Controller Synthesis for Data Center

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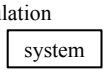
Background

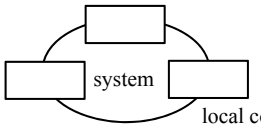
Control Systems
- aerospace
- automotive industry
- advanced robotic systems

} control objective

stabilization
output regulation



regulate global behavior



formally specify the desired system behavior

Specifications
- coordination and synchronization of individual modules
- sequencing of tasks
- reconfigurability and adaptability of components

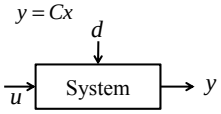
Background

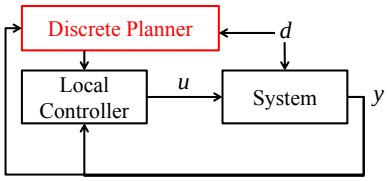
Hierarchical Approach

System

$\dot{x} = Ax + Bu + Ed$

$y = Cx$





Discrete Planner: generates control signals to the system to ensure that the system satisfies specifications

Synthesis

synthesizing a discrete planner that computes a discrete plan satisfying the specification based on the abstract, finite-state model of the system

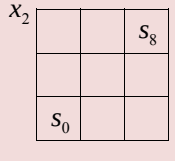
Introduction

Control with Temporal Logic

System Model
- State(discrete, continuous)
- dynamics

Proposition

Proposition s_i on partition



Specification

Initial State, Stability
Convergence, Safety
Sequence, Coverage

Linear Temporal Logic

Proposition
State(discrete)

Operator $\neg, \wedge, \vee, \rightarrow, \Box, \Diamond, next, until$

TuLiP[5]
- **Check Realizability**
- **Compute Strategy**

Problem

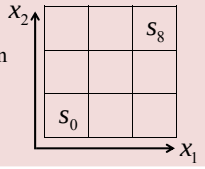
System Model(Linear Time Invariant System)

$x(k+1) = Ax(k) + Bu(k) + Ed(k)$ Initial $x(0) \in X$

Input $u(k) \in U$

Disturbance $d(k) \in D$

ex. $x := [x_1 \quad x_2]^T$



Proposition
- continuous state partition
- discrete state

$s_i(k) \in boolean$

$s(k) := s_0(k)s_1(k)s_2(k)\cdots$

Specification
- Initial State
- Desired Behavior
- Assumption

} Φ Specification(LTL)

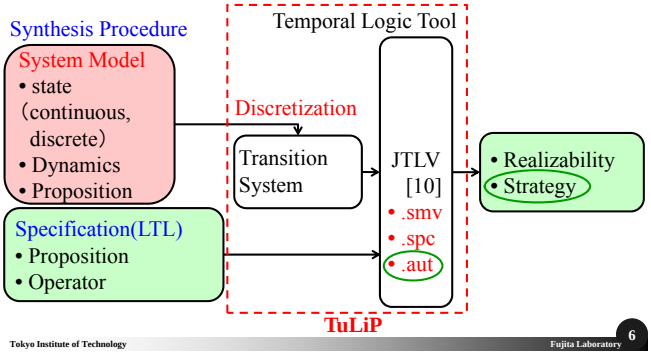
Proposition s
Operator $\neg, \wedge, \vee, \rightarrow, \Box, \Diamond, next, until$

TuLiP

Temporal Logic Planning (TuLiP) Toolbox

collection of Python-based code for automatic synthesis of correct-by-construction embedded control software

Synthesis Procedure



- Control(No Disturbance)
- Control(Disturbance)

Example

System Model

$$x(k+1)=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}x(k)+\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}u(k)$$

