Sensor Localization and Target Estimation in Visual Sensor Networks

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Introduction

Visual Sensor Networks
A network consisting of spatially distributed smart cameras

Application
• Environmental monitoring
• Surveillance

The information of the location of the sensor is needed

Network Localization
Determining the relative poses of each sensor in sensor networks

Preliminaries
Consider a Visual Sensor Network with Objects
Communication graph $G=(V,E)$
Set of cameras: $V=\{1,\ldots,N\}$
Edge $(i,j)\in E$  $j$ can get $i$’s information
Neighbor set: $X_i = \{ j \in V | (i,j) \in E \}$

Undirected graph

Pose Representation
Pose of camera $i: g_{wi} = (p_{wi}, R_{wi}) \in SE(3)$
Position: $p_{wi} \in \mathbb{R}^3$
Orientation: $R_{wi} \in SO(3)$

Pose of object: $g_{w{o}} = (p_{w{o}}, R_{w{o}})$

Pose of object relative to camera: $g_{o{i}} = g_{w{o}}^{-1}g_{w{i}} = (p_{o{i}}, R_{o{i}})$

Relative pose of the camera $i$ and $j$: $g_{ij} = g_{wi}^{-1}g_{wj} = (p_{ij}, R_{ij})$

Annual Schedule of my Research

• Survey and Problem Settings
  Presented in the FL seminar on May 11th
• First Trial and Evaluation of Proposed Method
  Would like to finish by June
• Simulation (Experimental Verification)
  Would like to finish by the middle presentation on July 25th

Localization in Camera Networks [5]
Distributed estimation of the pose of cameras
• Use feature points to get the neighboring pose
• Apply only in static scene
• Nonsimultaneous measure and estimation

Objective of my Research
• Use an object to get the relative pose of the camera
• Not only determine the relative pose of each sensor but also estimate the pose of the object

Preliminaries

**Rigid Body Motion**

\[ \gamma_i : \text{wedge } \mathbb{R}^3 \to SO(3) \]

\[ V_i^B = \text{body velocity} \]

**Relative Rigid Body Motion (RRBM)**

\[ \dot{\gamma}_i = -V_i^B \dot{\gamma}_{i-1} + \dot{\gamma}_{i-1} V_i^B \]

**Visual Measurements**

Position of feature points relative to object frame \( p_{i,o} = \ldots \)

Position of feature points relative to camera frame \( p_{i,v} = \ldots \)

**Perspective projection**

\[ f_i = \lambda_i \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \]

Estimate \( g_{i,v} \) from \( f_i \) by Visual Motion Observer \[8\]

Definition of Localization

Consider a visual sensor network. Let the world frame be the camera frame. The network is localized when the relative poses of the camera \( g_{i,v} \) for all \( i \in [2, \ldots, N] \) are uniquely determined.

Motivation of Localization Problem

- Estimates \( g_{i,v} \) are taken from their own camera frame
- Estimates \( g_{i,v} \) (measurements) are corrupted by noise
- Object pose needs to be expressed in a common frame (world frame)
- Object pose relative to world frame \( g_{i,w} \) must be unique

Objective

Determine the poses of the camera \( g_{i,v} \) and object relative to the camera \( g_{i,v} \) which are ...

- As close as the estimates
- Satisfy the uniqueness of the object pose

Cost Function

Cost Function

\[ \Phi = \sum_{i=1}^{N} \sum_{j=1}^{N} v(\dot{\gamma}_{i,v}, \dot{\gamma}_{j,v}) \]

Euclidean distance:

\[ \rho(\dot{\gamma}_i, \dot{\gamma}_j) = \frac{1}{2} \| \dot{\gamma}_i - \dot{\gamma}_j \|^2 \]

To avoid the solution will be trivial

\[ g_{i,v} = \hat{g}_{i,v}; \] Pose of the object estimated by camera \( i \)

Fix variables (known pose): \( \hat{g}_{i,v} \)

Decision variables: \( g_{i,v}, \) \( i \in V \) \( V = \{ 3, 4, \ldots, N \} \)

Outline

- Introduction
- Problem Settings
- First Trial in a Simple Model
- Analysis of the Trial

Definition of Localization

Consider 3 cameras and estimate only orientation

Cost function

\[ \phi = \frac{1}{2} || \dot{\gamma}_i - \dot{\gamma}_i ||^2 + \frac{1}{2} || \dot{\gamma}_i - \dot{\gamma}_i ||^2 + \frac{1}{2} || R_{i,v} R_{i,v} \| \}

