of $O(\epsilon^2)$ and $O(\epsilon/n)$ as $n \gg 1$.

$f_i \cong 1 - \epsilon \qquad \therefore \phi \cong 1 - 2\epsilon$

From the balance condition lemma, one concludes that

$$\mu = g_i(\bar{x}) \cong \frac{\epsilon}{(1-w_{ij})(1-\epsilon)} \quad \text{or} \quad \mu = O(\epsilon)$$

thus $\mu \ll 1$