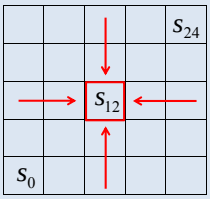
$$x=[x_1 \quad x_2]^T$$

$$-5<x_1(k)<5$$

$$-5<x_2(k)<5$$

Input $-0.4<u(k)<0.4$ Disturbance $d(k)=0$

Specification

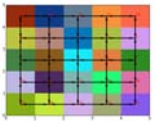


move and stay in s_{12} $\Diamond\Box s_{12}$

Transition System

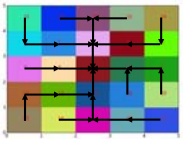
Transition System

Transition System from System Dynamics



Strategy

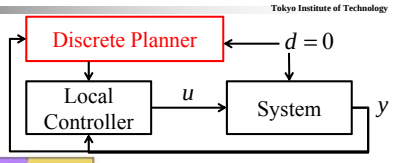
Strategy compute next state based on current state

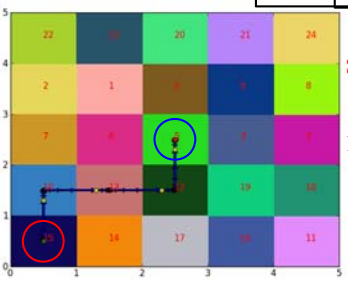


Simulation

Simulation

Initial State: 15, Goal : 5





Strategy

calculate next state

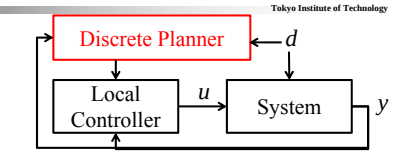
Input

feedback control input from current state to next state

- Control(No Disturbance)
- Control(Disturbance)

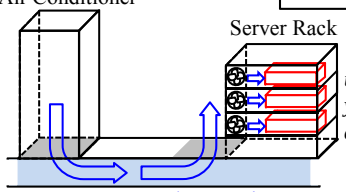
Data Center Cooling

Data Center



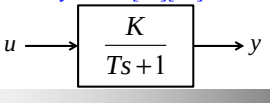
Air Conditioner

Server Rack



u: Air Conditioner temperature
y: Rack Temperature
d: CPU heat in Rack

Temperature Control Dynamics[11][12]

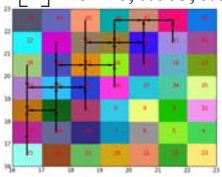


Problem(Disturbance)

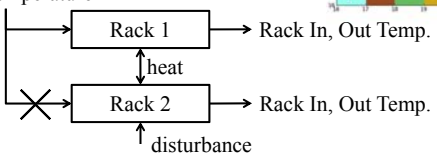
Discretization under Disturbance

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-0.005 & 0.001\\ 0.01 & -0.01\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0.004\\ 0\end{bmatrix}u+\begin{bmatrix}0\\ 1\end{bmatrix}d \qquad d=0, 0.005, 0.01$$

discretize(sampling time 250s)

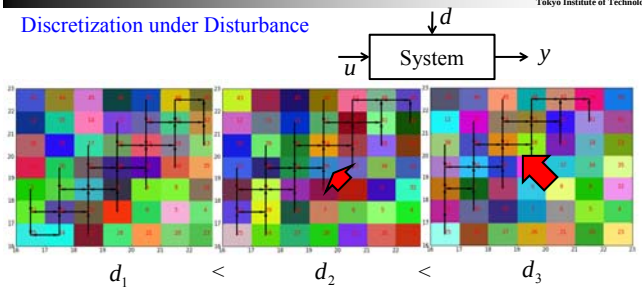


CRAC Temperature



Problem(Disturbance)

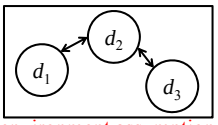
Discretization under Disturbance



$d_1 < d_2 < d_3$

Specification

Desired behavior +



environment assumption

Two Rack System

Two Rack System

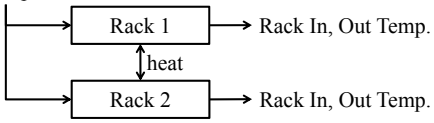
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-\frac{1}{1210} & 0.001\\ 0.001 & -\frac{1}{690}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}\frac{5.7}{1210}\\ \frac{5.86}{690}\end{bmatrix}u \qquad \begin{matrix}0<x<7\\ -3<u<10\end{matrix}$$

$$\Rightarrow \quad \begin{matrix}3<x_1<4\\ 3<x_2<4\end{matrix}$$

discretize(sampling time 250s)

