Design of Visual Motion Observer with Spherical Camera

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Outline

- Introduction and Background
  - Visual Motion Observer
  - Cameras
- Spherical Camera Model
  - Consideration from Theory (Spherical Camera Model)
  - Omnidirectional Camera using Spherical Camera Model
- Conclusion and Feature Works

Introduction

Visual Sensor

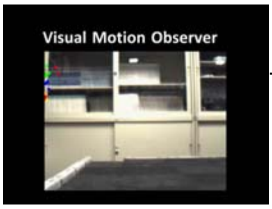
Camera sensor (visual information)

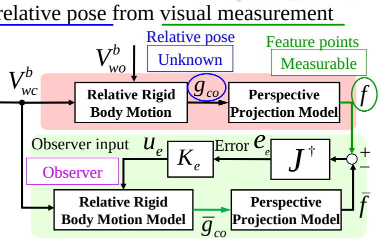
Advantages
- Rich information
- Easy understanding

Application
- Target tracking
- Monitoring

Visual Motion Observer (VMO)[1]

An observer to estimate the relative pose from visual measurement

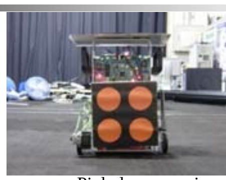




[1] M. Fujita, H. Kawai and M. W. Spong, *IEEE Transactions on Control Systems Technology*, Vol. 15, No.1, pp. 40-52, 2007.

Introduction : Camera

Pinhole Camera
- A simple camera
- A view like picture



Pinhole camera image

Catadioptric Camera

Combination of a pinhole camera and an omni (catadioptric)mirror

Advantage
- A wide view image
- One camera uses

Catadioptric camera image

Review : Pinhole Camera Model

Pinhole Camera Model

The center of pinhole camera $\boldsymbol{C}$

A point in a space : $\boldsymbol{X} = (X,\ Y,\ Z)^T$

A point on image plane : $\boldsymbol{f} = (u,\ v)^T$

Focus length : λ

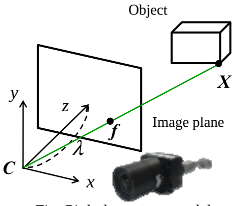


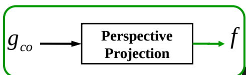
Fig. Pinhole camera model

Relation from $\boldsymbol{X}$ to $\boldsymbol{m}$

$$\begin{pmatrix} \boldsymbol{f} \\ 1 \end{pmatrix} = \frac{1}{Z} \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{X} \\ 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} u \\ v \end{pmatrix} = \frac{\lambda}{Z} \begin{pmatrix} X \\ Y \end{pmatrix}$$

A block of Perspective Projection



Review : Catadioptric Camera Model

Catadioptric Camera Model

Combination of pinhole camera and Catadioptric mirror

A point in a space : $\boldsymbol{X} = (X,\ Y,\ Z)^T$

A point on hyperboloid C^2: $\boldsymbol{x} = (x,\ y,\ z)^T$

A point on image plane : $\boldsymbol{f} = (u,\ v)^T$

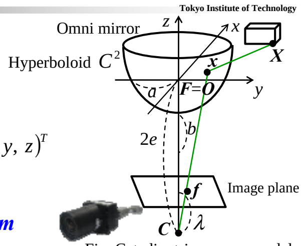


Fig. Catadioptric camera model

Relation from $\boldsymbol{X}$ to $\boldsymbol{x}$ and from $\boldsymbol{x}$ to $\boldsymbol{m}$

$$\begin{pmatrix} \boldsymbol{f} \\ 1 \end{pmatrix} = \frac{1}{z+2e} \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \\ 1 \end{pmatrix}, \quad \boldsymbol{x} = \frac{a^2}{b|\boldsymbol{X}| - eZ} \boldsymbol{X}$$

$$\left(e = \sqrt{a^2 + b^2} \right)$$

a, b : Parameters of hyperbola

e : Focus length of hyperbola

A feature point on image plane

$$u = \frac{\lambda a^2 X}{(a^2 - 2e^2)Z + 2be|\boldsymbol{X}|}, \quad v = \frac{\lambda a^2 Y}{(a^2 - 2e^2)Z + 2be|\boldsymbol{X}|}$$

Previous Work : Visual Motion Observer

Pinhole Camera

A pinhole camera

Perspective Projection
Image Jacobian $J^{\dagger}$: Pinhole camera model

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE Transactions on Control Systems Technology*, Vol. 15, No1, pp. 40-52, 2007.