Orientation

Gradient of the cost function

\[ \nabla g_{i,v} \phi = -R_{i,v}(sk R_{i,v} R_{i,v}) + sk R_{i,v} R_{i,v} + sk R_{i,v} R_{i,v} \]

Gradual estimates (discrete)

\[ R_{i,v}(t+1) = \exp_{R_{i,v}}(-\epsilon \nabla g_{i,v} \phi) \]

\[ R_{i,v}(t+1) = \exp_{R_{i,v}}(-\epsilon \nabla g_{i,v}(t)) \]
### Simulation Settings

Object & 3 Cameras: Stationary

Exponential expression:

\[ R_{0} = e^{	heta_{0}} \]

True orientation:

\[
\begin{align*}
\xi \sin(\theta_{x}) &= [0.2222 \quad 0.4364 \quad 0.7640] \\
\xi \sin(\theta_{y}) &= [-0.6572 \quad 0.2720 \quad 0.4755] \\
\xi \sin(\theta_{z}) &= [0.6257 \quad 0.6937 \quad 0.6975] \\
\end{align*}
\]

Fixed orientation:

\[
R_{0} = \begin{bmatrix}
0.7355 & 0.5092 & 0.0785 \\
0.0785 & 0.7355 & -0.5092 \\
-0.5092 & 0.0785 & 0.7355
\end{bmatrix}
\]

Estimated orientation: \( R_{est} \), \( R_{true} \), \( R_{true} \), Step size: 0.001

True orientation + Random constant noise (Gaussian with mean 0 variance 0.0001)

### Simulation Results

Plot of orientation \( R_{est} \):.

- New estimates \( R_{est} \)
- Estimated by VMO (measurements) \( R_{vmo} \)
- True orientation \( R_{true} \)

**Cost function** \( \phi \):

\[
\phi = 0.7651 \quad 0.7664
\]

**Time [s]**:

\[
2.627e^{-6}
\]

### Discussion & Problems

1. **What are the physical meanings of the estimates?**

   Distance between true and measured orientation:

   \[
   \frac{1}{2}||R_{true} - R_{0}||^2 + \frac{1}{2}||R_{0} - R_{true}||^2 + \frac{1}{2}||R_{0} - R_{true}||^2 = 7.50 \times 10^{-4}
   \]

   Distance between true and calculated orientation:

   \[
   \frac{1}{2}||R_{true} - R_{est}||^2 + \frac{1}{2}||R_{est} - R_{true}||^2 + \frac{1}{2}||R_{est} - R_{true}||^2 = 7.47 \times 10^{-4}
   \]

   New estimates seem to reduce the effectiveness of the noise.

2. **The pose of cameras change when the object moves**.

### Outline

- Introduction
- Problem Settings
- First Trial in Simple Models
- Analysis of the Trial
Lyapunov Method (Position)

**Regard cost function as Lyapunov candidate function**

Lyapunov function considering only position (Orientation fixed)

\[ V = \frac{1}{2} \sum_{i \in V} \| p_i - p_{\text{ref}} \|^2 + \frac{1}{2} \sum_{i \in V, j \in V} \| p_{ij} - p_{\text{ref},ij} \|^2 \]

\[ \hat{p}_{\text{ref}} = R_{\text{ref}} p_i + p_{\text{ref}} \]

Time derivative of the Lyapunov function

\[ \dot{V} = \sum_{i \in V} \left( \dot{p}_i - \dot{p}_{\text{ref}} \right)^T R_{\text{ref},ii} \left( \dot{p}_i - \dot{p}_{\text{ref}} \right) + \sum_{i \in V, j \in V} \left( p_{ij} - p_{\text{ref},ij} \right)^T R_{\text{ref},ij} \left( \dot{p}_{ij} - \dot{p}_{\text{ref},ij} \right) \]

Gradient method

\[ \hat{p}_i = \frac{\partial V}{\partial p_i} = \sum_{i \in V, j \in V} \left( p_{ij} - p_{\text{ref},ij} \right)^T R_{\text{ref},ij} \]