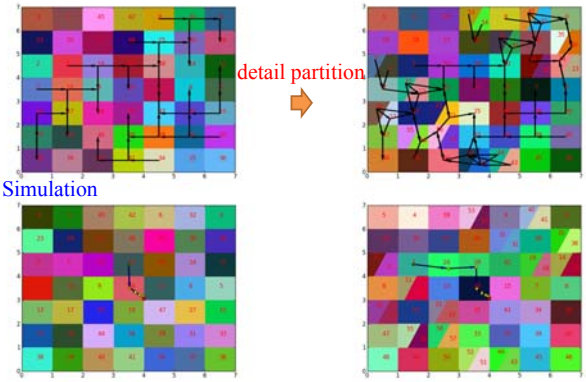
$$\begin{bmatrix}x_1(k+1)\\ x_2(k+1)\end{bmatrix}=\begin{bmatrix}0.8376 & 0.1903\\ 0.1903 & 0.7191\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}+\begin{bmatrix}1.295\\ 1.921\end{bmatrix}u$$

CRAC Temperature



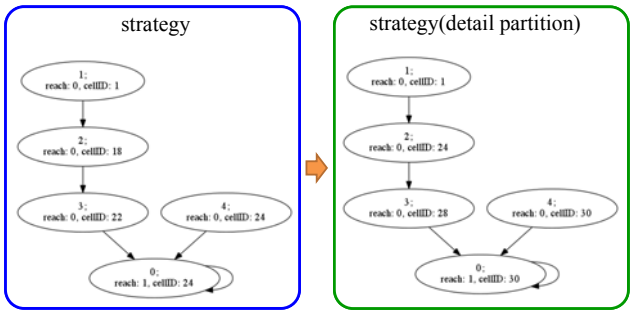
Two Rack System

Transition System(Two Rack System) 1step(k→k+1)



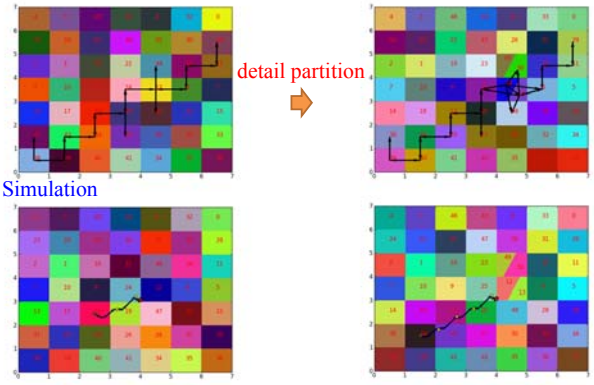
Two Rack System

Control Strategy 1step(k→k+1)



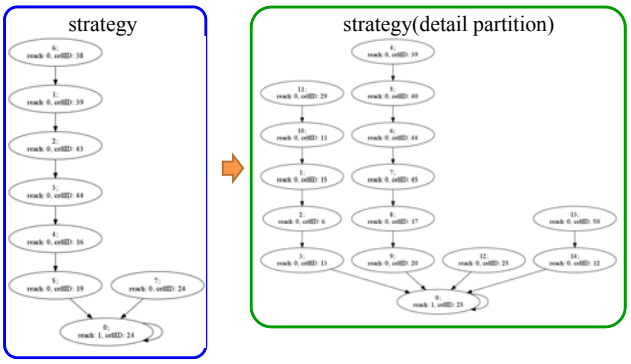
Two Rack System

Transition System(Two Rack System) 5step(k→k+5)



Two Rack System

Control Strategy 5step(k→k+5)



Summary

Summary

- Test TuLiP function
- Data Center Model
- Controller Synthesis Problem(under disturbance model)

Future Work

- Survey Discretization
- Data Center Controller Synthesis Problem

Reference

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