Catadioptric Camera

An omni mirror

Perspective Projection
Image Jacobian $J^{\dagger}$: Catadioptric camera model

[2] H. Kawai, T. Murao and M. Fujita, "Visual Motion Observer-based Pose Control with Panoramic Camera via Passivity Approach," *Proc. of the 2010 American Control Conference*, Baltimore, Maryland, USA, 12nd, June-July, pp. 4534-4539, 2010

+
A pinhole camera

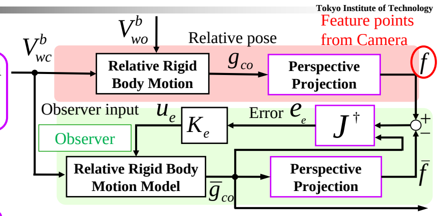
Note : The other perspective projection and image jacobian used

Is there a standard camera model ?

Approaches to Modified Visual Motion Observer

Approach 1

- Perspective projection
  Spherical Camera



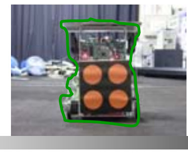
Approach 2

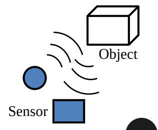
- Catadioptric Camera

- Modification of my previous work : Speeded-Up Robust Features

Others

- Snake （Detect object's edge)
- Sonic wave sensor





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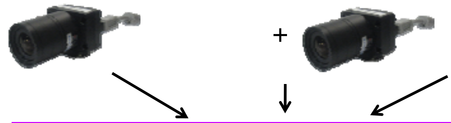
Central Camera

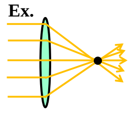
Pinhole Camera | **Catadioptric Camera** | **Dioptric Camera**

Pinhole camera | Omni mirror + Pinhole camera | Ex.



Central camera :
A collection of all rays incident to one points

Ex.



Is there a standard camera model ?

A standard camera model expressing all central camera

Spherical Camera Model[3]

[3]鳥居, 井宮, "画像理解のための中心カメラ系の解析," 情報処理学会研究報告. CVIM, [コンピュータビジョンとイメージメディア] 2006(51), pp. 243-258, 一般社団法人 情報処理学会, May 18, 2006.

Approaches to Modified Visual Motion Observer

Spherical Camera

[3]鳥居, 井宮, "画像理解のための中心カメラ系の解析," 情報処理学会研究報告. CVIM, [コンピュータビジョンとイメージメディア] 2006(51), pp. 243-258, 一般社団法人 情報処理学会, May 18, 2006.

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Spherical Camera Model [3]

Definition : Spherical Camera Model

A spherical camera consists of a camera center and a surface of a unit sphere whose center is the camera center.

The spherical camera collects rays in a space at the camera center and generates an image on the surface of the unit sphere.

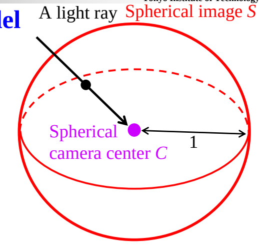


Fig. Spherical camera model

The center of spherical camera : **Spherical camera center : C**

The surface of a unit sphere : **Spherical image : S**

[3]鳥居, 井宮, "球面カメラの多視点幾何学,"電子情報通信学会技術研究報告. NLC, 言語理解とコミュニケーション 105(300), pp. 29-34, 社団法人 電子情報通信学会, Sep. 15, 2005 他[4], [5], [6]

