Tokyo Institute of Technology

Visual Feedback Attitude Synchronization Integrating a Velocity Observer

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Introduction: Background

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Social Problem w.r.t. Energy Generation

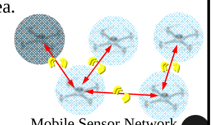
- Environmental Issues
  Nuclear: Air/Oceanic pollution caused by radioactive contamination (3.11)
  Thermal: Global warming, ecosystem change caused by CO_2, NO_X, SO_X
- Search and Rescue in the Time of Disaster
- Surveillance at Hazard Area

Nuclear Plant
Thermal Plant
Air Pollution
Oceanic Pollution

Measurement of such a damage level is an urgent need.

However, such pollution spreads through a wide area.

Mobile Sensor Network
Consisting of Networked Multiple Mobile Sensors
Network: good performance, robustness against failures
Mobile: adaptation to environment changes


Mobile Sensor Network

Introduction: Research Objective

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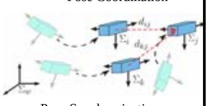
Control of MSN ➡ Pose Coordination

Cooperative Control of Robotic Networks
A distributed control strategy using local information so that the aggregate system achieves specified tasks or behaviors

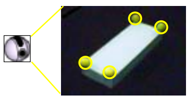
Pose (Position and Attitude) Synchronization
To lead all rigid bodies' poses to a common/desired value by distributed control strategies

Pose Coordination

Pose Synchronization

Information Flow for Cooperative Control
Most of works do NOT consider how to get necessary information to implement control laws

It is desired for agents to be distributed and use only relative sensing devices

➡ we use only vision


Visual Information

Today's Outline

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- Introduction
- Previous Works and Schedule Plan
- Problem Settings
- Main Results
  - Visual Feedback Attitude Synch. Integrating a Velocity Observer
  - Visual Feedback Pose Synch. Integrating a Velocity Observer
- Conclusions and Future Works

Previous Works

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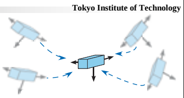
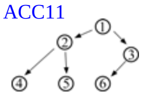
Pose Synchronization: CDC12
- Pose synchronization under general fixed digraphs
- Based on only relative information
- NOT consider how to obtain necessary information
- NO collision avoidance algorithm

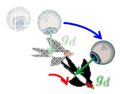


Visual Feedback Pose Synchronization: CDC11, 10, 09, ACC11
- Based on only visual information
- Leader-following visibility topologies
- Synchronization for the static leader
- Unnatural gain conditions



Visual Feedback Pose Control Integrating a Velocity Observer: CDC12
- Integrating a velocity observer for the target object
- NOT consider multiple rigid bodies case

Today's Challenge



Other Challenges
- Relative information-based flocking algorithm
- VFPS under a wider class of visibility structure
- Other configuration problems such as coverage or pursuit-evasion control

Schedule Plan

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4	5	6	7	8	9	10	11	12	1	2	3

○FL Seminar ○FL Seminar ○FL Seminar ○FL Seminar
○ACC Submission
CDC Submission○
○CDC Final Submission

Study of Flocking Algorithm, Wider Class of Visibility Structure, Pursuit Evasion ➡ Study of ･･･ Visibility Maintenance

Learning of Control/Management of Renewable Energies, Game Theory Approach

○D. Submission

Writing Doctor Thesis ○D. 1st ○D. Final

Present Plan of Doctor Thesis Outline
"Passivity-based Visual Feedback Pose Synchronization (Flocking Algorithm)"
- Introduction
- Problem Settings
- Passivity-based Pose Synchronization (Flocking Algorithm)
- Passivity-based Visual Feedback Pose Synchronization
- Passivity-based VFPS Integrating a Velocity Observer
- VFPS under a Wider Class of Visibility Structure (Flocking Algorithm)
- Conclusions

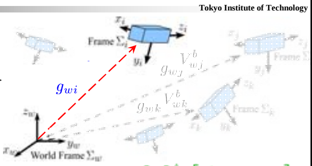
Visual Robotic Network: Rigid Body Motion

Kinematics of Rigid Bodies

Pose $(p_{wi}, e^{\hat{\xi}\theta_{wi}}) \in SE(3),\ i \in \mathcal{V}$

Rigid Body Set $\mathcal{V} := \{1, \cdots, n\}$

Exponential Coordinate for Rotation
- $\xi_{wi} \in \mathcal{R}^3$: rotation axis
- $\theta_{wi} \in \mathcal{R}$: rotation angle



