Inter-Vehicular Distance Control in Vehicle Networks

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FL11-8-2
8th, June, 2011

Background

Highway congestion

Highway congestion is imposing an intolerable burden on urban residents

Congestion occurs when vehicle's velocity variation propagates to following vehicles

It is difficult for human drivers to recognize tiny changing of the precede vehicle's velocity

Approaches

There are various approaches to improve congestion
They can be classified as
macro perspective and micro perspective

Macro perspective: On-ramp control, Transportation Network
Micro perspective: Vehicle Platoon Control

P. Varaiya, "Smart Cars on Smart Roads: Problems of Control", *IEEE Transactions on Automatic Control*, Vol. 38, No. 2, Feb. 1993

Problem Description

Situation

・Considering n vehicles in an one lane highway

Assumption
(A1)Vehicles run only forward
(A2)Homogeneous vehicles platoon

Intelligent vehicle(IV)[1],[2]

A vehicle equipped with control systems that can sense the environment around the vehicles and that result in a more efficient vehicle operation by assisting the driver or by taking complete control of the vehicle

Assumption
(A3)Vehicles take complete control

Objective

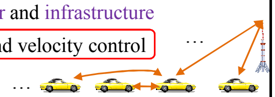
Using above situation,
we'd like to derive analytically optimal platoon control strategies

Optimal Strategy

Platoon control

・Vehicles have sensors to measure position and velocity data of own vehicle
・Vehicles can communicate each other and infrastructure

Considering platoon position and velocity control



Setting ith vehicle's input from above

$$u_i = K_p(s)(e_{x,i} - S_i) + K_v(s)e_{v,i}$$

S_i : Reference of relative distance from infrastructure

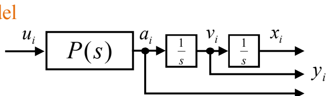
$$e_{x,i} = \sum_{k=1}^{n}\delta_k x_k - \left(\sum_{k=1}^{n}\delta_k\right)x_i \quad e_{v,i} = \sum_{k=1}^{n}\delta_k v_k - \left(\sum_{k=1}^{n}\delta_k\right)v_i \quad \delta_k = \begin{cases} 1 : \text{Communicating} \\ 0 : \text{Others} \end{cases}$$

Objective'

Using the situation,
we'd like to derive analytically optimal relative distance control strategies in a platoon

Modeling

Vehicle model



Assumption
(A4)Don't consider vehicle dynamics
$\Leftrightarrow P(s) = 1$, $a_i = u_i$

$$\Rightarrow \begin{bmatrix} \dot{x}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_i \Leftrightarrow \dot{q}_i = Aq_i + Bu_i \quad q_i = [x_i, v_i]^T$$

$$(\text{A1}) \Leftrightarrow q_i \geq 0$$

Platoon model

$$(\text{A2}) \Leftrightarrow \begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix}u \Leftrightarrow \dot{q} = A'q + B'u \quad q = [x, v]^T, \quad q \geq 0$$

$x = [x_1, \cdots, x_n]^T \quad u = [u_1, \cdots, u_n]^T$
$v = [v_1, \cdots, v_n]^T$

Modeling

Control input

$$u_i = K_p(s)(e_{x,i} - S_i) + K_v(s)e_{v,i} \quad e_{x,i} = \sum_{k=1}^{n}\delta_k x_k - \left(\sum_{k=1}^{n}\delta_k\right)x_i \quad e_{v,i} = \sum_{k=1}^{n}\delta_k v_k - \left(\sum_{k=1}^{n}\delta_k\right)v_i$$

Assumption
(A5) K_p, K_v are constant gain which satisfy (A1)

There exists some L_g such that

$$u = K_p(e_x - S) + K_v e_v$$
$$e_x = L_g x \quad e_v = L_g v$$

ex) $e_{x,1} = (-x_1 + x_3)$, $e_{x,2} = (x_1 - x_2)$, $e_{x,3} = (x_1 - x_3) + (x_2 - x_3)$ $\Rightarrow e_x = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}x$

Assumption
(A6) L_g is time invariant

Platoon model

$$\dot{q} = A'q + B'u$$

$$\Leftrightarrow \dot{q} = \begin{bmatrix} 0 & I \\ K_p L_g & K_v L_g \end{bmatrix}q - K_p\begin{bmatrix} 0 \\ I \end{bmatrix}S \Leftrightarrow \dot{q} = A''q - B''S, \quad q \geq 0$$

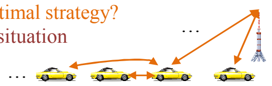
Platoon model

$$\dot{q} = A''q + B''S \quad q = [x^T, v^T]^T \geq 0 \quad S \leq 0$$

Optimization Problem

What is optimal strategy?

