Midterm Report on Passivity-based Event-driven Pose Synchronization on *SE*(3)

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Background

These days, Event-driven [-based, -triggered] Control gets much attention

12ACC
  9 related presentations

TUM Workshop on Oct. 2012
  "Event-based Control and Optimization"
  Prof. Fujita attended and talked

NecSys12
  5 related presentations

CDC12
  3 related sessions
  "Event-triggered and Self-triggered Control" (Tutorial Session)
  "Networked Event-based Control"
  "Event-based Control"

Many Recent Journal Articles

Background

Technical Issues on Implementation of Feedback Control Laws

Most of works assume ideal continuous-time feedback, i.e. do NOT consider

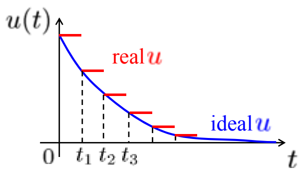
Time of
- computation of feedback control laws with embedded microprocessors
- sensor action: collecting and processing information
- actuating the controller updates
- digital communication (for in particular cooperative control)

It is important to assess to what extent we can increase the functionality of these embedded devices through novel real-time scheduling algorithms.

Event-driven Control [1-6]

The control signals are kept constant unti condition on certain signals triggers the r

Compared with *time-driven* control (i.e. s and fast sampling rate is applied to guara scenario, the possibility of reducing the n of transmissions, while guaranteeing desi event driven control very appealing.



Background and Research Objective

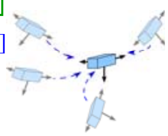
Technical Issues on Implementation of *Cooperative* Control Laws [7-12]

Although each agent actually acts in an asynchronous manners, most of works assume a synchronous implementation strategy regarding the control action updates and the scheduling of data transmissions among the coupled agents.

⇨ Event-driven Cooperative Control

It is favorable that trigger conditions is defined by local information of the same neighbor agents as those of laws.

Previous Works: Attitude/Pose Synchronization [13-15]



[13,14] consider communication delay, but does NOT consider other delay elements

Research Objective

To propose a event-driven pose synchronization law, conduct convergence analysis and clarify the remaining issues to be solved

Today's Outline

- Background
- Simple Introduction to Event-driven Control
- Introduction to Event-driven Cooperative Control
  - Event-driven Consensus Problem under Bidirectional Graphs
- Passivity-based Event-driven Pose Synchronization
  - Passivity-based Event-driven Position Synchronization under Strongly Connected Digraphs
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- Conclusions and Next Challenges

Introduction to Event-driven Control

The simplest example (based on [2,3])

Dynamics | State Feedback Law | ISS Lyapunov Function

$$\begin{cases}\dot{x} = u \\ y = x\end{cases} \quad x, u, y \in \mathcal{R} \qquad u = -ky,\ k > 0 \qquad V(x) = \frac{1}{2}x^2 \ \Rightarrow\ \lim_{t\to\infty} x(t) = 0$$

The implementation of the state feedback law on an embedded processor is typically done by sampling the state at time instants $t_0, t_1, t_2, \cdots$, computing $u(t_i) = k(x(t_i))$ and updating the actuator values at time instants $t_0 + \Delta, t_1 + \Delta, t_2 + \Delta, \cdots$, where $\Delta \geq 0$ represents the time required to read the state from the sensors, compute the control law and update the actuators.

In this talk, we assume Δ is negligible, i.e. $\Delta << t_{i+1} - t_i$

Therefore, the actual state feedback law becomes

$$u(t) = -ky(t_i),\ t \in [t_i, t_{i+1}). \qquad \text{(constant for } [t_i, t_{i+1}))$$

Measurement Error

$$e(t) := y(t_i) - y(t),\ t \in [t_i, t_{i+1}) \ \Rightarrow\ y(t_i) = y(t) + e(t)$$

Closed Loop System

$$\dot{x} = -ky(t_i) = -ky(t) - ke(t) = -kx - ke$$

Introduction to Event-driven Control

Convergence Analysis

ISS Lyapunov Function

$V(x) = \frac{1}{2}x^2$ ⇨ $\dot{V} = x\dot{x} = -kx^2 - kxe \le -\frac{3}{4}k|x|^2 + k|e|^2$

