Flocking Algorithms for Autonomous Mobile Agents

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FL11-9-2
15th, June, 2011

Introduction

Multi-agent System
A system composed of massive autonomous mobile agents

Advantage
- Efficiency
- Large Scale
- Robustness

Application
- Mobile Sensor Network
- Intelligent Transportation Network
- Self-Assembly

Mobile Sensor Network

Transportation Network

one of behavior with local interaction

Flocking
A form of a Collective Behavior of interacting agents with a Common Group Objective

Self-Assembly

How to achieve? ⇨ Cooperative Control

Background

Cooperative Control
A distributed control strategy using local information so that multi-agent system achieve common group objective

Previous Work
Passivity-based Output Synchronization and Flocking Algorithm in SE(3)
Group Objective
Implement flocking algorithm: alignment, cohesion and separation[2]

Formation Control based Approach
Group Objective
Micro: relative distance between agents
relative position between agents[3][4]
Macro: flock position, attitude and shape[5]-[7]

Other Approach
Group Objective
Keep both cohesion and scale-free property[8][9]

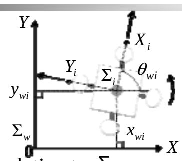
Outline

- Background
- Flocking Algorithm in Fujita lab.
- Interaction-based research
- Consider the Whole Flock research
- Other Model research
- Summary and Future Works

Y. Igarashi[1]

System Description

$i \in \{1, \dots, n\}$
$p_{wi} \in R^3$: position
Σ_w: world frame
Σ_i: agent i's frame
$e^{\hat{\xi}_{wi}\theta_{wi}}$: rotation matrix Σ_i relative to Σ_w
$\xi_{wi} \in R^3, \theta_{wi} \in R$: direction and angle of rotation
$\xi_{wi}^T \xi_{wi} = I_3$



Pose

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}_{wi}\theta_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix}$$

Rigid Body Motion

$V_i^b := (v_i^b, \omega_i)$: body velocity

$$\hat{V}_i^b = \begin{bmatrix} \hat{\omega}_i & v_i^b \\ 0 & 1 \end{bmatrix}, \dot{g}_{wi} = g_{wi}\hat{V}_i^b$$

: rigid body motion

$$y_{wi} = \begin{bmatrix} e^{\hat{\xi}_{wi}\theta_{wi}} & p_{wi} + d_{wi} \\ 0 & 1 \end{bmatrix}$$: output

Energy function

$$\psi(y_{wi}) := \frac{1}{2}\|q_{wi}\|^2 + \frac{1}{2}\mathrm{tr}\left(I_3 - e^{\hat{\xi}_{wi}\theta_{wi}}\right) \quad q_{wi} = p_{wi} + d_{wi}$$

$$\psi(y_{wi}) = 0 \Leftrightarrow y_{wi} = I_4$$

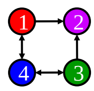
Definition: Output Synchronization

$$\lim_{t \to \infty} \psi(y_{wi}^{-1} y_{wj}) = 0, \forall i, j \in \{1, \dots, n\}$$

Y. Igarashi[1]

Communication Graph

$\mathcal{V} := \{1, \dots, n\}$: node set (agent)
$\mathcal{E} := \mathcal{V} \times \mathcal{V}$: edge set (communication)
$\mathcal{N}_i := \{j \in \mathcal{V} \mid (j,i) \in \mathcal{E}\}$: neighbor set (local information)



Output Synchronization Control Law

$v_d, e^{\hat{\xi}_d\theta_d}\omega_d$: desired velocity

$$V_i^b = -K_i \sum_{j \in \mathcal{N}_i} \left(\begin{bmatrix} e^{-\hat{\xi}_{wi}\theta_{wi}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} q_{wi} - q_{wj} \\ \mathrm{sk}\left(e^{-\hat{\xi}_{wj}\theta_{wj}} e^{\hat{\xi}_{wi}\theta_{wi}}\right)^{\vee} \end{bmatrix} \right) + \begin{bmatrix} e^{-\hat{\xi}_{wi}\theta_{wi}} & 0 \\ 0 & e^{-\hat{\xi}_{wi}\theta_{wi}} \end{bmatrix} \begin{bmatrix} v_d \\ e^{\hat{\xi}_d\theta_d}\omega_d \end{bmatrix}, i \in \{1, \dots, n\}$$