Straightforward interpretation

\[ \dot{V} = \sum_{i \in V} \left( \frac{\partial V}{\partial p_i} \right)^T \frac{\partial V}{\partial p_i} - \sum_{i \in V} \left( \frac{\partial V}{\partial p_i} \right)^2 \leq 0 \]

**Analysis in the case**

\[ \begin{bmatrix} 1 \ 2 \ 3 \ 4 \end{bmatrix}, V' = \{ 3, 4, \ldots , N \} \]

\[ L = \begin{bmatrix} \alpha & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \alpha \end{bmatrix} \]

\[ R_{\text{ref}} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]

\[ p_{\text{ref}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

**Conclusion**

- Problem Settings
  - Setting the cost function
- Orientation Simulation in a Simple Model
  - The cost function is minimized
- Question about what is the meaning of the converge value
- Analysis of the position cost function
  - Converge values are weighted average of the estimates

**Future Works**

- Analysis of the orientation cost function
- Analytical simulations
References


Appendix

Definition of Localization

Problem 6 (Frame localizability) [4]
Given a relative sensing network with reference node 1. The reference frame transformation \( [R_i, \mathbf{r}_i] \) for all \( i \in [2, \ldots, n] \) are uniquely determined by the relative measurements \( r_{1i} \), \( i \)'s orientation relative to node 1 \( p_{1i} \), \( i \)'s position

Definition 1 (Localized network) [5]
A network is said to be localized if there is a set of relative transformations \( \mathbf{a} \), such that, when the reference frame of the first node is fixed to \( \mathbf{a} \), the other absolute poses are uniquely determined.
For any path \( l \) from node 1 to node \( i \), we have \( \mathbf{a} = \mathbf{a}_{1i} \).

Calculation of the Gradient

Consider \( \mathbf{SO}(3) \) is submanifold of \( \mathbb{R}^{3 \times 3} \):

Define \( \phi : \mathbf{SO}(3) \to \mathbb{R} \)

Projection:

\( P_{\text{R}} : \mathbb{R}^{3 \times 3} \to \mathbf{SO}(3) \)

\( P_{\text{R}} \mathbf{Z} = \text{Rk}(\mathbf{R}^T \mathbf{Z}) \)

Tangent space:

\( T_{\text{R}} \mathbf{SO}(3) = \{ \mathbf{R}X \in \mathbb{R}^{3 \times 3} | X \in \mathfrak{so}(3) \} \)

Gradient:

\( \langle \nabla_{\mathbf{R}} \phi(\mathbf{R}), \mathbf{Z} \rangle = D\phi(\mathbf{R})[\mathbf{Z}] \) (2)

Inner product:

\( \langle \mathbf{Z}_1, \mathbf{Z}_2 \rangle = \text{tr}(\mathbf{Z}_1^T \mathbf{Z}_2) \)

From (2) and (3)

\( \nabla_{\mathbf{R}} \phi(\mathbf{R}) = -\mathbf{Q} \)

\( \nabla_{\mathbf{R}} \phi(\mathbf{R}) = P_{\text{R}} \nabla_{\mathbf{R}} \phi(\mathbf{R}) = P_{\text{R}}(-\mathbf{Q}) = -\text{Rk}(\mathbf{R}^T \mathbf{Q}) \)
Simulation: Moving Object

Simulation Settings

3 Cameras: Stationary
Object: Move

New estimates
Estimated by VMO
True orientation

Cost function $\phi$

Simulation: Only Position

Simulation Results

Lyapunov Method (Orientation)

Lyapunov function considering only orientation

V_R = \frac{1}{2} \sum_{i \in V} ||R_{i0} - R_{0i}||^2 + \frac{1}{2} \sum_{i \in V} \sum_{j \neq i} ||R_{i0} - R_{j0}||^2

\xi(R^T Q) = \text{tr}(I - R^T Q) = \frac{1}{2} ||R - Q||^2

From Lemma 4 in [11]

V_R \leq \frac{1}{2} \sum_{i \in V} \sum_{j \neq i} \text{tr}(R_{i0}^T R_{j0} - R_{i0}^T R_{j0} R_{j0}^T R_{i0}^T)