V is NOT continuously differentiable

If $|e|^2 \le \frac{3\sigma}{4}|x|^2,\ \sigma \in (0,1)$ (*) holds, then we get

$\dot{V} \le -\frac{3}{4}k(1-\sigma)|x|^2 \le 0.$

Therefore, we set the following trigger condition

$|e| > \frac{\sqrt{3\sigma}}{2}|x|.$ (**)

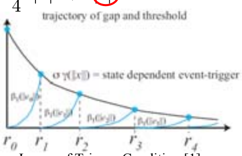


Image of Trigger Condition [1]

If (**) is satisfied at time t_i, then $u(t)$ is updated to $u(t) = -ky(t_i)$. Namely, $e(t_i) = y(t_i) - y(t_i) = 0$ holds, and thus (*) is automatically satisfied.

[3] claims that the simple execution rule (**) guarantees global asymptotic stability by construction. ⇨ ?? (Future Work)

Question on Feasibility of the Scheduling Policy

Is there undesired behavior like Zeno bahavior [16]?

Introduction to Event-driven Control

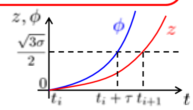
Theorem ([3, Theorem III.1]

There exists time $\tau > 0$ such that the inter-execution time $\{t_i - t_{i-1}\}_{i\in\mathcal{N}}$ implicitly defined by the execution rule (**) is lower bounded by

$t_i - t_{i-1} \ge \tau\ {}^\forall i \in \mathcal{N}.$

Proof:

Since $|\dot{x}| = |-kx - ke| \le k|x| + k|e|$ holds, we get



$$\frac{d}{dt}\frac{|e|}{|x|} = \frac{e\dot{e}}{|e||x|} - \frac{x\dot{x}|e|}{|x|^3} \le \frac{|e||\dot{x}|}{|e||x|} + \frac{|x||\dot{x}||e|}{|x|^3}$$

$$= \left(1 + \frac{|e|}{|x|}\right)\frac{|\dot{x}|}{|x|} \le \left(1 + \frac{|e|}{|x|}\right)\frac{k|x| + k|e|}{|x|} = k\left(1 + \frac{|e|}{|x|}\right)^2 \text{ for } t \in [t_i, t_{i+1}).$$

Thus, if we define $z := |e|/|x|$, we have $\dot{z} \le k(1+z)^2\ (z(t_i) = 0)$.

Now, find $\phi(t)$ s.t. $\dot{\phi} = k(1+\phi)^2, \phi(t_i) = 0$ satisfying $z(t) \le \phi(t)$.

Then, we obtain $\phi(t) = \frac{k(t-t_i)}{1 - k(t-t_i)},\ t \ge t_i$.

Since $\phi(t_i + \tau) = \frac{\sqrt{3\sigma}}{2}$, we get $\tau = \frac{\sqrt{3\sigma}}{k(2 + \sqrt{3\sigma})}$.

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Introduction to Event-driven Cooperative Control

The simplest example (based on [4])

Simple: Consensus, Bidirectional connected graphs, Synchronous approach

Dynamics: $\dot{x}_i = u_i,\quad x_i, u_i, \in \mathcal{R}\quad i \in \{1,\cdots,n\}$

Control Law: $u_i = \sum_{j\in\mathcal{N}_i}(x_j - x_i)$

Lyapunov Function Candidate: $V(x) = \frac{1}{2}x^T L x$

⇨ $\lim_{t\to\infty}|x_i(t) - x_j(t)| = 0\ {}^\forall i,j \in \{1,\cdots,n\}$

Event-driven Consensus Law

$u_i(t) = \sum_{j\in\mathcal{N}_i}(x_j(t_i) - x_i(t_i)),\ t \in [t_i, t_{i+1})$

(constant for $[t_i, t_{i+1})$)

