Sensor Localization and Target Estimation in Visual Sensor Networks

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FL11-8-3
8th, June, 2011

Annual Schedule of my Research

- Survey and Problem Settings

  Presented in the FL seminar on May 11th

- First Trial and Evaluation of Proposed Method

  Would like to finish by June

- Simulation (Experimental Verification)

  Would like to finish by the middle presentation on July 25th

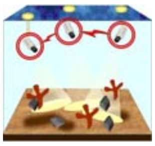
Introduction

Visual Sensor Networks

A network consisting of spatially distributed smart cameras

Application

- Environmental monitoring



- Surveillance

The information of the location of the sensor is needed

Network Localization

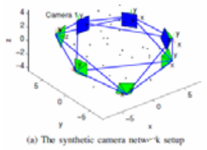
Determining the **relative poses of each sensor** in sensor networks

Introduction

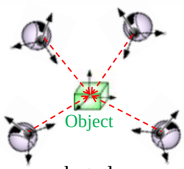
Localization in Camera Networks [5]

Distributed estimation of the pose of cameras

- Use feature points to get the neighboring pose
- Apply only in static scene
- Nonsimultaneous measure and estimation



Objective of my Research



- Use an object to get the relative pose of the camera
- Not only determine the relative pose of each sensor but also estimate the pose of the object

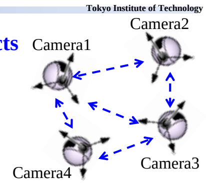
[5] R. Tron and R. Vidal, "Distributed Image-based 3-D Localization of Camera Sensor Networks," *Proc. of the 48th IEEE Conference on Decision and Control*, pp. 901-908, 2009.

Outline

- Introduction
- Problem Settings
- First Trial in a Simple Model
- Analysis of the Trial

Preliminaries

Consider a Visual Sensor Network with Objects



Communication graph $G=(V,E)$

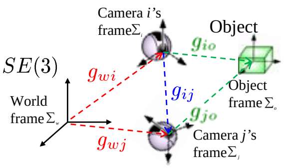
Set of cameras: $V=\{1,\cdots,N\}$

Edge: $(i,j)\in E$ j can get i's information

Neighbor set: $\mathcal{N}_i=\{j\in V\,|\,(i,j)\in E\}$

⇨ Undirected graph

Pose Representation



Pose of camera i : $g_{wi}=(p_{wi},R_{wi})\in SE(3)$

Position: $p_{wi}\in\mathbb{R}^3$

Orientation: $R_{wi}\in SO(3)$

Pose of object : $g_{wo}=(p_{wo},R_{wo})$

Pose of object relative to camera i: $g_{io}=g_{wi}^{-1}g_{wo}=(p_{io},R_{io})$

Relative pose of the camera i and j: $g_{ij}=g_{wi}^{-1}g_{wj}=(p_{ij},R_{ij})$

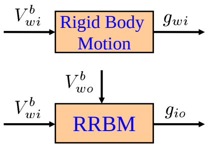
Preliminaries

Rigid Body Motion "∧" : wedge $\mathbb{R}^3 \to so(3)$

$\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi}$ V^b_{wi} : body velocity

Relative Rigid Body Motion (RRBM)

$\dot{g}_{io} = -\hat{V}^b_{wi} g_{wi} + g_{wi}\hat{V}^b_{wo}$



Visual Measurements

Position of feature points relative to object frame p_{ol} $l = 1, \cdots, m$

Position of feature points relative to camera frame

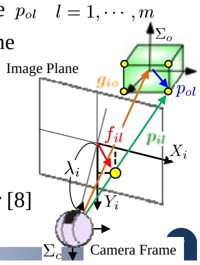
$$p_{il} = g_{io} p_{ol} = \begin{bmatrix} x_{il} & y_{il} & z_{il} \end{bmatrix}^T$$

Perspective projection λ_i :focal length $m \geq 4$

$$f_{il} = \frac{\lambda_i}{z_{il}} \begin{bmatrix} x_{il} \\ y_{il} \end{bmatrix} \quad f_i = \begin{bmatrix} f_{i1}^T & f_{i2}^T & \cdots & f_{im}^T \end{bmatrix}^T$$

Estimate g_{io} from f_i by Visual Motion Observer [8]

Estimated pose: $\bar{g}_{io} = (\bar{p}_{io}, \bar{R}_{io})$



Definition of Localization

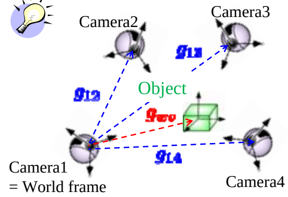
Definition of Localization

Consider a visual sensor network. Let the world frame be the camera1 frame. The network is localized when the relative poses of the camera g_{1i} for all $i \in \{2, \cdots, N\}$ are uniquely determined.

(Inspired by Bullo et al. [4] and Vidal et al. [5])

Motivation of Localization Problem

- Estimates $\bar{g}_{io}$ are taken from their own camera frame
- Estimates $\bar{g}_{io}$ (measurements) are corrupted by noise
- Object pose needs to be expressed in a common frame (world frame)
- Object pose relative to world frame g_{wo} must be unique



Objective

Objective

Determine the poses of the camera $\tilde{g}_{wi}$ and object relative to the camera $\tilde{g}_{io}$ which are ….