Spherical Camera Model

Spherical coordinate system

A point in a space : $\boldsymbol{X} = (X,\ Y,\ Z)^T \in \boldsymbol{R}^3$

A point on spherical image $\boldsymbol{S}$: $\xi = (x,\ y,\ z)^T$

The intersection of the light ray and spherical image yields a point :

$$\xi = \boldsymbol{X}/|\boldsymbol{X}|$$

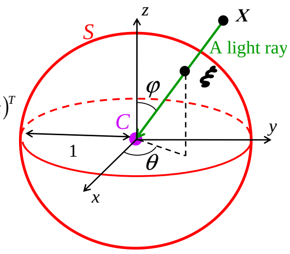


Fig. Spherical coordinate system

A point $\boldsymbol{x} = (x,\ y,\ z)^T$ expressed the spherical coordinate system

$$\boldsymbol{x} = (\cos\theta\sin\varphi,\ \sin\theta\sin\varphi,\ \cos\varphi)^T \quad 0 \le \theta < 2\pi,\ 0 \le \varphi < \pi$$

$$\rightarrow I(\theta,\ \varphi)$$

[3]鳥居, 井宮, "球面カメラの多視点幾何学,"電子情報通信学会技術研究報告. NLC, 言語理解とコミュニケーション 105(300), pp. 29-34, 社団法人 電子情報通信学会, Sep. 15, 2005 他[4], [5], [6]

Spherical Image Transform for Standardization

Pinhole-to-spherical image transform

A nonlinear function h :

The one-to-one mapping from a point $\boldsymbol{f} \in \boldsymbol{R}^2$ to a point $\xi \in S^2$ on the unit sphere

$$h : \boldsymbol{f} \rightarrow \xi$$

We set the center of a unit sphere C on the center of pinhole camera C_p

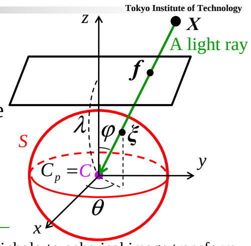


Fig. Pinhole-to-spherical image transform

We get follow result from pinhole camera model and spherical coordinate expression

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \lambda\tan\varphi\cos\theta \\ \lambda\tan\varphi\sin\theta \end{pmatrix} \quad \because \left(\begin{array}{l} (X,\ Y,\ Z) = (\cos\theta\sin\varphi,\ \sin\theta\sin\varphi,\ \cos\varphi), \\ \begin{pmatrix} u \\ v \end{pmatrix} = \dfrac{\lambda}{Z}\begin{pmatrix} X \\ Y \end{pmatrix} \end{array} \right)$$

The pinhole-camera image $I(u,v)$ is transformed to the image $I_S(\theta,\varphi)$ on a sphere

$$I(u,v) = I(\lambda\tan\varphi\cos\theta, \lambda\tan\varphi\sin\varphi) = I_S(\theta,\varphi)$$

Pinhole to Spherical Image Transformation

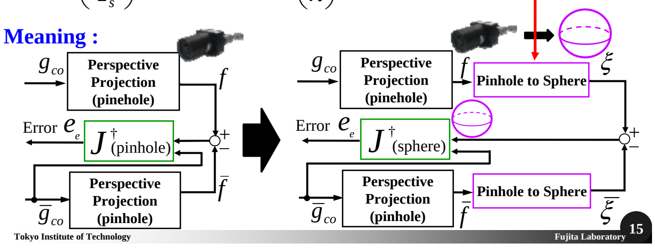
Relation between

a point on the image plane $f = (u \quad v)^T = (\lambda\tan\varphi\cos\theta \quad \lambda\tan\varphi\sin\theta)^T$

and a point on sphere surfacea $\xi = (\cos\theta\sin\varphi,\ \sin\theta\sin\varphi,\ \cos\varphi)$

$$\xi = \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} = \frac{1}{\sqrt{u^2 + v^2 + \lambda^2}} \begin{pmatrix} u \\ v \\ \lambda \end{pmatrix}$$

Meaning :



Spherical Image Transform for Standardization

Catadioptric-to-spherical image transform

A nonlinear function h :

The one-to-one mapping from a point $\boldsymbol{p} \in C^2$ to a point $\xi \in S^2$ on the unit sphere

$$h : \boldsymbol{p} \rightarrow \xi$$

We set the center of a unit sphere C at the focus of the quadric mirror F

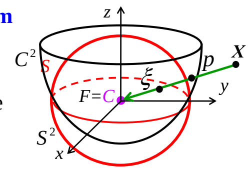


Fig. Catadioptric-to-spherical image transform

We get follow result from Catadioptric camera model and spherical coordinate expression

$$u = \frac{\lambda a^2\cos\theta\sin\varphi}{(a^2 - 2e^2)\cos\varphi + 2be}, \quad v = \frac{\lambda a^2\sin\theta\sin\varphi}{(a^2 - 2e^2)\cos\varphi + 2be}$$