Current situation



There are many connection vehicle to vehicle or to infrastructure

Want to reduce communication

⇨ $S = [\cdots, 0, S_i, 0 \cdots]^T \quad L_g = \begin{bmatrix} \ddots & & 0 \\ 1 & -1 & \\ & \ddots & \ddots \end{bmatrix}$

Proposition 4-1[3]

Consider the finite horizon optimization problem(*) under the dynamics(**).
The Nth stage Optimal value of the DP iteration is (***).
The optimal control at time k, for k=0,1,…,N-1 is $u^*_{N-k} = (u^*_{1,N-k}, \cdots, u^*_{n,N-k})$
where $u^*_{i,N-k} = \begin{cases} M & i = \arg\max(h_{N-k}) \\ 0 & otherwise \end{cases}$

This can be used for the strategy

$$\max_u \left\{ J(x) = vx_N + \sum_{k=0}^{N-1} \beta^k (vx_k - cu_k) \right\} \quad (*)$$

$$J_N(x) = v^T \widetilde{A}_N x + \sum_{k=0}^{N-1} \beta^k h^T_{N-k} u_{N-k} \quad (***)$$

$$\left. \begin{aligned} & x_{k+1} = Ax_k + Bu_k \\ & x_k \geq 0 \quad u_k \geq 0 \\ & \sum_i^n u^l_{i,k} \leq M \end{aligned} \right\} (**)$$

Optimization Problem

Objective''

Confirming proposition 4-1 is applicable to the platoon model

Platoon model

$\dot{q} = A''q + B''S \quad q \geq 0 \quad S \leq 0$

Discretization ⇨ $\dot{q}_k = A''_d q_k + B''_d S_k \quad q_k \geq 0 \quad S_k \leq 0$

Assumption

(A7) $\sum_{i=1}^{n} -S_{i,k} \leq M \quad M > 0$

(**)'

Optimization problem

$$\min_s \left\{ J_N = c^T q_N + \sum_{k=0}^{N-1} (\|Cq_k\|_1 + \|RS_k\|_1) \right\} \ (*)'$$

$$\Leftrightarrow \max_{s'} \left\{ J_N = c^T q_N + \sum_{k=0}^{N-1} (\|Cq_k\|_1 + \|RS'_k\|_1) \right\} \quad S \geq 0$$

⇨ $J_N = \widetilde{A}''_{d,N} q_0 + \sum_{k=1}^{N} h_k S_{N-k}$ (Appendix 1)(***)'

Optimization Problem

Check sufficient condition

DP iteration has an assumption $J_N \geq J_{N-1} \geq \cdots \geq J_0$

⇨ $J_k - J_{k-1} = c^T q_k + r^T S_{k-1} \geq 0$

The objective function always satisfy the assumption

Proposition 4-1'

Consider the finite horizon optimization problem(*)'under the dynamics(**)'.
The Nth stage Optimal value of the DP iteration is (***)'.
The optimal control at time k, for k=0,1,…,N-1 is $S^*_{N-k} = (S^*_{1,N-k}, \cdots, S^*_{n,N-k})$
where $S^*_{i,N-k} = \begin{cases} M & i = \arg\max(h_k) \\ 0 & otherwise \end{cases}$

Meaning of the optimal solution

$$\arg\max(h_k) = \max\left(\left(\sum_{l=0}^{k} c^T A''^l_d \right) B''_d + r^T \right)$$

The largest value of h is
the vehicle which connects most numbers of other vehicles

Summary

Assumption

(A1)Vehicles run only forward
(A2)Homogeneous vehicles platoon
(A3)Vehicles take complete control
(A4)Don't consider vehicle dynamics
(A5) K_p, K_v are constant gain controller which satisfy (A1)
(A6) L_g is time invariant
(A7) $\sum_{i=1}^{n} -S_{i,k} \leq M \quad M > 0$

Result

$S^*_{i,N-k} = \begin{cases} M & i = \arg\max(h_k) \\ 0 & otherwise \end{cases}$

Vision

(A7) $\sum_{i=1}^{n} S_{i,k} \leq M \quad \sum_{i=1}^{n} S_{i,k} \leq M(t) \rightarrow \sum_{i=1}^{n} S_{i,k} \leq M(x,v)$?

(A4) $\rightarrow P(s)$

Appendix

Appendix1

$$\max_{s'} \left\{ J_N = c^T q_N + \sum_{k=0}^{N-1} (\|Cq_k\|_1 + \|RS'_k\|_1) \right\}$$

$$\Leftrightarrow \max_{s'} \left\{ J_N = c^T q_N + \sum_{k=0}^{N-1} (c^T q_k + r^T S'_k) \right\} \quad C = diag(c^T) \quad c, r \geq 0, \quad R = diag(r^T)$$

$$c^T q_N + \sum_{k=0}^{N-1} c^T q_k + r^T S_k = c^T q_0 + \sum_{J=1}^{N} \left(c^T A''^J_d q_0 + c^T \sum_{k=0}^{J-1} (A''^k_d B''_d S_{N-k-1}) \right) + r^T \sum_{k=0}^{N-1} S_k$$

Let $\widetilde{A}''_{d,l} = \sum_{k=0}^{l} c^T A''^k_d$ then

$$= \widetilde{A}''_{d,N} q_0 + \sum_{k=1}^{N} (\widetilde{A}''_{d,k-1} B''_d + r^T) S_{N-k}$$

$$= \widetilde{A}''_{d,N} q_0 + \sum_{k=1}^{N} h_k S_{N-k} \qquad h_k = \widetilde{A}''_{d,k-1} B''_d + r^T$$

$$J_N = \widetilde{A}''_{d,N} q_0 + \sum_{k=1}^{N} h_k S_{N-k}$$