Lyapunov Method (Orientation)

\begin{align*}
\dot{\theta}_x &= -\frac{1}{2} \text{tr}(R_{i0}^T R_{j0} - R_{i0}^T R_{j0} R_{j0}^T R_{i0}^T) \\
\dot{\theta}_y &= -\frac{1}{2} \text{tr}(R_{i0}^T R_{j0} - R_{i0}^T R_{j0} R_{j0}^T R_{i0}^T) \\
\dot{\theta}_z &= -\frac{1}{2} \text{tr}(R_{i0}^T R_{j0} - R_{i0}^T R_{j0} R_{j0}^T R_{i0}^T)
\end{align*}

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Lyapunov function considering only orientation

V_R = \frac{1}{2} \sum_{i \in V} ||R_{i0} - R_{0i}||^2 + \frac{1}{2} \sum_{i \in V} \sum_{j \neq i} ||R_{i0} - R_{j0}||^2

\text{True position}

p_{x0} = \begin{bmatrix} 0.9572 & 0.4854 & 1.3000 \end{bmatrix}^T

p_{y0} = \begin{bmatrix} 0.7809 & 0.6624 & 0.6827 \end{bmatrix}^T

p_{z0} = \begin{bmatrix} -0.1147 & 0.2007 & 1.3340 \end{bmatrix}^T

p_{x3} = \begin{bmatrix} 0.4074 & 0.4329 & 0.0635 \end{bmatrix}^T

Fixed position and orientation

\dot{\theta}_x = \left( \frac{\dot{L}_1}{0} \right)

\xi \sin(\theta_{x0}) = \begin{bmatrix} 0.2222 & 0.4364 & 0.7640 \end{bmatrix}^T

\xi \sin(\theta_{y0}) = \begin{bmatrix} 0.6874 & 0.1123 & 0.6914 \end{bmatrix}^T

\dot{\theta}_x = \frac{1}{2} \text{tr}(R^T Q) = \frac{1}{2} ||R - Q||^2

\xi \sin(\theta_{x3}) = \begin{bmatrix} 0.1186 & 0.3526 & 0.7655 \end{bmatrix}^T

\begin{align*}
\dot{\theta}_x &= -\frac{1}{2} \text{tr}(R_{i0}^T R_{j0} - R_{i0}^T R_{j0} R_{j0}^T R_{i0}^T) \\
\dot{\theta}_y &= -\frac{1}{2} \text{tr}(R_{i0}^T R_{j0} - R_{i0}^T R_{j0} R_{j0}^T R_{i0}^T) \\
\dot{\theta}_z &= -\frac{1}{2} \text{tr}(R_{i0}^T R_{j0} - R_{i0}^T R_{j0} R_{j0}^T R_{i0}^T)
\end{align*}

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\end{align*}

\begin{align*}
\xi \sin(\theta_{x3}) &= \begin{bmatrix} 0.1186 & 0.3526 & 0.7655 \end{bmatrix}^T
\end{align*}
Lyapunov Method (Orientation)

Consideration

\[ \dot{V}_N \leq 0 \quad \text{if} \quad \sum_{N,N\neq M} \left( \sum_{M,M\neq N} \left( \sum_{M,M\neq N} d(R_{NM}, R_{MN}) + d(R_{MN}, R_{NM}) \right) \right) \geq 0 \]

Converge to the set??

\[ \Omega_A := \left\{ (R_{MN}, R_{NM}) \mid \sum_{N,N\neq M} \left( \sum_{M,M\neq N} d(R_{NM}, R_{MN}) + d(R_{MN}, R_{NM}) \right) \leq 0 \right\} \]

\[ \Omega_B := \left\{ (R_{MN}, R_{NM}) \mid \sum_{N,N\neq M} \left( \sum_{M,M\neq N} d(R_{NM}, R_{MN}) + d(R_{MN}, R_{NM}) \right) < 0 \right\} \]

Time Derivative of Orientation Lyapunov Function

Simple model (Ex.1)

\[ \dot{V}_N \leq - \sum_{M,M\neq N} \left( d(R_{NM}, R_{MN}) + d(R_{MN}, R_{NM}) \right) \sum_{N,N\neq M} \left( d(R_{NM}, R_{MN}) + d(R_{MN}, R_{NM}) \right) \]

Graph settings

Simple model (Ex.2)

\[ \dot{V}_N \leq - \sum_{M,M\neq N} \left( d(R_{NM}, R_{MN}) + d(R_{MN}, R_{NM}) \right) \sum_{N,N\neq M} \left( d(R_{NM}, R_{MN}) + d(R_{MN}, R_{NM}) \right) \]

Graph settings