- As close as the estimates
- Satisfy the uniqueness of the object pose

Cost Function

$$\Psi = \sum_{i \in V} (\psi(\tilde{g}_{io}^{-1}\bar{g}_{io}) + \sum_{j \in \mathcal{N}_i} \psi(\tilde{g}_{woi}^{-1}\tilde{g}_{woj}))$$

Euclidean distance:

$$\psi(g_1^{-1}g_2) = \frac{1}{2}\|g_1 - g_2\|_F^2 = \frac{1}{2}\|p_1 - p_2\|^2 + \frac{1}{2}\|R_1 - R_2\|_F^2$$

$g_{woi} = g_{wi} g_{io}$: Pose of the object estimated by camera i

To avoid the solution will be trivial

$\tilde{g}_{io} = \bar{g}_{io}$ $\tilde{g}_{wi}\tilde{g}_{io} = \tilde{g}_{wj}\tilde{g}_{jo}$ ⇨ $\Psi = 0$

Fix variables (known pose): $\tilde{g}_{w1} = (I, 0), \tilde{g}_{w2}$

Decision variables: $\tilde{g}_{wi}$ $i \in V'$ $V' = \{3, 4, \cdots, N\}$

Problem Settings

Optimization Problem

$$\min_{\tilde{g}_{io}, \tilde{g}_{wi}} \Psi = \min_{\tilde{g}_{io}, \tilde{g}_{wi}} \sum_{i \in V} (\varphi(\tilde{g}_{io}^{-1}\bar{g}_{io}) + \sum_{j \in \mathcal{N}_i} \varphi(\tilde{g}_{woi}^{-1}\tilde{g}_{woj}))$$

such that $\tilde{g}_{w1} = (I, 0)$ and $\tilde{g}_{w2}$ are known

Procedure of minimization: Gradient method

$$\dot{\tilde{g}}_{io} = -\mathrm{grad}_{\tilde{g}_{io}} \Psi \quad i \in V$$

$$\dot{\tilde{g}}_{wi} = -\mathrm{grad}_{\tilde{g}_{wi}} \Psi \quad i \in V' \quad V' = \{3, 4, \cdots, N\}$$

Division into Position and Orientation Parts

Position

$$\min_{\tilde{p}_{io}, \tilde{p}_{wi}} \varphi = \min_{\tilde{p}_{io}, \tilde{p}_{wi}} \frac{1}{2} \sum_{i \in V} \|\tilde{p}_{io} - \bar{p}_{io} + \tilde{p}_{woi} - \tilde{p}_{woj}\|^2$$

Orientation

$$\min_{\tilde{R}_{io}, \tilde{R}_{wi}} \Phi = \min_{\tilde{R}_{io}, \tilde{R}_{wi}} \frac{1}{2} \sum_{i \in V} \|\tilde{R}_{io} - \bar{R}_{io} + \tilde{R}_{woi} - \tilde{R}_{woj}\|_F^2$$

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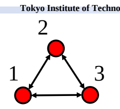
Trial with Simple Models

Consider 3 cameras and estimate only orientation



Graph settings

Cost function

$$\Phi = \frac{1}{2}\|\tilde{R}_{1o} - \bar{R}_{1o}\|_F^2 + \frac{1}{2}\|\tilde{R}_{2o} - \bar{R}_{2o}\|_F^2 + \frac{1}{2}\|\tilde{R}_{3o} - \bar{R}_{3o}\|_F^2 + \frac{1}{2}\|\tilde{R}_{wo1} - \tilde{R}_{wo2}\|_F^2 + \frac{1}{2}\|\tilde{R}_{wo2} - \tilde{R}_{wo3}\|_F^2 + \frac{1}{2}\|\tilde{R}_{wo3} - \tilde{R}_{wo1}\|_F^2$$

$R_{woi} = R_{wi} R_{io}$: Orientation of the object estimated by camera i

Gradient of the cost function

$$\mathrm{grad}_{\tilde{R}_{1o}} \Phi = -\tilde{R}_{1o}(\mathrm{sk}(\tilde{R}_{1o}^T \bar{R}_{1o}) + \mathrm{sk}(\tilde{R}_{wo1}^T \tilde{R}_{wo2} + \mathrm{sk}(\tilde{R}_{wo1}^T \tilde{R}_{wo3}))$$

$$\mathrm{grad}_{\tilde{R}_{w3}} \Phi = -\tilde{R}_{w3}(\mathrm{sk}(\tilde{R}_{w3}^T \tilde{R}_{wo1} \tilde{R}_{3o}^T) + \mathrm{sk}(\tilde{R}_{w3}^T \tilde{R}_{wo2} \tilde{R}_{3o}^T))$$

Update the estimates (discrete)

$$\tilde{R}_{io}(l+1) = \exp_{\tilde{R}_{io}(l)}(-\epsilon \, \mathrm{grad}_{\tilde{R}_{io}(l)} \Phi)$$

$$\tilde{R}_{wi}(l+1) = \exp_{\tilde{R}_{wi}(l)}(-\epsilon \, \mathrm{grad}_{\tilde{R}_{wi}(l)} \Phi)$$

$\tilde{R}_{io}(l)$: Estimates at iteration l

ε : Step size

Simulation Settings

Object & 3 Cameras : Stationary

Exponential expression

$R_{1o} = e^{\hat{\xi}\theta_{1o}} \quad \xi\sin(\theta_{1o}) = (\mathrm{sk}(R_{1o}))^{\vee}$

True orientation

$\xi\sin(\theta_{1o}) = \begin{bmatrix} 0.2222 & 0.4364 & 0.7640 \end{bmatrix}^T$

$\xi\sin(\theta_{2o}) = \begin{bmatrix} -0.6572 & 0.2739 & 0.4755 \end{bmatrix}^T$

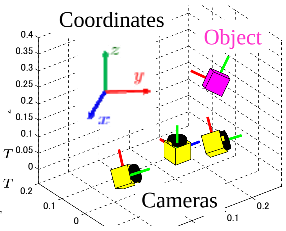
$\xi\sin(\theta_{3o}) = \begin{bmatrix} -0.6869 & 0.0336 & 0.4872 \end{bmatrix}^T$

$\xi\sin(\theta_{w3}) = \begin{bmatrix} 0.6257 & 0.6957 & 0.0975 \end{bmatrix}^T$

Fixed orientation

$R_{w1} = I_3 \quad \xi\sin(\theta_{w2}) = \begin{bmatrix} 0.7355 & 0.5092 & 0.0785 \end{bmatrix}^T$