Homogeneous Representation

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{4\times4}$$

$\wedge$: $\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$

$\vee$: Inverse Operator of $\wedge$

Body Velocity

$V^b_{wi} = \begin{bmatrix} v^b_{wi} \\ \omega^b_{wi} \end{bmatrix} \in \mathcal{R}^6$

$\hat{V}^b_{wi} := g^{-1}_{wi}\dot{g}_{wi} = \begin{bmatrix} \hat{\omega}^b_{wi} & v^b_{wi} \\ 0 & 0 \end{bmatrix} \in \mathcal{R}^{4\times4}$

$v^b_{wi} \in \mathcal{R}^3$: linear velocity

$\omega^b_{wi} \in \mathcal{R}^3$: angular velocity

Rigid Body Motion

$$\boxed{\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi}}$$

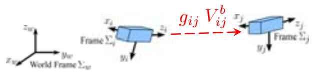
Rigid Body Motion

$V^b_{wi} \rightarrow \boxed{\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi}} \rightarrow g_{wi} = (p_{wi}, e^{\hat{\xi}\theta_{wi}})$

Visual Measurement

Relative Pose

$g_{ij} := g^{-1}_{wi}g_{wj} = (p_{ij}, e^{\hat{\xi}\theta_{ij}}) \in SE(3)$

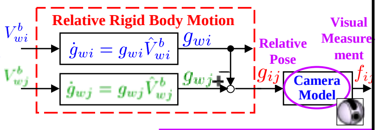


Body Velocity

$\hat{V}^b_{ij} := g^{-1}_{ij}\dot{g}_{ij}$

Relative Rigid Body Motion

$$\boxed{V^b_{ij} = -\mathrm{Ad}_{(g^{-1}_{ij})}V^b_{wi} + V^b_{wj}}$$



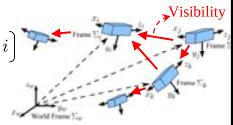
Visual Measurements f_{ij}

Visibility Structure among Rigid Bodies

Rigid Body Set $\mathcal{V} := \{1, \cdots, n\}$

Visibility Set (Edges in Graph Theory)

$\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ $\big((j,i) \in \mathcal{E}$: body j is visible from body $i\big)$



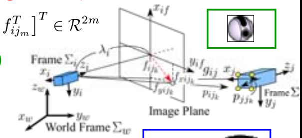
Visible Body Set (Neighbors in Graph Theory)

$$\boxed{\mathcal{N}_i := \{j \in \mathcal{V} \mid (j,i) \in \mathcal{E}\}}$$

Visual Measurement

Visual Measurement (Relative Sensing: Vision)

$\boxed{f_i = (f_{ij})_{j\in\mathcal{N}_i} \quad (8)}$ $f_{ij} := [f^T_{ij_1} \ \cdots \ f^T_{ij_m}]^T \in \mathcal{R}^{2m}$



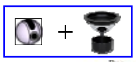
Pinhole Camera (Perspective Projection)

$$f_{ij_k} = \frac{\lambda_i}{z_{ij_k}}\begin{bmatrix} x_{ij_k} \\ y_{ij_k} \end{bmatrix} \in \mathcal{R}^2$$

$\lambda_i \in \mathcal{R}$: focal length

$k \in \{1, \cdots, m\}$

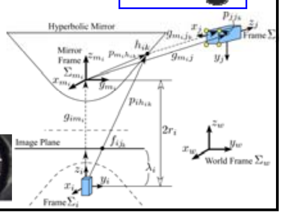
$p_{ij_k} = \begin{bmatrix} x_{ij_k} \\ y_{ij_k} \\ z_{ij_k} \end{bmatrix}$: position of k th feature point of body j relative to body i

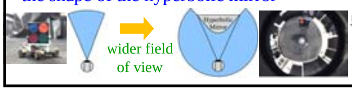


Generalized Camera (Panoramic Vision)

$$f_{ij_k} = \frac{\lambda_i c(p_{m_ij_k})}{2r_i + c(p_{m_ij_k})z_{m_ij_k}}\begin{bmatrix} x_{ij_k} \\ y_{ij_k} \end{bmatrix} \in \mathcal{R}^2$$

i.e. perspective projection scaled by the shape of the hyperbolic mirror





Visual Robotic Network

Visual Robotic Network $\boldsymbol{\Sigma}$

n Rigid Bodies

$$\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi}, \quad i \in \{1, \cdots, n\}$$