Goal

$\lim_{t\to\infty}|x_i(t) - x_j(t)| = 0\ {}^\forall i,j$

Measurement Error

$e_i(t) := x_i(t_i) - x_i(t),\ t \in [t_i, t_{i+1})$ ⇨ $x_i(t_i) = x_i(t) + e_i(t)$

Closed Loop System

$\dot{x} = -Lx(t_i)$

$= -L(x + e)$

$L \in \mathcal{R}^{n\times n}$: graph Laplacian

$x \in \mathcal{R}^n$: stacked state vector

$e \in \mathcal{R}^n$: stacked error vector

Introduction to Event-driven Cooperative Control

Convergence Analysis

$V_c(x) = \frac{1}{2}x^T L x$ ⇨ $\dot{V}_c = x^T L\dot{x} = -||Lx||^2 - x^T L^T L e \le -||Lx||^2 + ||Lx||||L||||e||$

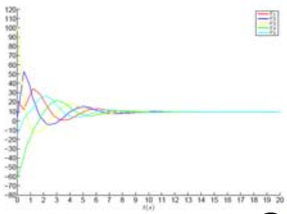
If $||e|| \le \sigma\frac{||Lx||}{||L||},\ \sigma \in (0,1)$ (*) holds, then we get $\dot{V}_c \le -(1-\sigma)||Lx||^2 \le 0.$

Therefore, we set the following trigger condition

$||e|| > \sigma\frac{||Lx||}{||L||}.$ (**)

V_c is NOT continuously differentiable

[4] claims that the simple execution rule (**) guarantees average consensus by using LaSalle's Invariance Principle of hybrid systems [16]. ⇨ ?? (Future Work)



Average Consensus

$\lim_{t\to\infty} x_i(t) = \frac{1}{n}\sum_{i=1}^{n} x_i(0)\ {}^\forall i$

Introduction to Event-driven Cooperative Control

Theorem ([4, Theorem 1])

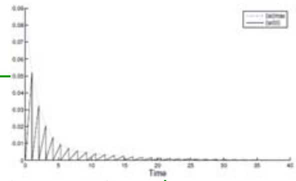
The inter-event time $\{t_i - t_{i-1}\}_{i\in\mathcal{N}}$ implicitly defined by the rule (**) is lower bounded by a strictly positive time τ.

Proof:

$$\frac{d}{dt}\frac{||e||}{||Lx||} = -\frac{e^T\dot{x}}{||e||||Lx||} - \frac{(Lx)^T L\dot{x}||e||}{||Lx||^3} \le \frac{||e||||\dot{x}||}{||e||||Lx||} + \frac{||\dot{x}||||L||||e||}{||Lx||^2}$$

$$= \left(1 + \frac{||L||||e||}{||Lx||}\right)\frac{||\dot{x}||}{||Lx||} \le \left(1 + \frac{||L||||e||}{||Lx||}\right)\frac{||Lx|| + ||Le||}{||Lx||} \le \left(1 + \frac{||L||||e||}{||Lx||}\right)^2$$

Thus, from the same analysis as in p. 8, we get $\tau = \frac{\sigma}{||L||(1+\sigma)}$.



We next challenge the following issues:

- Bidirectional → Strongly Connected
- Synchronous → Asynchronous
- Vector Space → $SE(3)$
- Error Energy → Individual Energy (Passivity Approach)

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Passivity-based Event-driven Position Synch.

Interconnection Topology: Strongly Connected Digraphs

Dynamics

$\dot{p}_i = v_i, \quad p_i, v_i \in \mathcal{R}^3$
$i \in \{1, \cdots, n\}$

Event-driven Position Synchronization Law

$v_i(t) = k_i \sum_{j\in\mathcal{N}_i} (\bar{p}_j(t) - \bar{p}_i(t)), \ k_i > 0$

$\bar{p}_i(t) = p_i(t_k^i), \ t \in [t_k^i, t_{k+1}^i)$ represents the last broadcasted output information of i at its event time t_k^i.
$\bar{p}_j(t) = p_j(t_{k'}^j), \ t \in [t_{k'}^j, t_{k'+1}^j)$ denotes the last transmitted output information of j at its event time $t_{k'}^j$.