$R_{w1}R_{1o} = R_{w2}R_{2o} = R_{w3}R_{3o}$



Estimated orientation R_{1o}, R_{2o}, R_{3o}

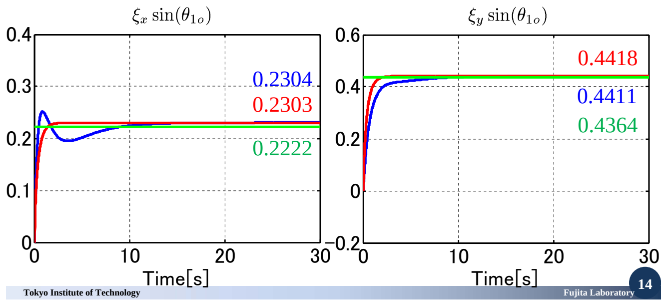
Step size :0.001

True orientation + Random constant noise
(Gaussian with mean 0 variance 0.0001)

Simulation Results

Plot of orientation R_{1o}

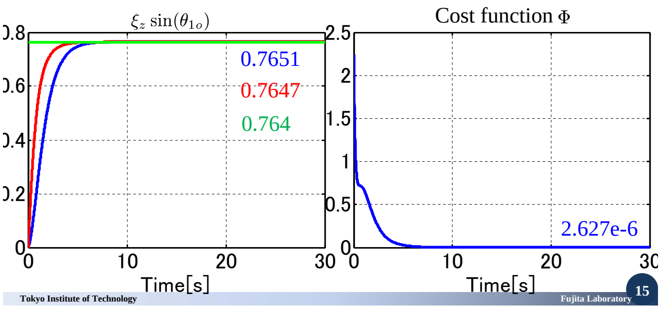
- New estimates $\tilde{R}_{1o}$
- Estimated by VMO (measurements) $\bar{R}_{1o}$
- True orientation R_{1o}



Simulation Results

Plot of orientation R_{1o}

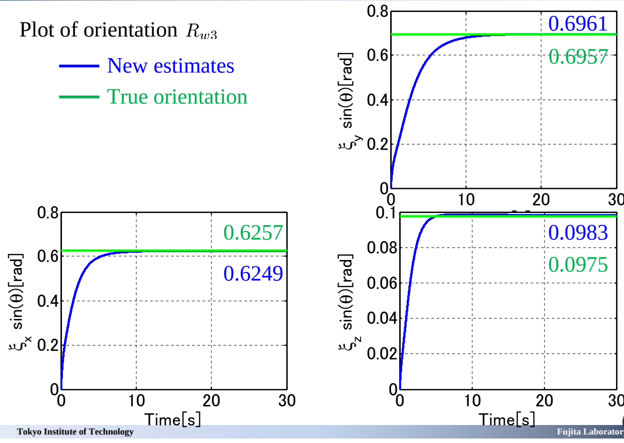
- New estimates $\tilde{R}_{1o}$
- Estimated by VMO (measurements) $\bar{R}_{1o}$
- True orientation R_{1o}



Simulation Results

Plot of orientation R_{w3}

- New estimates
- True orientation



Discussion & Problems

1. **What are the physical meanings of the estimates?**

   Distance between true and measured orientation

   $$\frac{1}{2}\|\bar{R}_{1o} - R_{1o}\|_F^2 + \frac{1}{2}\|\bar{R}_{2o} - R_{2o}\|_F^2 + \frac{1}{2}\|\bar{R}_{3o} - R_{3o}\|_F^2 = 7.50 \times 10^{-4}$$

   Distance between true and calculated orientation

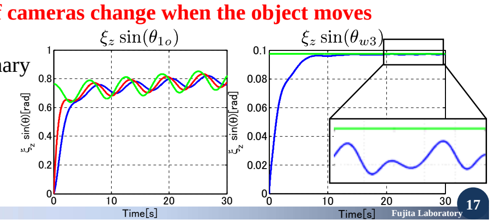
   $$\frac{1}{2}\|\tilde{R}_{1o} - R_{1o}\|_F^2 + \frac{1}{2}\|\tilde{R}_{2o} - R_{2o}\|_F^2 + \frac{1}{2}\|\tilde{R}_{3o} - R_{3o}\|_F^2 = 7.47 \times 10^{-4}$$

   ⇨ New estimates seem to reduce the effectiveness of the noise

2. **The pose of cameras change when the object moves**

Cameras: Stationary

Object :Move



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Lyapunov Method (Position)

Regard cost function as Lyapunov candidate function

Lyapunov function considering only position (Orientation fixed)

$$V_p = \frac{1}{2}\sum_{i\in V}\|\tilde{p}_{io}-\bar{p}_{io}\|^2 + \frac{1}{2}\sum_{i\in V}\sum_{j\in\mathcal{N}_i}\|\tilde{p}_{woi}-\tilde{p}_{woj}\|^2$$

$$p_{woi} = R_{wi}p_{io} + p_{wi}$$

Time derivative of the Lyapunov function

$$\dot{V}_p = \sum_{i\in V}(\tilde{p}_{io}-\bar{p}_{io})^T\dot{\tilde{p}}_{io} + \sum_{i\in V}\sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj})^T\tilde{R}_{wi}\dot{\tilde{p}}_{io} + \sum_{i\in V'}\sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj})^T\dot{\tilde{p}}_{wi} \qquad V' = \{3,4,\cdots,N\}$$

Gradient method

$$\dot{\tilde{p}}_{io} = -\frac{\partial V_p}{\partial\tilde{p}_{io}} \qquad \frac{\partial V_p}{\partial\tilde{p}_{io}} = \tilde{p}_{io}-\bar{p}_{io}+\sum_{j\in\mathcal{N}_i}(\tilde{p}_{io}+\tilde{R}_{wi}^T(\tilde{p}_{wi}-\tilde{R}_{wj}\tilde{p}_{jo}-\tilde{p}_{wj})$$

$$\dot{\tilde{p}}_{wi} = -\frac{\partial V_p}{\partial\tilde{p}_{wi}} \qquad \frac{\partial V_p}{\partial\tilde{p}_{wi}} = \sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj})$$