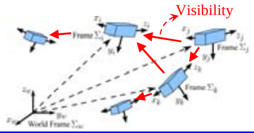
Input: V^b_{wi} (body velocity)

Visual Measurement

$$f_{ij_k} = \frac{\lambda_i c(p_{m_ij_k})}{2r_i + c(p_{m_ij_k})z_{m_ij_k}}\begin{bmatrix} x_{ij_k} \\ y_{ij_k} \end{bmatrix} \in \mathcal{R}^2$$

Visibility Structure

$$\mathcal{N}_i := \{j \in \mathcal{V} \mid (j,i) \in \mathcal{E}\}$$

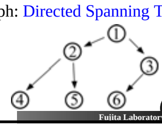


Assumption 1 (Leader-follower Type Visibility Structure)

- there exists a leader which has no visible body ($\mathcal{N}_1 = \emptyset$)
- the other bodies have a fixed visible body ($|\mathcal{N}_i| = 1$, and $\mathcal{N}_i$ is fixed $^\forall i \in \mathcal{V} \setminus \{1\}$)
- there exists a visibility path from each body to the leader ($^\forall i \in \mathcal{V} \setminus \{1\}$, $^\exists v_1, \cdots, v_r \in \mathcal{V}$ s.t. $v_1 = 1$, $v_r = i$ $(v_k, v_{k+1}) \in \mathcal{E}$ $^\forall k \in \{1, \cdots, r-1\}$)

➡ $G := (\mathcal{V}, \mathcal{E})$: Graph

- Graph: Directed Spanning Tree



Visual Feedback Attitude Synchronization

Definition 1: Visual Feedback Attitude Synchronization

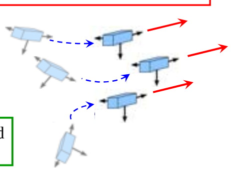
A visual robotic network $\sum$ is said to achieve visual feedback attitude synchronization, if each velocity input consists of only visual measurement (8) $(V^b_{wi}(f_i))$ and

$$\begin{cases} \lim_{t\to\infty}(v^b_{wi} - v^b_{wj}) = 0 \\ \lim_{t\to\infty}\phi(e^{\hat{\xi}\theta_{ij}}) = 0 \end{cases} \quad ^\forall i, j \in \mathcal{V} \qquad (1)$$

$$\phi(e^{\hat{\xi}\theta_{wi}}) := \frac{1}{2}\mathrm{tr}(I_3 - e^{\hat{\xi}\theta_{wi}}) \geq 0$$

$$\boxed{\phi(e^{\hat{\xi}\theta_{wi}}) = 0 \Leftrightarrow e^{\hat{\xi}\theta_{wi}} = I_3}$$

Eq. (1) ➡ $e^{\hat{\xi}\theta_{wi}} \rightarrow e^{\hat{\xi}\theta_*}$ $^\forall i \in \mathcal{V}$



Definition 1 can be readily extended to desired relative orientations $e^{\hat{\xi}\theta_{dij}}$

Assumption 2

- the leader (rigid body 1) moves with a constant body velocity $V = (v, 0)$ (can be readily extended to a finite Fourier series expansion)

Visual Feedback Attitude Synchronization Law

Control Error: $e^{\hat{\xi}\theta_{eij}} := e^{\hat{\bar{\xi}}\bar{\theta}_{ij}}$ Estimated Value $\quad (e^{\hat{\xi}\theta_{eij}} := e^{-\hat{\xi}\theta_{dij}}e^{\hat{\bar{\xi}}\bar{\theta}_{ij}})$