Assumption
- Graph is fixed and strongly connected
- There exists $e^{\hat{\xi}_\alpha\theta_\alpha}$ such that $e^{\hat{\bar{\xi}}_{wi}\bar{\theta}_{wi}} := e^{\hat{\xi}_d\theta_d} e^{-\hat{\xi}_\alpha\theta_\alpha} e^{\hat{\xi}_{wi}\theta_{wi}}, \forall i$ are positive definite

Theorem: Output Synchronization
Consider the n agent represented previously.
Under the upper assumption,
the velocity control law achieves output synchronization

Y. Igarashi[1]

Flocking Algorithm

Reynolds[2] introduced three rules to achieve flocking

- alignment
- cohesion(flock centering)
- separation(collision avoidance)

The alignment and cohesion rules are already incorporated in the proposed control law, however the separation is not

Definition: Collision

$$\|p_{wi} - p_{wj}\| \le r, r > 0$$

Sensing Graph

$\mathcal{E}_C := \{(j,i) \in \mathcal{V} \times \mathcal{V} | r < \|p_{wi} - p_{wj}\| \le R\}$: edge set (sensing) R: sensing radius

$\mathcal{N}_{Ci} := \{j \in \mathcal{V} | (j,i) \in \mathcal{E}_C\}$: neighbor set (local information)

Y. Igarashi[1]

Collision Avoidance

Assumption under initial condition, collision does not occur

Collision Avoidance Control

$$\frac{\partial U_{ij}}{\partial p_{wi}} = \begin{cases} 0 & \text{if } R \le \|p_{wi} - p_{wj}\| \\ s_{ij} & \text{if } r < \|p_{wi} - p_{wj}\| < R \\ \text{not defined} & \text{if } \|p_{wi} - p_{wj}\| = r \\ 0 & \text{if } \|p_{wi} - p_{wj}\| < r \end{cases}, s_{ij} = 4\frac{(R^2 - r^2)\left(\|p_{wi} - p_{wj}\|^2 - R^2\right)}{\left(\|p_{wi} - p_{wj}\|^2 - r^2\right)^3}$$

Velocity Control Law with Collision Avoidance

$$V_i^b = \begin{bmatrix} e^{-\hat{\xi}_{wi}\theta_{wi}} & 0 \\ 0 & e^{-\hat{\xi}_{wi}\theta_{wi}} \end{bmatrix} \begin{bmatrix} v_d \\ e^{\hat{\xi}_d \theta_d}\omega_d \end{bmatrix} - K_{pi} \begin{bmatrix} e^{-\hat{\xi}_{wi}\theta_{wi}} & 0 \\ 0 & I \end{bmatrix} \left(\sum_{j \in \mathcal{N}_i} \begin{bmatrix} q_{wi} - q_{wj} \\ \mathrm{sk}\left(e^{-\hat{\xi}_{wj}\theta_{wj}} e^{\hat{\xi}_{wi}\theta_{wi}}\right)^{\vee} \end{bmatrix} + \sum_{j \in \mathcal{N}_{Ci}} \begin{bmatrix} \frac{\partial U_{ij}}{\partial p_{wi}} \\ 0 \end{bmatrix} \right), i \in \{1, \ldots, n\}$$

Modified Output Synchronization Assumption

- Communication Graph is fixed ,undirected and connected
- $e^{\hat{\bar{\xi}}_{wi}\bar{\theta}_{wi}}$ are positive definite

Theorem : Flocking

The control law achieve attitude synchronization while avoiding collision ⇨ Flocking