The Catadioptric-camera image $I(u,v)$ is transformed to the image $I_S(\theta,\varphi)$ on a sphere

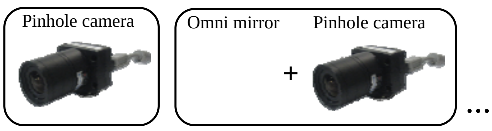
$$I(u,v) = I_S(\theta,\varphi)$$

To be continued …

Recap : Sphere Camera Model

Sphere Camera Model

A standard camera model expressing all central camera



A standard Visual Motion Observer obtained?!

Application to Visual Motion Observer

- Theory from Perspective Projection to Image Jacobian
- Derivation of Passivity

Approaches to Modified Visual Motion Observer

Catadioptric Camera

[7]G. Lopez-Nicolas, J.J. Guerrero, C. Sagues, "Multiple homographies with omnidirectional vision for robot homing," *Robotics and Autonomous Systems*, Vol.58, Issue6, pp. 773-783, 30 June, 2010.

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Setting

The geometry of the imaging system

The motion of the robot :

- x-y plane linear movement
- Rotation on z axis

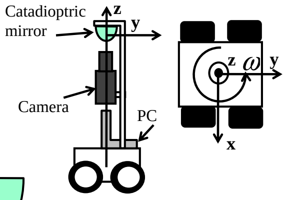
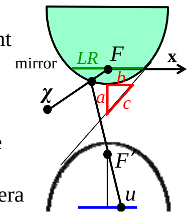


Fig. Machine Setting and Frame

The omni-directional camera model

$\chi \in R^3$: A world point

$\boldsymbol{u} = (u \quad v \quad 1)^T$
A point on the image acquired by the omnidirectional camera



Mirror parameters

$a = 28.0950\text{mm}$

$b = 23.4125\text{mm}$

$c = \sqrt{a^2 + b^2} = 36.5715\text{mm}$

$\overline{FF'} = 2c = 28.0950\text{mm}$

$LR = 2b^2/a = 39.0208\text{mm}$

Camera Model

Unified Sphere Model

χ_s : A point on the unit sphere

χ is projected in a unit sphere centered on C as $\chi_s \in R^3$

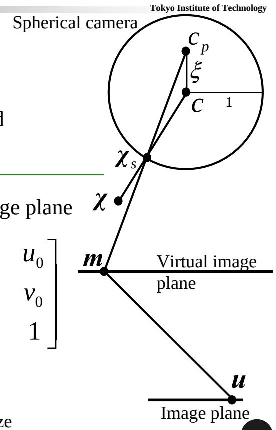
$\boldsymbol{m} = (x \quad y \quad 1)^T$: A point on a virtual image plane

$\boldsymbol{m} = \boldsymbol{K}^{-1}\boldsymbol{u} = (x \quad y \quad 1)$

Intrinsic parameters of the camera : $\boldsymbol{K} = \begin{bmatrix} \gamma & \gamma s & u_0 \\ 0 & \gamma & v_0 \\ 0 & 0 & 1 \end{bmatrix}$

$(u_0 \quad v_0)$: The center of optic axis

γ : focul length divides pixel size



Camera Model

Unified Sphere Model

Model parameters

$\xi = 0.9685\text{mm} \quad \gamma = -399.1505\text{pixels}$

$u_0 = 513.9324\text{pixels} \quad v_0 = 400.7654\text{pixels}$

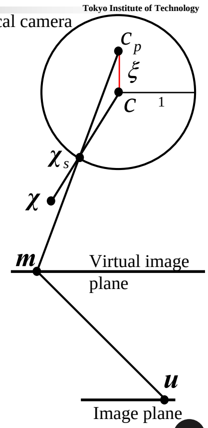
ξ : Distortion distance (??)