Measurement Error

$e_i(t) := p_i(t) - \bar{p}_i(t), \ t \in [t_k^i, t_{k+1}^i)$

$\Rightarrow p_i = e_i + \bar{p}_i$

Goal

$\lim_{t\to\infty} \|p_i(t) - p_j(t)\| = 0 \ {}^\forall i, j$

Trigger Conditions

(a): based on [10, 12]

$\|e_i\| > \frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{\|\sum_{j\in\mathcal{N}_i} (\bar{p}_j - \bar{p}_i)\|}, \ \sigma \in (0, 0.5)$

(b): more restrictive

$\|e_i\| > \frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{\sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|}$

Passivity-based Event-driven Position Synch.

Convergence Analysis (based on [10, 12])

$\gamma_i > 0$: determined by the strong connectivity

Lyapunov Function Candidate: $V_p = \frac{1}{2}\sum_{i=1}^n \frac{\gamma_i}{k_i}\|p_i\|^2$

$$\Rightarrow \dot{V}_p = \sum_{i=1}^n \frac{\gamma_i}{k_i} p_i^T \dot{p}_i = \sum_{i=1}^n \sum_{j\in\mathcal{N}_i} \gamma_i p_i^T (\bar{p}_j - \bar{p}_i) = \sum_{i=1}^n \sum_{j\in\mathcal{N}_i} \gamma_i (e_i + \bar{p}_i)^T (\bar{p}_j - \bar{p}_i)$$

$$= \sum_{i=1}^n \sum_{j\in\mathcal{N}_i} \gamma_i \left(e_i^T (\bar{p}_j - \bar{p}_i) - \frac{1}{2}\|\bar{p}_j - \bar{p}_i\|^2 \right)$$ (calc. process is omitted)

$$\le \sum_{i=1}^n \gamma_i \left(\|e_i\| \left\| \sum_{j\in\mathcal{N}_i} (\bar{p}_j - \bar{p}_i) \right\| - \frac{1}{2} \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2 \right)$$

V_p is NOT continuously differentiable

If $\|e_i\| \le \frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{\|\sum_{j\in\mathcal{N}_i} (\bar{p}_j - \bar{p}_i)\|}, \ \sigma \in (0, 0.5)$ holds, then we get

$$\dot{V}_p \le -\left(\frac{1}{2} - \sigma\right) \sum_{i=1}^n \gamma_i \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2 \le 0.$$ $\Rightarrow$ Trigger Condition (a)

[10, 12] claim that since $V_p \ge 0$ and $\dot{V}_p \le 0$ hold, one can further conclude that $\lim_{t\to\infty} \dot{V}_p = 0$, i.e. $\lim_{t\to\infty} \|\bar{p}_j(t) - \bar{p}_i(t)\| = 0 \ {}^\forall i, j.$ $\Rightarrow$?? (Future Work)

Passivity-based Event-driven Position Synch.

[10, 12] moreover claim that we get $\lim_{t\to\infty} e_i(t) = 0 \ {}^\forall i$ from $\lim_{t\to\infty} \|\bar{p}_j(t) - \bar{p}_i(t)\| = 0 \ {}^\forall i, j$ and $\|e_i\| \le \frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{\|\sum_{j\in\mathcal{N}_i} (\bar{p}_j - \bar{p}_i)\|}$.

Discussion: When $\lim_{t\to\infty} (x, y, z) = 0, \ x, y, z \in \mathcal{R}$, we can NOT obtain

$\lim_{t\to\infty} \frac{|x|^2 + |y|^2 + |z|^2}{|x + y + z|} = 0.$ (i.e. $\lim_{t\to\infty} e_i(t) \neq 0 \ {}^\forall i$)

Counter Ex.) $x + y + z = 0$

Thus, we should use (more restrictive) trigger condition (b). Then, we get

$$\|e_i\| \le \frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{\sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|} \le \frac{\sigma (\sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|)^2}{\sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|} = \sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|.$$

Therefore, we can get $\lim_{t\to\infty} \|e_i(t)\| \le \lim_{t\to\infty} \sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j(t) - \bar{p}_i(t)\| = 0.$

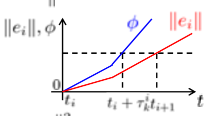
Then, we conclude $\lim_{t\to\infty} \|p_i(t) - p_j(t)\| = 0 \ {}^\forall i, j$: Position Synchronization

Passivity-based Event-driven Position Synch.