Lyapunov Method (Position)

$$\begin{aligned}\dot{V}_p &= -\sum_{i\in V}(\tilde{p}_{io}^T-\bar{p}_{io}^T+\sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj})^T\tilde{R}_{wi})(\tilde{p}_{io}-\bar{p}_{io}+\sum_{j\in\mathcal{N}_i}\tilde{R}_{wi}^T(\tilde{p}_{woi}-\tilde{p}_{woj}))\\ &\quad -\sum_{i\in V'}\sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj})^T(\tilde{p}_{woi}-\tilde{p}_{woj})\\ &= -\sum_{i\in V}P_i^TP_i - \sum_{i\in V'}Q_i^TQ_i \\ &\le 0\end{aligned}$$

$$P_i = \tilde{p}_{io}-\bar{p}_{io}+\sum_{j\in\mathcal{N}_i}\tilde{R}_{wi}^T(\tilde{p}_{woi}-\tilde{p}_{woj})$$

$$Q_i = \sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj})$$

Straightforward interpretation

$$\begin{aligned}\dot{V}_p &= \sum_{i\in V}\frac{\partial V_p}{\partial\tilde{p}_{io}}\dot{\tilde{p}}_{io} + \sum_{i\in V'}\frac{\partial V_p}{\partial\tilde{p}_{wi}}\dot{\tilde{p}}_{wi}\\ &= -\sum_{i\in V}\left(\frac{\partial V_p}{\partial\tilde{p}_{io}}\right)^2 - \sum_{i\in V'}\left(\frac{\partial V_p}{\partial\tilde{p}_{wi}}\right)^2\\ &\le 0\end{aligned}$$

Lyapunov Method (Position)

Analysis in the case $\dot{V}_p = 0$

$$P_i = \tilde{p}_{io}-\bar{p}_{io}+\sum_{j\in\mathcal{N}_i}\tilde{R}_{wi}^T(\tilde{p}_{woi}-\tilde{p}_{woj})$$

$$\dot{V}_p = \underbrace{\sum_{i\in V}P_i^TP_i}_{(1)} + \underbrace{\sum_{i\in V'}Q_i^TQ_i}_{(2)} = 0 \qquad Q_i = \sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj})$$

$i\in V$

$$\tilde{p}_{io}-\bar{p}_{io}+\sum_{j\in\mathcal{N}_i}\tilde{R}_{wi}^T(\tilde{p}_{woi}-\tilde{p}_{woj}) = 0 \quad (1)'$$

$$\Leftrightarrow \tilde{R}_{wi}(\tilde{p}_{io}-\bar{p}_{io})+\sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj}) = 0$$

$i\in V' \quad V' = \{3,4,\cdots,N\}$

$$\sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj}) = 0 \quad (2)'$$

Since $V'\subset V$, substitute (2)' to (1)'

$$\tilde{p}_{io} = \bar{p}_{io}$$

$$\Leftrightarrow \tilde{R}_{wi}(\tilde{p}_{io}-\bar{p}_{io}) + \tilde{p}_{wi} - \tilde{p}_{wi} + \sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj}) = 0$$

$$\Leftrightarrow \tilde{p}_{woi}-\bar{p}_{woi}+\sum_{j\in\mathcal{N}_i}(\tilde{p}_{woi}-\tilde{p}_{woj}) = 0$$

$$\tilde{p}_{woi} = \tilde{R}_{wi}\tilde{p}_{io}+\tilde{p}_{wi}$$

$$\bar{p}_{woi} = \tilde{R}_{wi}\bar{p}_{io}+\tilde{p}_{wi}$$

Lyapunov Method (Position)

Let $x_i, \bar{x}_i$ be the element of $\tilde{p}_{woi}, \bar{p}_{woi}$ $\qquad \tilde{p}_{woi} = \begin{bmatrix} x_i \\ \circ \\ \circ \end{bmatrix}$

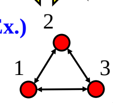
$$x_i - \bar{x}_i + \sum_{j\in\mathcal{N}_i}(x_i - x_j) = 0$$

Let $x = \begin{bmatrix} x_1 & \cdots & x_N \end{bmatrix}^T$ $\bar{x} = \begin{bmatrix} \bar{x}_1 & \cdots & \bar{x}_N \end{bmatrix}^T$ $\qquad V=\{1,\cdots,N\}$

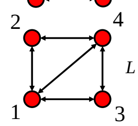
$x - \bar{x} + Lx = 0 \qquad L$: Graph Laplacian

$$\Leftrightarrow (I+L)x = \bar{x} \quad \Leftrightarrow x = (I+L)^{-1}\bar{x}$$

Ex.)



$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad (I+L)^{-1} = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} \quad \tilde{p}_{wo1} = \frac{1}{2}\bar{p}_{wo1}+\frac{1}{4}\bar{p}_{wo2}+\frac{1}{4}\bar{p}_{wo3}$$



$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} \quad (I+L)^{-1} = \begin{bmatrix} 7/15 & 1/5 & 1/5 & 2/15 \\ 1/5 & 2/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 2/5 & 1/5 \\ 2/15 & 1/5 & 1/5 & 7/15 \end{bmatrix}$$

Weighted average

Checked by the simulation (Appendix)

Lyapunov Method (Position)

Characteristic of $(I+L)^{-1}$ (Inspired by Hatanaka [7] , [11])

$$x = (\alpha I + L)^{-1}\bar{x} \qquad \alpha > 0 \qquad \alpha\to 0 \quad (\alpha I + L)^{-1} \to \frac{1}{\alpha n}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$$

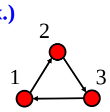
$$V_p = \frac{1}{2}\alpha\sum_{i\in V}\|\tilde{p}_{io}-\bar{p}_{io}\|^2 + \frac{1}{2}\sum_{i\in V}\sum_{j\in\mathcal{N}_i}\|\tilde{p}_{woi}-\tilde{p}_{woj}\|^2$$

$$\dot{\tilde{p}}_{io} = -\alpha(\tilde{p}_{io}-\bar{p}_{io}) - \sum_{j\in\mathcal{N}_i}(\tilde{p}_{io}+\tilde{R}_{wi}^T(\tilde{p}_{wi}-\tilde{R}_{wj}\tilde{p}_{jo}-\tilde{p}_{wj}))$$