Visual Feedback Attitude Synchronization Law

$\bar{V}_i = (\bar{v}_i, 0)$: estimated body velocity

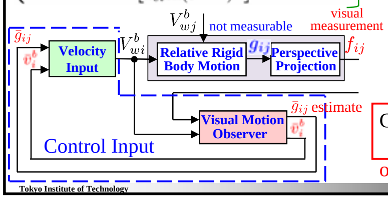
$$\left\{\begin{array}{l} v^b_{wi} = \bar{v}_i \\ \omega^b_{wi} = k_{ci}\mathrm{sk}(e^{\hat{\xi}\theta_{eij}})^\vee \\ \dot{\bar{v}}_i = u_{vvi} \\ u_{vvi} = k_{vi}p_{eij} \\ \bar{V}^b_{ij} := (\bar{g}^{-1}_{ij}\dot{\bar{g}}_{ij})^\vee = -\mathrm{Ad}_{(\bar{g}^{-1}_{ij})}V^b_{wi} + \bar{V}_i + u_{ei} \\ u_{ei} = k_{ei}e_{ij} - \begin{bmatrix} 0 \\ k_{ei}\mathrm{sk}(e^{\hat{\xi}\theta_{eij}})^\vee \end{bmatrix} \end{array}\right.$$

Velocity Input (first two equations); Velocity Observer (third and fourth); Pose Observer (last two)

$j \in \mathcal{N}_i,\ i \in \mathcal{V}$

$k_{ci}, k_{vi}, k_{ei} > 0$

$u_{vi} = (u_{vvi}, u_{vwi})$



Estimation Error:

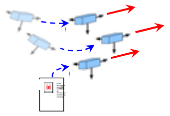
$g_{eij} := \bar{g}^{-1}_{ij}g_{ij}$

$e_{ij} := \begin{bmatrix} p_{eij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{eij}})^\vee \end{bmatrix}$

Control input (7) feedbacks only visual measurement

Visual Feedback Attitude Synchronization

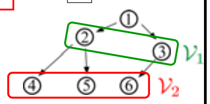
Theorem 1: Visual Feedback Attitude Synch.
Under Assumption 1 and 2, the present control law on the visual robotic network Σ achieves visual feedback attitude synchronization in the sense of (1).



Proof

Define subsets $\mathcal{V}_i \subset \mathcal{V},\ i \in \{1, \cdots, m\}$ as follows.

$$\mathcal{V}_1 := \{i \in \mathcal{V} \mid \mathcal{N}_i = 1\}$$
$$\mathcal{V}_i := \{i \in \mathcal{V} \mid \mathcal{N}_i \in \mathcal{V}_{i-1}\}$$



We first consider the rigid body group $\mathcal{V}_1$. Then, since each body in the present group sees only the leader (body 1) moving with V, all bodies achieve the same attitude and body velocity as the leader's one.
(The proof is omitted. refer to [1])

[1] T. Ibuki, T. Hatanaka and M. Fujita "Passivity-based Visual Pose Regulation for a Moving Target Object in Three Dimensions: Structure Design and Convergence Analysis", *Proc. of the 51st IEEE Conference on Decision and Control*, 2012 (submitted)

Visual Feedback Attitude Synchronization

We next consider the rigid body group $\mathcal{V}_2$. Then, each rigid body i in $\mathcal{V}_2$ sees a body in $\mathcal{V}_1$.
The orientation part of the total control/estimation error system of body i is formulated by

$$\begin{cases} \dot{\omega}_{wj}^b = k_{cj}\dot{e}_{cjk} \\ \omega_{cij}^b = -e^{\hat{\xi}\theta_{cij}}\omega_{wi}^b + u_{e\omega i} \\ \omega_{eij}^b = -e^{\hat{\xi}\theta_{eij}}u_{e\omega i} + \omega_{wj}^b \end{cases},\ k \in \mathcal{N}_j,\ j \in \mathcal{N}_i$$

$$e_{cij} := \mathrm{sk}(e^{\hat{\xi}\theta_{cij}})^\vee$$
$$e_{eij} := \mathrm{sk}(e^{\hat{\xi}\theta_{eij}})^\vee$$
$$u_{vi} = (u_{vvi}, u_{v\omega i})$$
$$u_{ei} = (u_{evi}, u_{e\omega i})$$