Then, $\boldsymbol{m}$ is projected in a unit sphere as χ_s

$$\chi_s = (\lambda x \quad \lambda y \quad \lambda - \xi)^T$$

$$\lambda = \frac{\xi + \sqrt{1 + (1-\xi^2)(x^2 + y^2)}}{x^2 + y^2 + 1}$$

Own problem : How derived ξ ?



From Estimation of homograph to Control

Estimation of Homograph

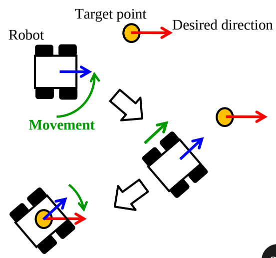
- Two image used
- RANSAC algorithm (to correspond points)

Derivation of direction

There exist infinite vertical virtual planes

Visual Control Law

1. Rotation to desired orientation
2. Linear movement toward the target
3. Rotation to target orientation



Stability Analysis

Lyapunov functions

Result and Recap

Experiment

Simulation data

- Points on plane and Points on 3D
- Estimation of direction

Previous method vs. Multiple homography estimation method

- Less estimation error (At most 2 degrees)
- Robust noise (against points on 3D)

Real data

- Catadioptric image used
- The Robot controlled to desired area

Previous method vs. Multiple homography estimation method

- Extraction of many matching points
- Estimation of real direction

Recap

Spherical camera model to **omni-directional camera system**

Method of feature matching algorithm

Conclusion and Feature Works

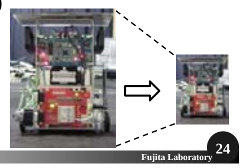
Conclusion

- Consider about Spherical Camera Model (SCM)
- Survey about Catadioptic Camera

Feature Works

- Derivation of theory about Visual Motion Observer using SCM
- Another methods Apply to visual motion observer
  - Contour pose estimation (using snakes)
  - Sonic wave
- Modification of previous work : Property

  Contraction of processing time



References

■ **Visual Motion Observer**

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE Transactions on Control Systems Technology*, Vol. 15, No1, pp. 40-52, 2007.

[2] H. Kawai, T. Murao and M. Fujita, "Visual Motion Observer-based Pose Control with Panoramic Camera via Passivity Approach," *Proc. of the 2010 American Control Conference*, Baltimore, Maryland, USA, 12nd, June-July, pp. 4534-4539, 2010

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[4]鳥居, 井宮, "球面カメラの多視点幾何学," 電子情報通信学会技術研究報告. NLC, 言語理解とコミュニケーション 105(300), pp. 29-34, 社団法人 電子情報通信学会, Sep. 15, 2005

[5]藤木, 赤穂, "球面カメラのエピポーラ幾何学とその計算," 電子情報通信学会技術研究報告. DE, データ工学 105(117), pp. 41-46, 社団法人 電子情報通信学会, June 10, 2005.

[6]藤木, "透視射影画像または球面カメラ画像からの3次元形状復元に向けて," 電子情報通信学会技術研究報告 NLC, 言語理解とコミュニケーション 105(300), pp. 35-40, 社団法人 電子情報通信学会, Sep. 15, 2005.

■ **Catadioptric Camera**

[7]G. Lopez-Nicolas, J.J. Guerrero, C. Sagues, "Multiple homographies with omnidirectional vision for robot homing," *Robotics and Autonomous Systems*, Vol.58, Issue6, pp. 773-783, 30 June, 2010.

References (of References)

■ **Catadioptric Camera (a Sub reference)**

[8] C. Geyer and K. Daniilidis, "A Unifying Theory for Central Panoramic Systems and Practical Implications," *ECCV 2000*, LNCS 1843, pp. 445-461, 2000.

Now reading

■ **Catadioptric Camera**

[9] C. Plagemann, C. Stachniss, J. hess, F. Endres and Nathan Franklin "A nonparametric leaning approach to range sensing from omnidirectional vision," *Robotics and Autonomous Systems*, Vol.58, Issue6, pp. 762-772, 30 June, 2010.