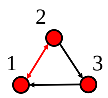
Assumption

The graph is connected and balanced ⇐ heuristic

Ex.)



$$(I+L)^{-1} = \begin{bmatrix} 4/7 & 2/7 & 1/7 \\ 1/4 & 4/7 & 2/7 \\ 2/4 & 1/7 & 4/7 \end{bmatrix}$$



$$(I+L)^{-1} = \begin{bmatrix} 2/3 & 2/9 & 1/9 \\ 1/3 & 4/9 & 2/9 \\ 1/3 & 1/9 & 5/9 \end{bmatrix}$$

Not balanced

Conclusion

Conclusion

- Problem Settings
  - Setting the cost function
- Orientation Simulation in a Simple Model
  - The cost function is minimized
  - Question about what is the meaning of the converge value
- Analysis of the position cost function
  - Converge values are weighted average of the estimates

Future Works

- Analysis of the orientation cost function
- Analytical simulations

References

[1] S. Thrun, W. Burgard and D. Fox, “Probabilistic Robotics,” The MIT Press, 2006.

[2] N. Trwmy, X. S. Zhou, K. X. Zhou and S. I. Roumeliotis, “3D Relative Pose Estimation from Distance-only Measurements,” *Proc. of the 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 1071-1078, 2007.

[3] J. Aspnes, T. Eren, D. K. Goldenberg, A. S. Morse, W. Whiteley, Y. R. Yang, B. D. O. Anderson and P. N. Belhumeur, “A Theory of Network Localization,” *IEEE Transactions on Mobile Computing*, Vol. 5, No. 12, pp. 1663-1678, 2006.

[4] G. Piovan, I. Shames, B. Fidan, F. Bullo and B. D. O. Anderson, “On Frame and Orientation Localization for Relative Sensing Networks,” Automatica, 2011. (submitted)

[5] R. Tron and R. Vidal, “Distributed Image-based 3-D Localization of Camera Sensor Networks,” *Proc. of the 48th IEEE Conference on Decision and Control*, pp. 901-908, 2009.

[6] R. Tron, R. Vidal and A. Terzis, “Distributed Pose Averaging in Camera Sensor Networks via Consensus on SE(3),” *Proc. of the International Conference on Distributed Smart Cameras*, 2008.

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Appendix

Definition of Localization

Definition of localization

Problem 6 (Frame localizability) [4]

Given a relative sensing network with reference node1.
The reference frame transformation $\{R_i^1, p_i^1\}$ for all $i \in \{2,\cdots,n\}$ are uniquely determined by the relative measurements

R_i^1 :i’s orientation relative to node1 p_i^1 :i’s position

Definition 1 (Localized network) [5]

A network is said to be localized if there is a set of relative transformations g_{ij} such that, when the reference frame of the first node is fixed to g_1, the other absolute poses g_i are uniquely determined.
For any path l from node 1 to node i, we have $g_i = g_l \circ g_1$

Calculation of the Gradient

Consider

$$\phi(R) = \frac{1}{2}\|R - Q\|_F^2 = \mathrm{trace}(I - R^TQ) \qquad \phi : SO(3) \to \mathbb{R}$$

SO(3) is submanifold of $\mathbb{R}^{3\times3}$

Define $\bar{\phi}(\cdot) = \phi(\cdot) \qquad \bar{\phi} : \mathbb{R}^{3\times3} \to \mathbb{R}$

$$\mathrm{grad}_R\phi(R) = P_R \mathrm{grad}_R\bar{\phi}(R) \qquad (1)$$

Projection: $P_R : \mathbb{R}^{3\times3} \to T_R SO(3) \qquad P_R Z = R\mathrm{sk}(R^TZ)$

Tangent space: $T_R SO(3) = \{RX \in \mathbb{R}^{3\times3} | X \in so(3)\}$

$$< \mathrm{grad}_R\bar{\phi}(R), Z >= D\bar{\phi}(R)[Z] \qquad (2)$$

$< \cdot, \cdot >$: Inner product $< Z_1, Z_2 >= \mathrm{trace}(Z_1^T Z_2) \quad Z_1, Z_2 \in \mathbb{R}^{3\times3}$

Calculation of the Gradient

Directional derivative

$$D\bar{\phi}(R)[Z] = \lim_{t\to0}\frac{\bar{\phi}(R + tZ) - \bar{\phi}(R)}{t} = -\mathrm{trace}(Z^TQ) \qquad (3)$$

From (2) and (3)

$$\mathrm{grad}_R\bar{\phi}(R) = -Q$$

$$\mathrm{grad}_R\phi(R) = P_R\mathrm{grad}_R\bar{\phi}(R) = P_R(-Q) = -R\mathrm{sk}(R^TQ)$$

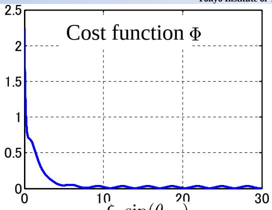
Simulation : Moving Object

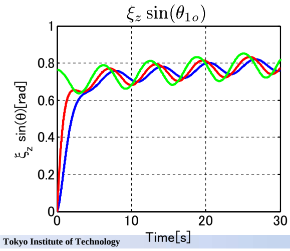
Simulation Settings

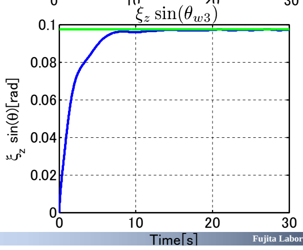
3 Cameras : Stationary

Object :Move

- New estimates
- Estimated by VMO
- True orientation







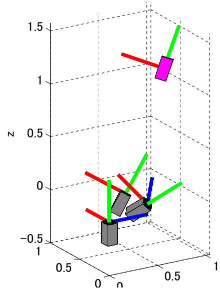
Simulation Settings: Only Position

True position

$$p_{1o} = \begin{bmatrix} 0.9572 & 0.4854 & 1.3003 \end{bmatrix}^T$$
$$p_{2o} = \begin{bmatrix} 0.7809 & 0.6624 & 0.8927 \end{bmatrix}^T$$
$$p_{3o} = \begin{bmatrix} -0.1147 & 0.2007 & 1.3340 \end{bmatrix}^T$$
$$p_{w3} = \begin{bmatrix} 0.4074 & 0.4529 & 0.0635 \end{bmatrix}^T$$

