Visual Feedback Attitude Synchronization Integrating a Velocity Observer

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Introduction: Research Objective

Control of MSN
Pose Coordination

Cooperative Control of Robotic Networks
A distributed control strategy using local information so that the aggregate system achieves specified tasks or behaviors

• Pose (Position and Attitude) Synchronization
To lead all rigid bodies’ poses to a common/desired value by distributed control

Information Flow for Cooperative Control
Most of works do NOT consider how to get necessary information to implement control laws

It is desired for agents to be distributed and use only relative sensing devices

Visual Feedback Pose Synchronization
Integrating a Velocity Observer

Problem Settings
• Pose synchronization under general fixed digraphs
  - Based on only relative information
  - NOT consider how to obtain necessary information
  - Network collision avoidance algorithm

Visual Feedback Pose Synchronization: CDC12, 10, 09, ACC11
• Based on only visual information
• Leader-following visibility topologies
• Synchronization for the static leader
• Unnatural gain conditions

Visual Feedback Pose Control Integrating a Velocity Observer: CDC12
• Integrating a velocity observer for the target object
• NOT consider multiple rigid bodies case

Present Plan of Doctor Thesis Outline

Today’s Outline

• Introduction
• Previous Works and Schedule Plan

• Problem Settings
• Main Results
  - Visual Feedback Attitude Synch. Integrating a Velocity Observer
  - Visual Feedback Pose Synch. Integrating a Velocity Observer

• Conclusions and Future Works

Schedule Plan

4 5 6 7 8 9 10 11 12 1 2 3
○FL Seminar ○FL Seminar ○FL Seminar ○ACC Submission○CDC Submission ○CDC Final Submission
Study of Flocking Algorithm, Wider Class of Visibility Structure, Pursuit Evasion
○Study of Visibility Maintenance
○Learning of Control Management of Renewable Energies, Game Theory Approach
○Writing Doctor Thesis ○D. 1st ○D. Final

FL Seminar 2011-5-2
Assumption 2

the leader (rigid body 1) moves with a constant body velocity $v_1 = (v_{1x}, v_{1y}, v_{1z})$

Kinematics of Rigid Bodies

Pose $\eta_i = (\mathbf{R}_i, \mathbf{t}_i), \ i \in V$ is a set of rigid bodies in SE(3).

Exponential Coordinate for Rotation

- Homogeneous Representation
  
  $g_{\alpha} = \begin{bmatrix} e^{\alpha_i} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \in \mathbb{R}^{4 \times 4}$

- Body Velocity
  
  $\dot{g}_{\alpha} = \begin{bmatrix} \dot{\alpha}_i & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \in \mathbb{R}^{4 \times 4}$

- Angular Velocity $\omega_i = [\omega_{ix}, \omega_{iy}, \omega_{iz}] \in \mathbb{R}^3$

- Linear Velocity $v_i = [v_{ix}, v_{iy}, v_{iz}] \in \mathbb{R}^3$

Visual Robotic Network Rigid Body Motion

Relative Pose

$
\eta_{ij} = \mathbf{R}_{ij} \mathbf{t}_{ij} \in SE(3)
$

Body Velocity

$\dot{\eta}_{ij} = \mathbf{R}_{ij} \mathbf{v}_{ij} \in \mathbb{R}^{4 \times 4}$

Relative Rigid Body Motion

$\dot{\eta}_{ij} = \mathbf{R}_{ij} \mathbf{v}_{ij} = \mathbf{R}_{ij} \mathbf{v}_{1j} - \mathbf{R}_{ij} \mathbf{v}_{1i}$

Visible Body Set (Neighbors in Graph Theory)

$\mathcal{N}_i := \{ j \in V \mid (i, j) \in E \}$

Visual Feedback Attitude Synchronization Law

Control Error $e_{\omega_{ij}} := \dot{\omega}_i - \dot{\omega}_j$ Estimated Value

$\hat{\omega}_i = \kappa_{\omega} \dot{\omega}(\hat{\eta}_{ij})$

Velocity Input $\dot{\omega}_i = \dot{\omega}_i$ Velocity Observer

Pose Observer $\dot{\eta}_{ij} = \mathbf{R}_{ij} \mathbf{v}_{ij} \in SE(3)$

Estimation Error $e_{\omega_{ij}} := \dot{\omega}_i - \dot{\omega}_j$

Relative Rigid Body Motion

$\hat{\eta}_{ij} = \mathbf{R}_{ij} \mathbf{v}_{ij} = \mathbf{R}_{ij} \mathbf{v}_{1j} - \mathbf{R}_{ij} \mathbf{v}_{1i}$