Lower Bound Analysis of the Inter-execution Time (based on [12])

For $t \in [t_k^i, t_{k+1}^i)$, $\frac{d}{dt}\|e_i\| \le \|\dot{e}_i\| = \|\dot{p}_i\| = k_i \left\| \sum_{j\in\mathcal{N}_i} (\bar{p}_j - \bar{p}_i) \right\|$ holds.

So, the evolution of $\|e_i\|$ during $[t_k^i, t_{k+1}^i)$ is bounded by the solution to $\dot{\phi} = k_i \left\| \sum_{j\in\mathcal{N}_i} (\bar{p}_j - \bar{p}_i) \right\|, \ \phi(t_k^i) = 0.$



Thus the time for $\|e_i\|$ to evolve from 0 to $\frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{\sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|}$ is lower bounded by the solution to $\phi(t_k^i + \tau_k^i) = \frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{\sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|}$, and we get

$$\tau_k^i = \frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{k_i (\sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|) \|\sum_{j\in\mathcal{N}_i} (\bar{p}_j - \bar{p}_i)\|} \left(\ge \frac{\sigma \sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|^2}{k_i (\sum_{j\in\mathcal{N}_i} \|\bar{p}_j - \bar{p}_i\|)^2} \right)$$ ✗

When $\|\bar{p}_j - \bar{p}_i\|$ goes to 0, how to assess τ_k^i?
Is there any constant value $a > 0$ satisfying $\tau_k^i \ge a$? (Future Work)

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Passivity-based Event-driven Attitude Synch.

Dynamics

$\dot{R}_i = R_i\omega_i^b,\ R_i \in SO(3),\ \omega_i^b \in \mathcal{R}^3$
$i \in \{1,\cdots,n\}$

$\bar{R}_i(t) = R_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i)$
$\bar{R}_j(t) = R_j(t_{k'}^j),\ t \in [t_{k'}^j, t_{k'+1}^j)$

Event-driven Attitude Synchronization Law

$\omega_i^b(t) = k_i \sum_{i\in\mathcal{N}_i} \mathrm{sk}(\bar{R}_i^T(t)\bar{R}_j(t))^\vee,\ k_i > 0$

Goal

$\lim_{t\to\infty} \psi(R_i^T(t)R_j(t)) = 0\ {}^\forall i,j$

$\psi(R) := \frac{1}{2}\|I - R\|_F^2 \geq 0$

Measurement Error and Its Norm

(a): $e_i(t) := \bar{R}_i^T(t)R_i(t) \in SO(3),\ t \in [t_k^i, t_{k+1}^i)$ ⇨ $R_i = \bar{R}_i e_i$

$\psi(e_i(t)) = \mathrm{tr}(I - \bar{R}_i^T(t)R_i(t)) \geq 0$

(b): $e_i(t) := \mathrm{sk}(R_i(t))^\vee - \mathrm{sk}(\bar{R}_i(t))^\vee \in \mathcal{R}^3,\ t \in [t_k^i, t_{k+1}^i)$

⇨ $\mathrm{sk}(R_i)^\vee = e_i + \mathrm{sk}(\bar{R}_i)^\vee$

$\|e_i(t)\| = \|\mathrm{sk}(R_i(t))^\vee - \mathrm{sk}(\bar{R}_i(t))^\vee\| \geq 0$

Since $\mathrm{sk}(R_i)^\vee = \xi \sin\theta$ holds for appropriate $\xi \in \mathcal{R}^3,\ \theta \in \mathcal{R}$, the measurement error (b) should be defined in $\theta \in [-\pi/2, \pi/2]$.