We define the following storage function

$$U_i := \phi(e^{\hat{\xi}\theta_{cij}}) + \phi(e^{\hat{\xi}\theta_{eij}})$$

Then, the time derivative of U_i satisfies

$$\begin{aligned} \dot{U}_i &= e_{cij}^T\omega_{cij}^b + e_{eij}^T\omega_{eij}^b \\ &= e_{cij}^T(-\omega_{wi}^b + u_{e\omega i}) + e_{eij}^T(-u_{e\omega i} + \omega_{wj}^b) \\ &= -[e_{cij}^T\ e_{eij}^T]^T \underbrace{\begin{bmatrix} (k_{ci}+k_{ei})I_3 & -k_{ei}I_3 \\ -k_{ei}I_3 & k_{ei}I_3 \end{bmatrix}}_{>0} \begin{bmatrix} e_{cij} \\ e_{eij} \end{bmatrix} + e_{eij}^T\underbrace{\omega_{wj}^b}_{\to 0} \end{aligned}$$

Visual Feedback Attitude Synchronization

$$\dot{U}_i = -[e_{cij}^T\ e_{eij}^T]^T \underbrace{\begin{bmatrix} (k_{ci}+k_{ei})I_3 & -k_{ei}I_3 \\ -k_{ei}I_3 & k_{ei}I_3 \end{bmatrix}}_{>0} \begin{bmatrix} e_{cij} \\ e_{eij} \end{bmatrix} + e_{eij}^T\underbrace{\omega_{wj}^b}_{\to 0}$$

We consider the case $\omega_{wj}^b = 0$. Since $e^{\hat{\xi}\theta_{cij}},\ e^{\hat{\xi}\theta_{eij}}$ are rotation matrices, there exists a positive scalar satisfying

$$\dot{U}_i = -e_{ceij}^T Q_i e_{ceij} \le -\alpha_i \|e_{ceij}\|^2 \qquad e_{ceij} := \begin{bmatrix} e_{cij} \\ e_{eij} \end{bmatrix},\ Q_i := \begin{bmatrix} (k_{ci}+k_{ei})I_3 & -k_{ei}I_3 \\ -k_{ei}I_3 & k_{ei}I_3 \end{bmatrix}$$

Since U_i monotonically decreases except for $e_{ceij} = 0$, there exists finite time $T > 0$ satisfying $U_i(T) < 1$. Then, $\phi(e^{\hat{\xi}\theta_{cij}}) < 1,\ \phi(e^{\hat{\xi}\theta_{eij}}) < 1$ hold, and thus there exist nonnegative scalars $\alpha_{mi},\ \alpha_{Mi}\ (\alpha_{mi} \le \alpha_{Mi})$ satisfying the following inequality [2].

$$\alpha_{mi}\|e_{ceij}(t)\|^2 \le U_i(t) \le \alpha_{Mi}\|e_{ceij}(t)\|^2\ {}^\forall t \ge T$$

Therefore, the following inequalities hold.

$$\frac{\dot{U}_i(t)}{U_i(t)} \le -\frac{\alpha_i\|e_{ceij}(t)\|^2}{\alpha_{Mi}\|e_{ceij}(t)\|^2}\ {}^\forall t \ge T \Rightarrow U_i(t) \le U_i(T)e^{-(\alpha_i/\alpha_{Mi})(t-T)}\ {}^\forall t \ge T$$

Namely, when $\omega_{wj}^b = 0$, the equilibrium point $e_{ceij} = 0$ of the orientation part of the total control/estimation error system is exponentially stable after time T.

[2] F. Bullo and R. Murray, "Tracking for Fully Actuated Mechanical Systems: a Geometric Framework," *Automatica*, Vol. 35, No. 1, pp. 17–34, 1999.

Visual Feedback Attitude Synchronization

We next write down control and estimation error systems in matrix representation;

$$\begin{cases} \dot{e}^{\hat{\xi}\theta_{cij}} = -k_{ci}\mathrm{sk}(e^{\hat{\xi}\theta_{cij}})e^{\hat{\xi}\theta_{cij}} + k_{ei}e^{\hat{\xi}\theta_{cij}}(\mathrm{sk}(e^{\hat{\xi}\theta_{eij}}) - \mathrm{sk}(e^{\hat{\xi}\theta_{cij}})) \\ \dot{e}^{\hat{\xi}\theta_{eij}} = -k_{ei}e^{\hat{\xi}\theta_{cij}}(\mathrm{sk}(e^{\hat{\xi}\theta_{eij}}) - \mathrm{sk}(e^{\hat{\xi}\theta_{cij}})) + e^{\hat{\xi}\theta_{eij}}\underline{\hat{\omega}_{wj}^b} \end{cases}$$