Outline

H. Yamaguchi[4]

Setting

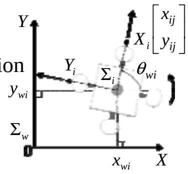
$i \in \mathcal{V} := \{1, \ldots, n\}$: agent

$p_{wi} = [x_{wi} \quad y_{wi}]^T$: position

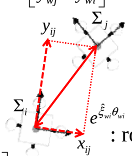
θ_{wi}: attitude

Σ_w: world frame

Σ_i : robot i's frame

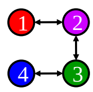


$\begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} = e^{-\hat{\xi}_{wi}\theta_{wi}} \begin{bmatrix} x_{wj} - x_{wi} \\ y_{wj} - y_{wi} \end{bmatrix}$:relative position in Σ_i



$e^{\hat{\xi}_{wi}\theta_{wi}}$: rotation matrix

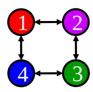
Assumption: Symmetrical attraction



graph laplacian ⇨

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Symmetric



⇨

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

$\mathcal{N}_i$: agent i's neighbor, agent reaching arrow to agent i

H. Yamaguchi[4]

Kinematics of Agents v_i^b : body velocity

$\dot{p}_{wi} = e^{\hat{\xi}_{wi}\theta_{wi}} v_i^b$: Kinematics

Flocking Algorithm stabilization vector

$$\begin{bmatrix} v_{xi}^b \\ v_{yi}^b \end{bmatrix} = \underbrace{\sum_{j \in \mathcal{N}_i} \tau_{ij} \begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix}}_{\text{cohesion}} + \underbrace{\begin{bmatrix} d_{xi}^b \\ d_{yi}^b \end{bmatrix}}_{\text{alignment}} + \begin{bmatrix} D_{xi}^b \\ D_{yi}^b \end{bmatrix} + \underbrace{\sum_{j \in \mathcal{V}} \delta_{ij} \left\{ \begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} - \frac{\Delta}{\left\| [x_{ij} \quad y_{ij}]^T \right\|} \begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix} \right\}}_{\text{separation}}, i \in \mathcal{V}$$

$$\delta_{ij} = \begin{cases} \delta, & \left\| [x_{ij} \quad y_{ij}]^T \right\| \le \Delta \\ 0, & \left\| [x_{ij} \quad y_{ij}]^T \right\| > \Delta \end{cases}$$

$[d_{xi}^b \quad d_{yi}^b]^T = e^{-\hat{\xi}_{wi}\theta_{wi}} [d_{wxi} \quad d_{wyi}]^T$: formation vector in Σ_i

$[D_{xi}^b \quad D_{yi}^b]^T = e^{-\hat{\xi}_{wi}\theta_{wi}} [D_{wxi} \quad D_{wyi}]^T$: stabilization vector in Σ

stabilization vector's detail

$$\begin{bmatrix} \dot{D}_{xi}^b \\ \dot{D}_{yi}^b \end{bmatrix} = -\begin{bmatrix} v_{xi}^b \\ v_{yi}^b \end{bmatrix} + \sum_{j \in \mathcal{N}_i} \tau_{ij} \left\{ e^{-\hat{\xi}_{wi}\theta_{wi}} e^{\hat{\xi}_{wj}\theta_{wj}} \begin{bmatrix} D_{xj}^b \\ D_{yj}^b \end{bmatrix} - \begin{bmatrix} D_{xi}^b \\ D_{yi}^b \end{bmatrix} \right\}, i \in \mathcal{V}$$

keep stability with formation vector imbalance

Stability and Formation controllability

- Stability of position and stabilization vector is independent of formation vector
- Formation is controllable by the formation vector