Fixed position and orientation

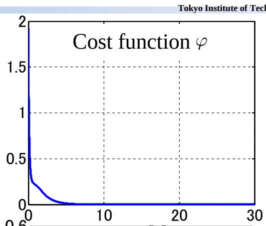
$$g_{w1} = (I_3, 0)$$
$$p_{w2} = \begin{bmatrix} 0.4567 & 0.3162 & 0.0488 \end{bmatrix}^T$$
$$\xi\sin(\theta_{w2}) = \begin{bmatrix} 0.2222 & 0.4364 & 0.7640 \end{bmatrix}^T$$
$$\xi\sin(\theta_{w3}) = \begin{bmatrix} 0.6874 & 0.1123 & 0.6914 \end{bmatrix}^T$$
$$p_{wo1} = p_{wo2} = p_{wo3} \qquad R_{wo1} = R_{wo2} = R_{wo3}$$
$$\xi\sin(\theta_{1o}) = \begin{bmatrix} 0.1186 & 0.3526 & 0.7655 \end{bmatrix}^T$$

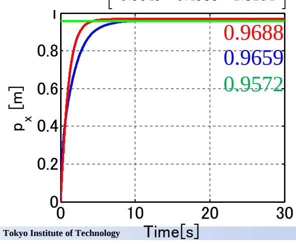


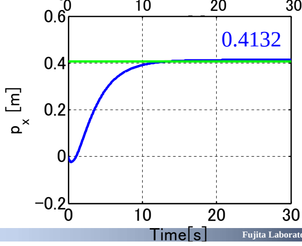
Simulation: Only Position

Simulation Results

$$\tilde{p}_{3o} = \bar{p}_{3o}$$
$$\tilde{p}_{wo1} = \frac{1}{2}\bar{p}_{wo1} + \frac{1}{4}\bar{p}_{wo2} + \frac{1}{4}\bar{p}_{wo3} = \begin{bmatrix} 0.9659 & 0.4941 & 1.3094 \end{bmatrix}^T$$
$$\tilde{p}_{wo2} = \frac{1}{4}\bar{p}_{wo1} + \frac{1}{2}\bar{p}_{wo2} + \frac{1}{4}\bar{p}_{wo3} = \begin{bmatrix} 0.9649 & 0.4938 & 1.3104 \end{bmatrix}^T$$







Lyapunov Method (Orientation)

Lyapunov function considering only orientation

$$V_R = \frac{1}{2}\sum_{i\in V}\|\tilde{R}_{io} - \bar{R}_{io}\|_F^2 + \frac{1}{2}\sum_{i\in V}\sum_{j\in\mathcal{N}_i}\|\tilde{R}_{woi} - \tilde{R}_{woj}\|_F^2$$
$$= \sum_{i\in V}\phi(\bar{R}_{io}^T\tilde{R}_{io}) + \sum_{i\in V}\sum_{j\in\mathcal{N}_i}\phi(\tilde{R}_{woi}^T\tilde{R}_{woj})$$

$$\phi(R^TQ) = \mathrm{tr}(I - R^TQ) = \frac{1}{2}\|R - Q\|_F^2 \qquad R_{woi} = R_{wi}R_{io}$$

Time derivative of the Lyapunov function $\qquad V' = \{3, 4, \cdots, N\}$

$$\dot{V}_R = -\sum_{i\in V}\mathrm{tr}(\dot{\tilde{R}}_{io}^T\bar{R}_{io} + \sum_{j\in\mathcal{N}_i}\dot{\tilde{R}}_{io}^T\tilde{R}_{wi}^T\tilde{R}_{woj}) - \sum_{i\in V'}\sum_{j\in\mathcal{N}_i}\mathrm{tr}(\tilde{R}_{io}^T\dot{\tilde{R}}_{wi}^T\tilde{R}_{woj})$$

Gradient method

$$\dot{\tilde{R}}_{io} = -\mathrm{grad}_{\tilde{R}_{io}}V_R$$
$$\mathrm{grad}_{\tilde{R}_{io}}V_R = -\tilde{R}_{io}\mathrm{sk}(\tilde{R}_{io}^T\bar{R}_{io}) - \tilde{R}_{io}\sum_{j\in\mathcal{N}_i}\mathrm{sk}(\tilde{R}_{woi}^T\tilde{R}_{woj})$$
$$\mathrm{grad}_{\tilde{R}_{wi}}V_R = -\tilde{R}_{wi}\sum_{j\in\mathcal{N}_i}\mathrm{sk}(\tilde{R}_{wi}^T\tilde{R}_{woj}\tilde{R}_{io}^T)$$