Since $\mathrm{sk}(\cdot)^\vee : \mathcal{R}^{3\times 3} \to \mathcal{R}^3$ is bijective for $e^{\hat{\xi}\theta},\ \theta \in (-\pi, \pi)$, the error systems can be written by

$$\begin{cases} \dot{e}_{cij} = f(e_{cij}, e_{eij}) \\ \dot{e}_{eij} = g(e_{eij}) + \mathrm{sk}(e^{\hat{\xi}\theta_{eij}}\underline{\hat{\omega}_{wj}^b})^\vee \end{cases}$$

Since the equilibrium point $e_{ceij} = 0$ of the nominal system ($\omega_{wj}^b = 0$) is exponentially stable and $\lim_{t\to\infty} \omega_{wj}^b = 0$ holds, we can conclude $\lim_{t\to\infty} e_{ceij} = 0$ from [3, lemma 9.6].

We finally consider the position part of the total control/estimation error system.

$$\begin{cases} \dot{v}_{wj}^b = k_{cj}\dot{p}_{ejk} \\ \dot{v}_{ei} = \dot{v}_{wj}^b - u_{vvi} \\ \dot{p}_{eij} = e^{\hat{\xi}\theta_{eij}}v_{eij}^b \end{cases},\ k \in \mathcal{N}_j,\ j \in \mathcal{N}_i \qquad v_{ei} := v_{wj}^b - \bar{v}_i$$

[3] H. K. Khalil, *Nonlinear Systems, Third Edition*, Prentice Hall, 2002.

Visual Feedback Attitude Synchronization

We substitute the present control law as follows.

$$\dot{v}_{ei} = -k_{vi}p_{eij} + k_{cj}\dot{p}_{ejk}$$
$$\begin{aligned} \dot{p}_{eij} &= -u_{evi} \underline{- \bar{v}_i} + \hat{p}_{eij}u_{e\omega i} - e^{\hat{\xi}\theta_{eij}}v_{wj}^b \\ &= -k_{ei}p_{eij} + \underline{v_{ei} - v_{wj}^b} + k_{ei}\hat{p}_{eij}(e_{eij} - e_{cij}) - e^{\hat{\xi}\theta_{eij}}v_{wj}^b \end{aligned}$$

Then, we obtain

$$\begin{bmatrix} \dot{v}_{ei} \\ \dot{p}_{eij} \end{bmatrix} = \underbrace{\begin{bmatrix} 0_3 & -k_{vi}I_3 \\ I_3 & -k_{ei}I_3 \end{bmatrix}}_{\text{Hurwits}} \begin{bmatrix} v_{ei} \\ p_{eij} \end{bmatrix} + \begin{bmatrix} k_{cj}\dot{p}_{ejk} \to 0 \\ \underbrace{(e^{\hat{\xi}\theta_{eij}} - I_3)}_{\to 0}v_{wj}^b + k_{ei}(\mathrm{sk}(\underbrace{I_3 - e^{\hat{\xi}\theta_{cij}}}_{\to 0}) - \mathrm{sk}(\underbrace{I_3 - e^{\hat{\xi}\theta_{eij}}}_{\to 0}))p_{eij} \end{bmatrix}$$

Thus, similarly to the orientation part, we can conclude $\lim_{t\to\infty} (v_{ei}, p_{eij}) = 0$.

Namely, all bodies eventually achieve the same attitude and body velocity as the first group's one.
The convergence analysis for the other groups $\mathcal{V}_i,\ i = 3, 4, \cdots, m$ are the same. $\square$

Discussion

Assumption 2'

- the leader (rigid body 1) moves with a constant body velocity $V = (v, \omega)$

Visual Feedback Attitude Synchronization Law

$\bar{V}_i = (\bar{v}_i, \bar{\omega}_i)$: estimated body velocity

$$\left\{\begin{aligned} &v_{wi}^b = \bar{v}_i - \hat{\bar{\omega}}_i p_{eij} \\ &\omega_{wi}^b = k_{ci}\mathrm{sk}(e^{\hat{\xi}\theta_{cij}})^\vee + e^{\hat{\xi}\theta_{cij}}\bar{\omega}_i \end{aligned}\right\}\text{Velocity Input}$$
$$\left\{\begin{aligned} &\dot{\bar{V}}_i = u_{vi} \\ &u_{vi} = k_{vi}e_{ij} \end{aligned}\right\}\text{Velocity Observer}$$
$$\left\{\begin{aligned} &\bar{V}_{ij}^b := (\bar{g}_{ij}^{-1}\dot{\bar{g}}_{ij})^\vee = -\mathrm{Ad}_{(\bar{g}_{ij}^{-1})}V_{wi}^b + \bar{V}_i + u_{ei} \\ &u_{ei} = k_{ei}e_{ij} - \begin{bmatrix} \hat{\bar{\omega}}_i p_{eij} \\ k_{ei}\mathrm{sk}(e^{\hat{\xi}\theta_{cij}})^\vee \end{bmatrix} \end{aligned}\right\}\text{Pose Observer}$$