H. Yamaguchi[4]

Formation Setting Independent of neighbor

Relative position vector $\begin{bmatrix} \phi_{xk} \\ \phi_{xk} \end{bmatrix} = \begin{bmatrix} x_{wj} \\ y_{wj} \end{bmatrix} - \begin{bmatrix} x_{wi} \\ y_{wi} \end{bmatrix}, k = 1, 2, \ldots, n-1$

$$\Phi_x = [\phi_{x1} \quad \phi_{x2} \quad \ldots \quad \phi_{x(n-1)}]^T, \Phi_y = [\phi_{y1} \quad \phi_{y2} \quad \ldots \quad \phi_{y(n-1)}]^T$$

$$\hat{\phi}_x = x_{w1} + x_{w2} + \ldots + x_{wn}, \hat{\phi}_y = y_{w1} + y_{w2} + \ldots + y_{wn}$$

$$\Psi_x = [\phi_{x1} \quad \phi_{x2} \quad \ldots \quad \phi_{x(n-1)} \quad \hat{\phi}_x]^T = [\Phi_x \quad \hat{\phi}_x]^T, \Psi_y = [\phi_{y1} \quad \phi_{y2} \quad \ldots \quad \phi_{y(n-1)} \quad \hat{\phi}_y]^T = [\Phi_y \quad \hat{\phi}_y]^T$$

Formation Decision

$A \in R^{(n-1) \times (n-1)}$
$P \in R^{(n-1) \times n}$

$$\begin{bmatrix} \dot{\Phi}_x \\ \dot{\Phi}_y \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} \Phi_x \\ \Phi_y \end{bmatrix} + \begin{bmatrix} Pd_x \\ Pd_y \end{bmatrix} \quad d_x = [d_{x1} \quad d_{x2} \quad \ldots \quad d_{xn}]^T, \; d_y = [d_{y1} \quad d_{y2} \quad \ldots \quad d_{yn}]^T$$

Formation Control

using $d_x = -P^T (PP^T)^{-1} A\Phi_{xd} + h_x v_1$ $v_1 = [1 \quad 1 \quad \ldots \quad 1]^T \in R^n$

$d_y = -P^T (PP^T)^{-1} A\Phi_{yd} + h_y v_1$ h_x, h_y: arbitrary scalars

in distributed Flocking Algorithm with only local information

$[\Phi_x \quad \Phi_y]^T$ converge to desired formation $[\Phi_{xd} \quad \Phi_{yd}]^T$

H. Yamaguchi[4]

Summary and Comments

Flocking Algorithm

- Similar Flocking Algorithm to Igarashi[1] et al.
- Stabilization vector and Collision Avoidance is different point
- Considering Imbalance between Formation vector

Formation Decision

- Formation is apart from stabilization
- Formation vector correspond to desired formation from formula
- Check availability of formation (distributed or not)

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F. Zhang et al.[5]

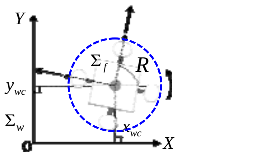
Setting

$i \in \mathcal{V} := \{1, \ldots, n\}$: agent Σ_w: world frame

$p_{wi} \in R^3$: position in Σ_w Σ_f: formation frame

m_i: mass

R: rotation matrix Σ_f relative to Σ_w



Formation position and attitude

Kinetic Energy of all agents

$$K^{tot} = \frac{1}{2}\sum_{i \in \mathcal{V}} m_i \|\dot{p}_{wi}\|^2$$

$M = \sum_{i \in \mathcal{V}} m_i$: total mass

$$\Rightarrow K^{tot} = \frac{1}{2}M\|\dot{p}_{wc}\|^2 + \frac{1}{2}\sum_{i \in \mathcal{V}} m_i \|\dot{p}_{ci}\|^2$$

$p_{wc} = \frac{\sum_{i \in \mathcal{V}} m_i p_{wi}}{M}$: center of mass in Σ_w

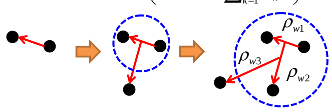
$p_{ci} = p_{wi} - p_{wc}$: position relative to center in Σ_w

F. Zhang et al.[5]

Jacobi coordinates

one of

Jacobi coordinates $\rho_{wi} = \sqrt{\mu_i}\left(p_{c(i+1)} - \frac{\sum_{k=1}^{i} m_k p_{ck}}{\sum_{k=1}^{i} m_k}\right), \frac{1}{\mu_i} = \frac{1}{\sum_{k=1}^{i} m_k} + \frac{1}{m_{i+1}}, i = 1, 2, \ldots, n-1$