Passivity-based Event-driven Attitude Synch.

Convergence Analysis 1: Measurement Error (a)

$\gamma_i > 0$: determined by the strong connectivity

Lyapunov Function Candidate: $V_R = \frac{1}{2}\sum_{i=1}^n \frac{\gamma_i}{k_i}\psi_i(R_i)$

$$\dot{V}_R = \sum_{i=1}^n \frac{\gamma_i}{k_i}(\mathrm{sk}(R_i)^\vee)^T\omega_i^b = \sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i(\mathrm{sk}(\underline{R_i})^\vee)^T\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee$$

$$= \sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i(\mathrm{sk}(\underline{\bar{R}_i e_i})^\vee)^T\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee$$

$$= -\frac{1}{2}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\left(\mathrm{tr}(\mathrm{sk}(\bar{R}_i e_i)\mathrm{sk}(\bar{R}_i^T\bar{R}_j))\right) \quad \left(a^T b = -\frac{1}{2}\mathrm{tr}(\hat{a}\hat{b}),\ a,b \in \mathcal{R}^3\right)$$

$$= -\frac{1}{8}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\left(\mathrm{tr}((\bar{R}_i e_i - e_i^T\bar{R}_i^T)(\bar{R}_i^T\bar{R}_j - \bar{R}_j^T\bar{R}_i))\right)$$

$$= -\frac{1}{8}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\mathrm{tr}(\bar{R}_i e_i\bar{R}_i^T\bar{R}_j - \bar{R}_i e_i\bar{R}_j^T\bar{R}_i - e_i^T\bar{R}_i^T\bar{R}_i^T\bar{R}_j + e_i^T\bar{R}_i^T\bar{R}_j^T\bar{R}_i)$$

Passivity-based Event-driven Attitude Synch.

Tool: $\mathrm{tr}(A) = \mathrm{tr}(A^T),\ \mathrm{tr}(A+B) = \mathrm{tr}(A) + \mathrm{tr}(B),\ \mathrm{tr}(AB) = \mathrm{tr}(BA)$

$$\dot{V}_R = -\frac{1}{8}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\mathrm{tr}(\bar{R}_i e_i\bar{R}_i^T\bar{R}_j - \bar{R}_i e_i\bar{R}_j^T\bar{R}_i - e_i^T\bar{R}_i^T\bar{R}_i^T\bar{R}_j + e_i^T\bar{R}_i^T\bar{R}_j^T\bar{R}_i)$$

$$= -\frac{1}{4}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\mathrm{tr}(\bar{R}_i e_i\bar{R}_i^T\bar{R}_j - \bar{R}_i e_i\bar{R}_j^T\bar{R}_i) \quad \begin{pmatrix}\mathrm{tr}(ABC) = \mathrm{tr}(BCA)\\ \mathrm{tr}(ABC) \neq \mathrm{tr}(ACB)\end{pmatrix}$$

$$= -\frac{1}{4}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\mathrm{tr}(\bar{R}_i e_i\bar{R}_i^T(I - \bar{R}_i) - \bar{R}_i e_i\bar{R}_j^T(I - \bar{R}_j) + \underline{\bar{R}_i e_i}(I - \bar{R}_j^T\bar{R}_i))$$

$\underline{\bar{R}_i e_i} = R_i$

$$= -\frac{1}{2}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\mathrm{tr}(\bar{R}_i e_i\bar{R}_i^T(I - \bar{R}_i) - \bar{R}_i e_i\bar{R}_j^T(I - \bar{R}_j))$$

$$-\frac{1}{4}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\mathrm{tr}((R_i + R_i^T)(I - \bar{R}_i^T\bar{R}_j))$$

$$\leq -\frac{1}{2}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\mathrm{tr}(\bar{R}_i e_i\bar{R}_i^T(I - \bar{R}_i) - \bar{R}_i e_i\bar{R}_j^T(I - \bar{R}_j))$$ if $R_i \geq 0$ holds

$$-\frac{1}{4}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\lambda_{min}(R_i + R_i^T)\mathrm{tr}(I - \bar{R}_i^T\bar{R}_j)$$

$(\mathrm{tr}(AB) \neq \mathrm{tr}(A)\mathrm{tr}(B))$ failed orz…

Passivity-based Event-driven Attitude Synch.