Lyapunov Method (Orientation)

$$\dot{V}_R = -\sum_{i\in V}\mathrm{tr}\{(\bar{R}_{io}^T\tilde{R}_{io} + \sum_{j\in\mathcal{N}_i}\tilde{R}_{woj}^T\tilde{R}_{woi})\mathrm{sk}(\tilde{R}_{io}^T\bar{R}_{io} + \sum_{j\in\mathcal{N}_i}\tilde{R}_{woi}^T\tilde{R}_{woj})\}$$
$$-\sum_{i\in V'}\sum_{j\in\mathcal{N}_i}\mathrm{tr}(\tilde{R}_{woj}^T\tilde{R}_{wi}\mathrm{sk}(\tilde{R}_{wi}^T\tilde{R}_{woj}\tilde{R}_{io}^T)\tilde{R}_{io}) \qquad \mathrm{sk}(M) = \frac{1}{2}(M - M^T)$$
$$= -\frac{1}{2}\sum_{i\in V}\mathrm{tr}\{(I - (\bar{R}_{io}^T\tilde{R}_{io})^2) + \sum_{j\in\mathcal{N}_i}(\bar{R}_{io}^T\tilde{R}_{io}\tilde{R}_{woi}^T\tilde{R}_{woj} - \bar{R}_{io}^T\tilde{R}_{io}\tilde{R}_{woj}^T\tilde{R}_{woi})$$
$$+\sum_{j\in\mathcal{N}_i}(\tilde{R}_{woj}^T\tilde{R}_{woi}\tilde{R}_{io}^T\bar{R}_{io} - \tilde{R}_{woj}^T\tilde{R}_{woi}\bar{R}_{io}^T\tilde{R}_{io})$$
$$+\sum_{j\in\mathcal{N}_i}\sum_{k\in\mathcal{N}_i}(\tilde{R}_{woj}^T\tilde{R}_{wok} - \tilde{R}_{woj}^T\tilde{R}_{woi}\tilde{R}_{wok}^T\tilde{R}_{woi})\}$$
$$-\frac{1}{2}\sum_{i\in V'}\sum_{j\in\mathcal{N}_i}\sum_{k\in\mathcal{N}_i}\mathrm{tr}(\tilde{R}_{woj}^T\tilde{R}_{wok} - \tilde{R}_{woj}^T\tilde{R}_{woi}\tilde{R}_{wok}^T\tilde{R}_{woi})$$

From Lemma4 in [11] $\quad \forall R_1, R_2, R_3 \in SO(3) \qquad \mathrm{sym}(M) = \frac{1}{2}(M + M^T)$

$$\frac{1}{2}\mathrm{tr}(R_1^TR_2 - R_1^TR_3R_2^TR_3) \geq \phi(R_1^TR_2) - \phi(R_1^TR_3) + \lambda_{min}(\mathrm{sym}(R_1^TR_3))\phi(R_3^TR_2)$$

Lyapunov Method (Orientation)

$$\dot{V}_R \leq -\sum_{i\in V}\{\phi(\tilde{R}_{io}^T\bar{R}_{io}) - \sum_{j\in\mathcal{N}_i}(\phi(\tilde{R}_{io}^T\bar{R}_{io}) + \phi(\tilde{R}_{woi}^T\tilde{R}_{woj}) - 2\phi(\bar{R}_{woi}^T\tilde{R}_{woj}))$$
$$-\sum_{j\in\mathcal{N}_i}\sum_{k\in\mathcal{N}_i}(\phi(\tilde{R}_{woi}^T\tilde{R}_{woj}) - \phi(\tilde{R}_{woj}^T\tilde{R}_{wok}))\}$$
$$-\sum_{i\in V'}\sum_{j\in\mathcal{N}_i}\sum_{k\in\mathcal{N}_i}(\phi(\tilde{R}_{woi}^T\tilde{R}_{woj}) - \phi(\tilde{R}_{woj}^T\tilde{R}_{wok}))$$
$$-\sum_{i\in V}\{\lambda_{min}(\mathrm{sym}(\tilde{R}_{io}^T\bar{R}_{io}))(\phi(\tilde{R}_{io}^T\bar{R}_{io}) + \sum_{j\in\mathcal{N}_i}\phi(\tilde{R}_{woi}^T\tilde{R}_{woj}))$$
$$-\sum_{j\in\mathcal{N}_i}\lambda_{min}(\mathrm{sym}(\tilde{R}_{woi}^T\tilde{R}_{woj}))(\phi(\tilde{R}_{io}^T\bar{R}_{io}) + \sum_{k\in\mathcal{N}_i}\phi(\tilde{R}_{woi}^T\tilde{R}_{wok}))\}$$
$$-\sum_{i\in V'}\sum_{j\in\mathcal{N}_i}\sum_{k\in\mathcal{N}_i}\lambda_{min}(\mathrm{sym}(\tilde{R}_{woi}^T\tilde{R}_{woj}))\phi(\tilde{R}_{woi}^T\tilde{R}_{wok})$$

$$\dot{V}_R < 0 \text{ if } \begin{cases} \sum_{i\in V}\sum_{j\in\mathcal{N}_i}\phi(\bar{R}_{io}^T\bar{R}_{io}) + \phi(\bar{R}_{woi}^T\bar{R}_{woj}) \geq 2\sum_{i\in V}\sum_{j\in\mathcal{N}_i}\phi(\bar{R}_{woi}^T\bar{R}_{woj}) \\ \sum_{i\in V}\sum_{j\in\mathcal{N}_i}\sum_{k\in\mathcal{N}_i}\phi(\bar{R}_{woi}^T\bar{R}_{woj}) \geq \sum_{i\in V}\sum_{j\in\mathcal{N}_i}\sum_{k\in\mathcal{N}_i}\phi(\bar{R}_{woj}^T\bar{R}_{wok}) \end{cases}$$

Lyapunov Method (Orientation)

Consideration

$$\dot{V}_R < 0 \ \text{if} \begin{cases} \sum_{i\in V}\sum_{j\in \mathcal{N}_i} \phi(\bar{R}_{io}^T\tilde{R}_{io}) + \phi(\tilde{R}_{woi}^T\tilde{R}_{woj}) \geq 2\sum_{i\in V}\sum_{j\in \mathcal{N}_i} \phi(\tilde{R}_{woi}^T\tilde{R}_{woj}) \\ \sum_{i\in V}\sum_{j\in \mathcal{N}_i}\sum_{k\in \mathcal{N}_i} \phi(\tilde{R}_{woi}^T\tilde{R}_{woj}) \geq \sum_{i\in V}\sum_{j\in \mathcal{N}_i}\sum_{k\in \mathcal{N}_i} \phi(\tilde{R}_{woj}^T\tilde{R}_{wok}) \end{cases}$$

Converge to the set??