Visual Measurements $f_{ij}$

Graph: Directed Spanning Tree

- $\exists$ leader that has no visible body $\mathcal{N}_i = \emptyset$

- the other bodies have a fixed visible body $\mathcal{N}_i \neq \emptyset$ and $\mathcal{N}_i \in V\setminus \{1\}$

- there exists a visibility path from each body to the leader $\forall i \in V \setminus \{1\}$, $\mathcal{N}_i \cap \mathcal{N}_j = \emptyset, n_{1i} = 1, n_{1j} = i$

Assumption 1 (Leader-follower Type Visibility Structure)

Visibility Structure among Rigid Bodies

$\mathcal{N}_i := \{ j \in V \mid (i, j) \in E \}$

Visual Robotic Network

Pinhole Camera (Perspective Projection)

$f_{ij} = \frac{\lambda_i}{\lambda_j} \frac{x_{ij}}{x_{0j}} \in \mathbb{R}^2$

Position of the feature point $p_{ij} = \frac{\lambda_i}{\lambda_j} (x_{ij}, y_{ij})$ to body $i$

Generalized Camera (Panoramic Vision)

$f_{ij} = \frac{\lambda_i}{\lambda_j} \frac{x_{ij}}{x_{0j}} \in \mathbb{R}^2$

Perspective projection scaled by the shape of the hyperbolic mirror

$\mathcal{N}_i := \{ j \in V \mid (i, j) \in E \}$

Visual Measurement

Input: $V_i := \{ 1, \ldots, n \}$

Output: $\mathbf{t}_{ij}, \mathbf{R}_{ij} \in \mathbb{R}^{4 \times 4}$

Visual Feedback Attitude Synchronization

Definition 1: Visual Feedback Attitude Synchronization

A visual robotic network $\Sigma$ is said to achieve visual feedback attitude synchronization, if each velocity input consists of only visual measurement $\dot{V}_i$ and

$v_i = \dot{g}_{\alpha_i} \mathbf{v}_{ij} \in \mathbb{R}^3$

Eq. (1) $e_{\alpha_{ij}} = \dot{\alpha}_j - \dot{\alpha}_i = 0$

Definition 1 can be readily extended to desired relative orientation $\hat{\mathbf{R}}_{ij}$

Assumption 2

the leader (rigid body 1) moves with a constant body velocity $v_1 = (v_{1x}, v_{1y}, v_{1z})$

(1) it can be readily extended to a finite Fourier series expansion.
Theorem 1: Visual Feedback Attitude Synchronization

Under Assumption 1 and 2, the present control law on the visual robotic network $\mathcal{V}$ achieves visual feedback attitude synchronization in the sense of (1).

Proof

Define subsets $V_i \subset \mathcal{V}$, $i \in \{1, \ldots, n\}$ as follows.

$$V_i := \{j \in \mathcal{V} | x_i = j\}$$

We first consider the rigid body group $V_i$. Then, since each body in the present group sees only the leader (body 1) moving with $\omega^l$, all bodies achieve the same attitude and body velocity as the leader’s one.

(The proof is omitted. refer to [1])


Visual Feedback Attitude Synchronization

We next consider the rigid body group $V_i$. Then, each rigid body $i$ in $V_i$ sees a body in $\psi_i$.

The orientation part of the total control/estimation error system of body $i$ is formulated by

$$\dot{\theta}_{ei} = \omega_{ei} + \Delta \theta_{ei}$$

where $\Delta \theta_{ei} = \theta_{ei} - \dot{\theta}_{ei}$.

We define the following storage function

$$V := \frac{1}{2} \theta_{ei}^T \theta_{ei}$$

Then, the time derivative of $V$ satisfies

$$\dot{V} = \theta_{ei}^T \dot{\theta}_{ei} = \theta_{ei}^T (\omega_{ei} + \Delta \theta_{ei}) = \theta_{ei}^T \Delta \theta_{ei}$$