Convergence Analysis 2: Measurement Error (b)

$\gamma_i > 0$: determined by the strong connectivity

Lyapunov Function Candidate: $V_R = \frac{1}{2}\sum_{i=1}^n \frac{\gamma_i}{k_i}\psi_i(R_i)$

$$\dot{V}_R = \sum_{i=1}^n \frac{\gamma_i}{k_i}(\mathrm{sk}(R_i)^\vee)^T\omega_i^b = \sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i(\mathrm{sk}(\underline{R_i})^\vee)^T\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee$$

$$= \sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i(\underline{e_i + \mathrm{sk}(\bar{R}_i)^\vee})^T\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee$$

$$= \sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i e_i^T\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee + \sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i(\mathrm{sk}(\bar{R}_i)^\vee)^T\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee$$

if $\bar{R}_i \geq 0$ holds

$$\leq \sum_{i=1}^n\gamma_i\|e_i\|\left\|\sum_{j\in\mathcal{N}_i}\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee\right\| - \frac{1}{4}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i}\gamma_i\lambda_{min}(\bar{R}_i + \bar{R}_i^T)\mathrm{tr}(I - \bar{R}_i^T\bar{R}_j)$$

Therefore, we have to guarantee $\bar{R}_i(t) > 0$ (i.e. $R_i(t) > 0$) ${}^\forall t \geq 0$ under the proposed event-driven control law (Future Work)

Passivity-based Event-driven Attitude Synch.

$$\dot{V}_R \leq \sum_{i=1}^n\gamma_i\|e_i\|\left\|\sum_{j\in\mathcal{N}_i}\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee\right\| - \frac{1}{4}\sum_{i=1}^n\gamma_i\lambda_{min}(\bar{R}_i + \bar{R}_i^T)\sum_{j\in\mathcal{N}_i}\mathrm{tr}(I - \bar{R}_i^T\bar{R}_j)$$

If $\|e_i\| \leq \dfrac{\sigma\sum_{j\in\mathcal{N}_i}\mathrm{tr}(I - \bar{R}_i^T\bar{R}_j)}{\lambda_{min}(\bar{R}_i + \bar{R}_i^T)\|\sum_{j\in\mathcal{N}_i}\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee\|}$, $\sigma \in (0, 0.25)$ holds, then we get

$$\dot{V}_R \leq -\left(\frac{1}{4} - \sigma\right)\sum_{i=1}^n\gamma_i\lambda_{min}(\bar{R}_i + \bar{R}_i^T)\sum_{j\in\mathcal{N}_i}\mathrm{tr}(I - \bar{R}_i^T\bar{R}_j) \leq 0.$$

V_R is NOT continuously differentiable

Therefore we obtain $\lim_{t\to\infty}\psi(\bar{R}_i^T(t)\bar{R}_j(t)) = 0\ {}^\forall i,j$.

We next check whether $\lim_{t\to\infty} e_i(t) = 0\ {}^\forall i$ holds. Similarly to p. 16, we utilize

Trigger Condition: $\|e_i\| > \dfrac{\sigma\sum_{j\in\mathcal{N}_i}\mathrm{tr}(I - \bar{R}_i^T\bar{R}_j)}{\lambda_{min}(\bar{R}_i + \bar{R}_i^T)\sum_{j\in\mathcal{N}_i}\|\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee\|}$, $\sigma \in (0, 0.25)$

Discussion: The condition implies

$\|e\| \leq a\dfrac{(1-\cos x) + (1-\cos y) + (1-\cos z)}{|\sin x| + |\sin y| + |\sin z|}$

l'Hôpital's Rule

$\lim_{\theta\to0+}\dfrac{1-\cos\theta}{\sin\theta} = \lim_{\theta\to0+}\dfrac{\sin\theta}{\cos\theta} = 0$

(Future Work)

Probably, we can show that $\lim_{t\to\infty} e_i(t) = 0\ {}^\forall i$, i.e. $\lim_{t\to\infty}\psi(R_i^T(t)R_j(t)) = 0\ {}^\forall i,j$.