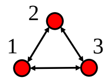
$$\Omega_1 := \{(\bar{R}_{io}, \bar{R}_{wi})_{i\in V} | \sum_{i\in V}\sum_{j\in \mathcal{N}_i} \phi(\bar{R}_{io}^T\bar{R}_{io}) + \phi(\bar{R}_{woi}^T\bar{R}_{woj}) \leq 2\sum_{i\in V}\sum_{j\in \mathcal{N}_i} \phi(\bar{R}_{woi}^T\bar{R}_{woj})\}$$

$$\Omega_2 := \{(\bar{R}_{io}, \bar{R}_{wi})_{i\in V} | \sum_{i\in V}\sum_{j\in \mathcal{N}_i}\sum_{k\in \mathcal{N}_i} \phi(\bar{R}_{woi}^T\bar{R}_{woj}) \leq \sum_{i\in V}\sum_{j\in \mathcal{N}_i}\sum_{k\in \mathcal{N}_i} \phi(\bar{R}_{woj}^T\bar{R}_{wok})\}$$

Time Derivative of Orientation Lyapunov Function

Simple model (Ex.1)

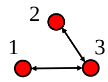
$$\begin{aligned} \dot{V}_R \leq & -\sum_{i\in V}(3\phi(\bar{R}_{io}^T\tilde{R}_{io}) + 2\sum_{j\in\mathcal{N}_i}\phi(\tilde{R}_{woi}^T\tilde{R}_{woj}) - \phi(\bar{R}_{io}^T\tilde{R}_{wi}^T\tilde{R}_{woj})) \\ & - 2(\phi(\tilde{R}_{wo1}^T\tilde{R}_{wo3}) + \phi(\tilde{R}_{wo2}^T\tilde{R}_{wo3}) - \phi(\tilde{R}_{wo1}^T\tilde{R}_{wo2})) \\ & - \sum_{i\in V}\lambda_{min}(\text{sym}(\bar{R}_{io}^T\tilde{R}_{io}))(\phi(\bar{R}_{io}^T\tilde{R}_{io}) + \sum_{j\in\mathcal{N}_i}\phi(\tilde{R}_{woi}^T\tilde{R}_{woj})) \\ & - \lambda_{min}(\text{sym}(\bar{R}_{wo1}^T\tilde{R}_{wo2}))(\phi(\bar{R}_{1o}^T\tilde{R}_{1o}) + \phi(\bar{R}_{2o}^T\tilde{R}_{2o}) + \phi(\tilde{R}_{wo1}^T\tilde{R}_{wo2}) + 3\phi(\tilde{R}_{wo1}^T\tilde{R}_{wo3}) + 2\phi(\tilde{R}_{wo2}^T\tilde{R}_{wo3})) \\ & - \lambda_{min}(\text{sym}(\bar{R}_{wo2}^T\tilde{R}_{wo3}))(\phi(\bar{R}_{2o}^T\tilde{R}_{2o}) + \phi(\bar{R}_{3o}^T\tilde{R}_{3o}) + \phi(\tilde{R}_{wo1}^T\tilde{R}_{wo2}) + 2\phi(\tilde{R}_{wo1}^T\tilde{R}_{wo3}) + 3\phi(\tilde{R}_{wo2}^T\tilde{R}_{wo3})) \\ & - \lambda_{min}(\text{sym}(\bar{R}_{wo1}^T\tilde{R}_{wo3}))(\phi(\bar{R}_{1o}^T\tilde{R}_{1o}) + \phi(\bar{R}_{3o}^T\tilde{R}_{3o}) + 2\phi(\tilde{R}_{wo1}^T\tilde{R}_{wo2}) + \phi(\tilde{R}_{wo1}^T\tilde{R}_{wo3}) + \phi(\tilde{R}_{wo2}^T\tilde{R}_{wo3})) \end{aligned}$$



Graph settings

Simple model (Ex.2)

$$\begin{aligned} \dot{V}_R \leq & -\sum_{i\in V}(2\phi(\bar{R}_{io}^T\tilde{R}_{io}) + 2\sum_{j\in\mathcal{N}_i}\phi(\tilde{R}_{woi}^T\tilde{R}_{woj}) - \phi(\bar{R}_{io}^T\tilde{R}_{wi}^T\tilde{R}_{woj})) \\ & - (3\phi(\tilde{R}_{wo1}^T\tilde{R}_{wo3}) + 3\phi(\tilde{R}_{wo2}^T\tilde{R}_{wo3}) - 4\phi(\tilde{R}_{wo1}^T\tilde{R}_{wo2})) \\ & - \sum_{i\in V}\lambda_{min}(\text{sym}(\bar{R}_{io}^T\tilde{R}_{io}))(\phi(\bar{R}_{io}^T\tilde{R}_{io}) + \sum_{j\in\mathcal{N}_i}\phi(\tilde{R}_{woi}^T\tilde{R}_{woj})) \\ & - \lambda_{min}(\text{sym}(\bar{R}_{wo1}^T\tilde{R}_{wo2}))(\phi(\bar{R}_{1o}^T\tilde{R}_{1o}) + \phi(\bar{R}_{2o}^T\tilde{R}_{2o}) + 2\phi(\tilde{R}_{wo1}^T\tilde{R}_{wo2}) + \phi(\tilde{R}_{wo1}^T\tilde{R}_{wo3}) + 2\phi(\tilde{R}_{wo2}^T\tilde{R}_{wo3})) \\ & - \lambda_{min}(\text{sym}(\bar{R}_{wo2}^T\tilde{R}_{wo3}))(\phi(\bar{R}_{2o}^T\tilde{R}_{2o}) + \phi(\bar{R}_{3o}^T\tilde{R}_{3o}) + 2\phi(\tilde{R}_{wo1}^T\tilde{R}_{wo2}) + \phi(\tilde{R}_{wo1}^T\tilde{R}_{wo3}) + 2\phi(\tilde{R}_{wo2}^T\tilde{R}_{wo3})) \end{aligned}$$



Graph settings