ρ_{wi} not all collinear

Relative displacement between $(i+1)$th agent and the center of mass of the sub-cluster of first i agents

Jacobi coordinates are not unique

There exists an orthogonal matrix h between any two Jacobi coordinates

$$\begin{bmatrix}\rho_{w1}^1 & \rho_{w2}^1 & \cdots & \rho_{w(n-1)}^1\end{bmatrix} = \begin{bmatrix}\rho_{w1}^2 & \rho_{w2}^2 & \cdots & \rho_{w(n-1)}^2\end{bmatrix} h, h \in O(n-1)$$

Kinetic Energy of all agents using Jacobi coordinates

$$K^{tot} = \frac{1}{2}M\|\dot{p}_{wc}\|^2 + \frac{1}{2}\sum_{i \in \mathcal{V}} \|\dot{\rho}_{wi}\|^2$$

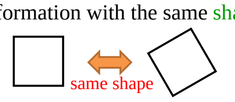
F. Zhang et al.[5]

shape coordinates

shape coordinates $s^j = s^j(\rho_{w1}, \rho_{w2}, \ldots, \rho_{w(n-1)}), j = 1, 2, \ldots, 3n-6$

$$s^j(R\rho_{w1}, R\rho_{w2}, \ldots, R\rho_{w(n-1)}) = s^j(\rho_{w1}, \rho_{w2}, \ldots, \rho_{w(n-1)}), \forall R \in SO(3)$$

the orientation of this formation with the same shape in Σ_w and in Σ_f



$\rho_{wi} = R\rho_i^b(s)$ R: rotation matrix from in Σ_w to in Σ_f

$s = \begin{bmatrix}s^1 & s^2 & \cdots & s^{3n-6}\end{bmatrix}^T$ ρ_i^b: Jacobi coordinates in Σ_f which only depend on shape coordinates

$$\Omega = R^T\dot{R}, I(s) = \sum_{i=1}^{n-1}\left(\|\rho_i^b\|^2 I_{3\times3} - \rho_i^b(\rho_i^b)^T\right), A_j(s) = I^{-1}\sum_{i=1}^{n-1}\rho_i^b \times \frac{\partial \rho_i^b}{\partial s^j}, G_{jk} = -A_j^T I A_k + \sum_{i=1}^{n-1}\frac{\partial \rho_i^b}{\partial s^j}\frac{\partial \rho_i^b}{\partial s^k}$$

Kinetic Energy of all agents using shape coordinates

$$K^{tot} = \frac{1}{2}M\|\dot{p}_{wc}\|^2 + \frac{1}{2}(\Omega + A\dot{s})^T I(\Omega + A\dot{s}) + \frac{1}{2}\dot{s}^T G\dot{s}$$

F. Zhang et al.[5]

Lagrange equation for formations in Σ_w

Lagrangian $L(p_w, \dot{p}_w) = K^{tot}(\dot{p}_w) - U(p_w)$ $U(p_w)$: potential energy

$p_w = \begin{bmatrix}p_{w1} & p_{w2} & \cdots & p_{w(n-1)}\end{bmatrix}^T$: position in Σ_w

Lagrange equations before coordinate transformation

$$m_i \ddot{p}_{wi} = u_{wi} - \frac{\partial U}{\partial p_{wi}}, i \in \mathcal{V}$$

u_{wi}: control forces in Σ_w

Lagrange equation for formations using shape coordinates

Lagrangian using shape coordinate

$$L(p_{wc}, R, s, \dot{p}_{wc}, \dot{R}, \dot{s}) = \frac{1}{2}M\|\dot{p}_{wc}\|^2 + \frac{1}{2}(\Omega + A(s)\dot{s})^T I(\Omega + A(s)\dot{s}) + \frac{1}{2}\dot{s}^T G\dot{s} - U(p_{wc}, R, s, \dot{p}_{wc})$$