Passivity-based Event-driven Attitude Synch.

Lower Bound Analysis of the Inter-execution Time (based on [12])

For $t \in [t_k^i, t_{k+1}^i)$,

$$\frac{d}{dt}\|e_i\| \leq \|\dot{e}_i\| = \left\|\frac{d}{dt}(\mathrm{sk}(R_i)^\vee)\right\| \overset{?}{=} \left\|\left(\frac{d}{dt}\mathrm{sk}(R_i)\right)^\vee\right\| = \frac{1}{2}\|(\dot{R}_i - \dot{R}_i^T)^\vee\|$$

$$= \frac{1}{2}\|(R_i\hat{\omega}_i^b - (\hat{\omega}_i^b)^T R_i^T)^\vee\| = \frac{1}{2}\|(R_i\mathrm{sk}(\bar{R}_i^T\bar{R}_j) - \mathrm{sk}(\bar{R}_i^T\bar{R}_j)^T R_i^T)^\vee\|$$

(omit $\sum_{j\in\mathcal{N}_i}$)

$$= \frac{1}{2}(\|(\bar{R}_i e_i\mathrm{sk}(\bar{R}_i^T\bar{R}_j) - \mathrm{sk}(\bar{R}_i^T\bar{R}_j)^T e_i^T\bar{R}_i^T)^\vee\|$$

$= ??$ (Future Work)

Technical Issues to Be Solved

1. Strict Convergence Analysis

Is it OK to claim the convergence to $\mathrm{Ker}(L)$ from $\dot{V} \leq -(1-\sigma)\|Lx\|^2 \leq 0$?

2. Strict Lower Bound Analysis of the Inter-execution Time

How to assess τ_k^i when $\|\bar{p}_j - \bar{p}_i\|$ goes to 0 ? $\frac{d}{dt}(\|e_i\|/\|\sum_{j\in\mathcal{N}_i}(\bar{p}_j - \bar{p}_i)\|)$?

3. Continuation of Convergence Analysis in the Measurement Error (a)

$e_i(t) := \bar{R}_i^T(t)R_i(t) \in SO(3)$ ⇨ $\dot{V}_R \leq$??

4. Guarantee of Positive Definiteness of Each Orientation

To prove that if $\bar{R}_i(0) > 0$, then $\bar{R}_i(t) > 0$ (i.e. $R_i(t) > 0$) $^\forall t \geq 0$ holds.

5. Continuation of Convergence Analysis in the Measurement Error (b)

$$\lim_{t\to\infty}\|e_i\| \leq \lim_{t\to\infty}\frac{\sigma\sum_{j\in\mathcal{N}_i}\mathrm{tr}(I - \bar{R}_i^T\bar{R}_j)}{\lambda_{min}(\bar{R}_i + \bar{R}_i^T)\|\sum_{j\in\mathcal{N}_i}\mathrm{sk}(\bar{R}_i^T\bar{R}_j)^\vee\|} = 0 \text{ ?}$$

6. Continuation of Lower Bound Analysis of the Inter-execution Time

$\frac{d}{dt}\|e_i\| = \cdots$ (in progress…)

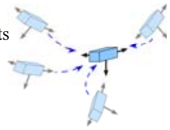
Conclusion and Next Challenges

Conclusions

- introduced event-driven (cooperative) control problems through the simplest scalar examples
- proposed an event-driven position synchronization law and proved convergence under strongly connected digraphs
- proposed an event-driven attitude synchronization law and analyzed convergence
- clarified the remaining issues to be solved

Next Challenges

- Detailed analysis of convergence to the set of equilibrium points
- Derivation of the lower bound of the inter-event time
- Verifications through simulation and experiments
- To finish writing Ph.D. thesis



⇨ Submit the conference paper to 13CDC
Submit a journal paper with further results next spring

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