Inter-Vehicular Distance Control in Vehicle Networks

Takuto Takagi
FL11-8-2
8th, June, 2011

Problem Description

Situation
• Considering n vehicles in an one lane highway
Assumption
(A1) Vehicles run only forward
(A2) Homogeneous vehicles platoon
Intelligent vehicle ([1],[2])

A vehicle equipped with control systems that can sense the environment around the vehicles and that result in a more efficient vehicle operation either by assisting the driver or by taking complete control of the vehicle.

Assumption
(A3) Vehicles take complete control

Objective
Using above situation, we'd like to derive analytically optimal platoon control strategies

Background

Highway congestion
Highway congestion is imposing an intolerable burden on urban residents.

Congestion occurs when vehicle's velocity variation propagates to following vehicles.
It is difficult for human drivers to recognize tiny changing of the preceding vehicle's velocity.

Approaches

There are various approaches to improve congestion.

They can be classified as:
• Macro perspective
• Micro perspective

Macro perspective:
• On-ramp control
• Transportation Network

Micro perspective:
• Vehicle Platoons Control

P. Varaiya, "Smart Cars on Smart Roads: Problems of Control".

Modeling

Vehicle model

\[
\begin{pmatrix}
\dot{x}_i \\
\dot{v}_i
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
x_i \\
v_i
\end{pmatrix} + \begin{pmatrix}
0 \\
B
\end{pmatrix} u_i
\]

Assumption
• Don't consider vehicle dynamics

\[ P(s) = \frac{1}{s} \]

Platoon model

\[
\begin{pmatrix}
\dot{x} \\
\dot{v}
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
x \\
v
\end{pmatrix} + \begin{pmatrix}
0 \\
B
\end{pmatrix} u
\]

\[ q = A'q + B'u \]

\[ q \geq 0 \]

Optimal Strategy

Platoon control

• Vehicles have sensors to measure position and velocity data of own vehicle.

• Vehicles can communicate each other and infrastructure.

Assumption

• Considering platoon position and velocity control

Setting ith vehicle's input from above

\[ u_i = K_i(s)(x_0 - S_i) + K_i(s)S_i \]

\[ S_i \text{ : Reference of relative distance from infrastructure} \]

\[ e_{ij} = \sum_{i=1}^{n} \delta_{ij} x_i - \sum_{i=1}^{n} \delta_{ij} x_i \]

\[ \delta_{ij} = \begin{cases} 1 & \text{if communicating} \\ 0 & \text{otherwise} \end{cases} \]

Objective

Using the situation, we'd like to derive analytically optimal relative distance control strategies in a platoon.

Control input

\[ u_i = K_i(s)(x_0 - S_i) + K_i(s)S_i \]

\[ e_{ij} = L_i x_i - L_i x_j \]

Assumption

\[ A' = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \]

There exists some \( K \) such that

\[ u = K_i(s)(x_0 - S_i) + K_i(s)S_i \]

Platoon model

\[ \begin{pmatrix}
\dot{x}_i \\
\dot{v}_i
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
x_i \\
v_i
\end{pmatrix} + \begin{pmatrix}
0 \\
B
\end{pmatrix} u_i
\]

\[ q = A'q + B'u \]

\[ q \geq 0 \]

Platoon model

\[ \dot{q} = A'q + B'S \]

\[ q \geq 0 \]

\( S \leq 0 \)
Consider the finite horizon optimization problem (*) under the dynamics (**).

Proposition 4-1: Consider the finite horizon optimization problem (*) under the dynamics (**).

The Nth stage Optimal value of the DP iteration is (***)

\[ J_N(s) = \min_{\{u'_N, ..., u'_1\}} \{ -D_Ns_N + B_Nu_N \} \]

where

\[ S_N = (S'_N, ..., S'_1) \]

The optimal control at time \( k \), for \( k = 0, 1, ..., N-1 \) is \( u'_N = (u'_N, ..., u'_1) \)

Meaning of the optimal solution

\[ \text{arg} \max \{ \sum_{i=1}^{N} c^i d'^i + r^i \} \]

The largest value of h is the vehicle which connects most numbers of other vehicles.