Lagrange equations using shape coordinates

u_{wc}, u_g, u_s: control forces to formation

formation position $M\ddot{p}_{wc} = -\frac{\partial U}{\partial p_{wc}} + u_{wc}$

formation orientation $\frac{d}{dt}(I(\Omega + A(s)\dot{s})) = -\Omega \times I(\Omega + A(s)\dot{s}) - R^{-1}\frac{\partial U}{\partial R} + u_g$

formation shape $\frac{d}{dt}(G\dot{s}) + A(s)^T\frac{d}{dt}(I(\Omega + A(s)\dot{s}))$ $G_{jk} = -A_j^T I A_k + \sum_i^{n-1}\frac{\partial \rho_{wi}}{\partial s^j}\frac{\partial \rho_{wi}}{\partial s^k}$

$$= \frac{1}{2}\left[\frac{\partial I}{\partial s}\right]^* : (\Omega, \Omega) + \frac{1}{2}\left[\frac{\partial G}{\partial s}\right]^*(\dot{s}, \dot{s}) - \frac{\partial U}{\partial s} + u_s$$

F. Zhang et al.[5]

Energy function

$$V_L = \frac{1}{2}\|s-s^0\|^2 + \frac{1}{2}(\Omega + A(s)\dot{s})^T I(\Omega + A(s)\dot{s}) + \frac{1}{2}\dot{s}^T G\dot{s}$$

s^0: desired shape

Shape Control Law

attitude control $u_g = -k_1\Omega$

shape control $u_s = \frac{\partial U}{\partial s} - (s - s^0) - \dot{s}$

shape control input need only shape information if $\frac{\partial U}{\partial s} = 0$

orientation control input need formation rotation velocity in Σ_w, Ω

to estimate Ω, need sensing fixed object

⇨ not only relative information

Theorem

Suppose the potential U is rigid motion invariant ,
by using Shape Control Law , shape s^0 is
locally asymptotically stabilized

F. Zhang et al.[5]

Summary and Comments

Factor of Whole

Position , Attitude and Shape

Control Law

Input to position, attitude and shape individually

Total energy function objective

Total energy converges to 0

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Y. P. Gunji[8][9]

Physarum

Physarum can solve several kinds of problem

graph theoretical problem

optimization problem

System emulating Physarum

A system composed of particles, which move and modified based upon particle transformation that contains the relationship between parts and the whole ⇨ emulate the network formed by Physarum

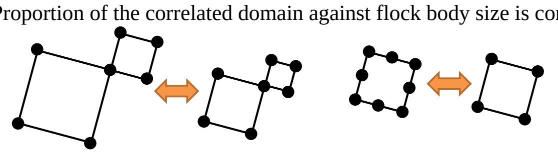
cannot move

Preserve cohesion property

Y. P. Gunji[8][9]

Scale-free sub-domain

Proportion of the correlated domain against flock body size is constant



Boids[2]

Either cohesion or scale-free property ⇨

Physarum

Two type bodies (parts and whole)

Both cohesion and scale-free property

Summary

Necessity of Scale-free sub-domain property

Scale-free sub-domain property is added to Boids[2]

but boids can have either cohesion or scale-free property

Other Model

Physarum model can have both cohesion and scale-free property

Outline

- Background
- Flocking Algorithm in Fujita lab.
- Micro-oriented research
- Macro-oriented research
- Other approach
- Summary and Future Works

Summary and Future Works

Previous Works in Fujita Lab.
Output Synchronization
Boids like Flocking Algorithm
Different Approach
Decide Formation
in Interaction Control Law
or in Using Input to Whole Flock
Other Flocking Algorithm
Include Scale-free sub-domain property
Formation Decision or Flocking
To introduce bird flock algorithm to real multi-agent system
need Scale-free sub-domain property
Flocking Objective
Keep both cohesion and scale-free property
How to achieve? ⇨ Adding another term to Boids like Flocking Algorithm

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