Since $\theta_{ei}$ is monotonically decreasing except for $\Delta \theta_{ei} = 0$, there exists finite time $T > 0$ satisfying $\dot{V} < 0$. Then, $\theta_{ei}^T \theta_{ei} < \theta_{ei}^T \theta_{ei}$ and $\theta_{ei}^T \theta_{ei}$ is hold, and thus there exist nonnegative scalars $\alpha_{ei}, \beta_{ei}, \gamma_{ei}$ satisfying the following inequality [2]:

$$\alpha_{ei} \theta_{ei}^T \theta_{ei} < \beta_{ei} \gamma_{ei}$$

Therefore, the following inequalities hold.

$$V(t) < \frac{\beta_{ei} \gamma_{ei}}{\alpha_{ei}}$$

Namely, when $\theta_{ei}^T \theta_{ei} = 0$, the equilibrium point $\theta_{ei} = 0$ is for the orientation part of the total control/estimation error system is exponentially stable after time $T$.

We next write down control and estimation error systems in matrix representation;

$$\dot{x}_{ei} = A x_{ei} + B u_{ei} + C e_{ei}$$

where

$$x_{ei} := \begin{bmatrix} \theta_{ei} \\ \omega_{ei} \end{bmatrix}, \quad u_{ei} := \begin{bmatrix} \omega_{ei} \\ \Delta \theta_{ei} \end{bmatrix}, \quad e_{ei} := \begin{bmatrix} \theta_{ei} - \dot{\theta}_{ei} \\ \omega_{ei} - \omega_{ei} \end{bmatrix}$$

Then, we obtain

$$\dot{x}_{ei} = \begin{bmatrix} \theta_{ei} \\ \omega_{ei} \end{bmatrix} \begin{bmatrix} \dot{\theta}_{ei} \\ \dot{\omega}_{ei} \end{bmatrix} = \begin{bmatrix} \theta_{ei} \\ \omega_{ei} \end{bmatrix} \begin{bmatrix} \dot{\theta}_{ei} \\ \dot{\omega}_{ei} \end{bmatrix}$$

Thus, similarly to the orientation part, we can conclude $\lim_{t \to \infty} e_{ei}(t) = 0$.

Namely, all bodies eventually achieve the same attitude and body velocity as the leader’s one.

Discussion

Assumption 2

• the leader (rigid body 1) moves with a constant body velocity $V = (v, \omega)$

Visual Feedback Attitude Synchronization Law

$$\dot{\theta}_{ei} = \dot{\theta}_{ei}$$

Velocity Input

$$\dot{\omega}_{ei} = k_1 (x_{ei} - \omega_{ei})$$

Velocity Observer

$$\dot{\theta}_{ei} = \theta_{ei}$$

Pose Observer

The orientation part of the total control/estimation error system is formulated by

$$\dot{e}_{ei} = \begin{bmatrix} \dot{\theta}_{ei} \\ \dot{\omega}_{ei} \end{bmatrix} = \begin{bmatrix} \theta_{ei} \\ \omega_{ei} \end{bmatrix}$$

where

$$e_{ei} := \begin{bmatrix} \theta_{ei} - \dot{\theta}_{ei} \\ \omega_{ei} - \omega_{ei} \end{bmatrix}$$
Discussion

We define the following storage function

\[ U_i = \frac{1}{2p_2} (e_i^T e_i + \theta_i^2) + \frac{1}{2k_i} \omega_i^2 \]

Then, the time derivative of \( U_i \) satisfies

\[ \dot{U}_i = e_i^T \dot{e}_i + \frac{1}{k_i} \omega_i \dot{\omega}_i + \frac{1}{k_i} \omega_i \theta_i \]

\[ = e_i^T (\omega_i \dot{e}_i + \dot{\omega}_i) + \theta_i \omega_i \dot{\theta}_i \]

\[ = -[e_i^T \dot{e}_i + \theta_i \dot{\theta}_i] \]

\[ = [e_i^T + \theta_i \dot{\theta}_i] \]

\[ = \frac{1}{k_i} \omega_i \dot{\omega}_i + \frac{1}{k_i} \omega_i \dot{\theta}_i \]

Namely, we have to take another approach for exponential stability of attitude synchronization.

Future Work

Conclusion

Conclusions

- Visual feedback pose synchronization under leader-following visibility structures
- Integrating a velocity observer
- Eliminating unnatural gain conditions

Next Challenges

- Acyclic Directed Spanning Tree
- Cyclic Digraph
- Completed Graph
- Study of Pursuit-evasion Problems

Local Report

\[ \omega_i = k_i \sum_{j \in N(i)} \theta_j \theta_j + \frac{2\theta_i \omega_i}{T} \]