$$j \in \mathcal{N}_i,\ i \in \mathcal{V}$$

The orientation part of the total control/estimation error system is formulated by

$$\begin{cases} \dot{\omega}_{wj}^b = k_{cj}\dot{e}_{cjk} + (e^{\hat{\xi}\theta_{cjk}}\bar{\omega}_j)^{\cdot} \\ \dot{\omega}_{ei} = \dot{\omega}_{wj}^b - u_{v\omega i} \\ \omega_{cij}^b = -e^{\hat{\xi}\theta_{cij}}\omega_{wi}^b + \bar{\omega}_i + u_{e\omega i} \\ \omega_{eij}^b = -e^{\hat{\xi}\theta_{eij}}(u_{e\omega i} + \bar{\omega}_i) + \omega_{wj}^b \end{cases},\ k \in \mathcal{N}_j,\ j \in \mathcal{N}_i \qquad V_{ei} := V_{wj}^b - \bar{V}_i = (v_{ei}, \omega_{ei})$$

Discussion

We define the following storage function

$$U_i := \phi(e^{\hat{\xi}\theta_{cij}}) + \phi(e^{\hat{\xi}\theta_{eij}}) + \frac{1}{2k_{vi}}\|\omega_{ei}\|^2$$

Then, the time derivative of U_i satisfies

$$\begin{aligned}
\dot{U}_i &= e_{cij}^T\omega_{cij}^b + e_{eij}^T\omega_{eij}^b + \frac{1}{k_{vi}}\omega_{ei}^T\dot{\omega}_{ei} \\
&= e_{cij}^T(-\omega_{wi}^b + \bar{\omega}_i + u_{ewi}) + e_{eij}^T(-u_{ewi} \underbrace{- \bar{\omega}_i + \omega_{wj}^b}_{-\omega_{ei}}) + \frac{1}{k_{vi}}\omega_{ei}^T(\underline{\dot{\omega}_{wj}^b} \underbrace{- u_{vwi}}_{k_{vi}e_{eij}}) \\
&= \underbrace{-[e_{cij}^T \ e_{eij}^T]^T \begin{bmatrix} (k_{ci}+k_{ei})I_3 & -k_{ei}I_3 \\ -k_{ei}I_3 & k_{ei}I_3 \end{bmatrix} \begin{bmatrix} e_{cij} \\ e_{eij} \end{bmatrix}}_{\text{No information about } -\|\omega_{ei}\|^2} + \frac{1}{k_{vi}}\omega_{ei}^T\underbrace{(k_{cj}\dot{e}_{cjk} + (\dot{e^{\hat{\xi}\theta_{cjk}}}\bar{\omega}_j)}_{\to 0 \text{ if } V \text{ is constant}}
\end{aligned}$$

Namely, we have to take another approach for exponential stability of attitude synchronization.

➡ Future Work

Conclusion

Conclusions

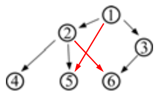
- Visual feedback pose synchronization under leader-following visibility structures
- Integrating a velocity observer
- Eliminating unnatural gain conditions

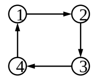
➡ ISCIE(JCMSI) or Tech. Report?

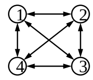
Next Challenges

- Acyclic Directed Spanning Tree
- Cyclic Digraph
- Completed Graph
- Study of Pursuit-evasion Problems

➡ Local Report







$$\omega_{wi}^b = k_{ci}\sum_{j\in\mathcal{N}_i} \mathrm{sk}(e^{\hat{\xi}\bar{\theta}_{ij}})^\vee + \underbrace{e^{\hat{\xi}\bar{\theta}_{ij}}}_{?}\bar{\omega}_i$$