Now reading

Appendix

Extra : Example of a Dioptric Camera

Dioptric Camera Images

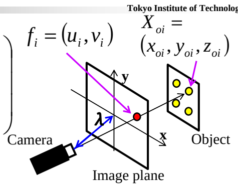
A half of Sphere plane image ≈

Image Jacobian of Spherical Camera Model

Measured Output

Perspective projection

$$f_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \quad \left(\begin{bmatrix} x_{ci} \\ y_{ci} \\ z_{ci} \end{bmatrix} = R_{co} X_{oi} + p_{co} \right)$$



f_i : i th feature point (pinhole camera)

Then,

$$\xi_i = \begin{pmatrix} x_{si} \\ y_{si} \\ z_{si} \end{pmatrix} = \frac{1}{\sqrt{u_i^2 + v_i^2 + \lambda^2}} \begin{pmatrix} u_i \\ v_i \\ \lambda \end{pmatrix}$$

While,

$$\xi_i = \frac{1}{\|X_i\|} X_i, \; X_i = \begin{pmatrix} x_{ci} \\ y_{ci} \\ z_{ci} \end{pmatrix} \quad \left(\begin{aligned} \|X\| &= (X^T X)^{\frac{1}{2}} \\ &= \sqrt{x_{ci}^2 + y_{ci}^2 + z_{ci}^2} \end{aligned} \right)$$

Image Jacobian of Spherical Camera Model

Using a first-order Taylor expansion

$$\xi_i - \bar{\xi}_i = \begin{pmatrix} \frac{\partial \xi_i}{\partial x_s} & \frac{\partial \xi_i}{\partial y_s} & \frac{\partial \xi_i}{\partial z_s} \end{pmatrix} (X_i - \bar{X}_i)$$

Visual data — Real data (on camera coordinate)

where

$$J_i = \begin{pmatrix} \frac{\partial \xi_i}{\partial x_{ci}} & \frac{\partial \xi_i}{\partial y_{ci}} & \frac{\partial \xi_i}{\partial z_{ci}} \end{pmatrix} \Bigg|_{X = \bar{X}} \quad \left(\begin{aligned} \|\bar{X}_c\| &= (\bar{X}_c^T \bar{X}_c)^{\frac{1}{2}} \\ &= \sqrt{\bar{x}_{ci}^2 + \bar{y}_{ci}^2 + \bar{z}_{ci}^2} \end{aligned} \right)$$

Image jacobian

$$= \frac{1}{\|\bar{X}\|} \begin{pmatrix} 1 - \frac{1}{\|\bar{X}\|^2} \bar{x}_{ci}^2 & -\frac{1}{\|\bar{X}\|^2} \bar{x}_{ci}\bar{y}_{ci} & -\frac{1}{\|\bar{X}\|^2} \bar{x}_{ci}\bar{z}_{ci} \\ -\frac{1}{\|\bar{X}\|^2} \bar{x}_{ci}\bar{y}_{ci} & 1 - \frac{1}{\|\bar{X}\|^2} \bar{y}_{ci}^2 & -\frac{1}{\|\bar{X}\|^2} \bar{y}_{ci}\bar{z}_{ci} \\ -\frac{1}{\|\bar{X}\|^2} \bar{x}_{ci}\bar{z}_{ci} & -\frac{1}{\|\bar{X}\|^2} \bar{y}_{ci}\bar{z}_{ci} & 1 - \frac{1}{\|\bar{X}\|^2} \bar{z}_{ci}^2 \end{pmatrix}$$

Homography [7]

Homography

- $\boldsymbol{H}$: Homography
- $\boldsymbol{n}$: Unit normal of the plane
- d : Distance along $\boldsymbol{n}$ between the plane and the reference position

$$\boldsymbol{H} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}\left(1 - \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}\frac{\boldsymbol{n}^T}{d}\right)$$

$$\left(\boldsymbol{n} = \begin{pmatrix} n_x & n_y & 0 \end{pmatrix}\right)$$

Control Low

$$\boldsymbol{H} = \begin{pmatrix} h_{11} & h_{12} & 0 \\ h_{21} & h_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$v = 0$$
$$\omega = k_1\sqrt{(h_{11} - \cos\varphi)^2 + (h_{12} + \sin\varphi)^2}$$
: Step1

$$v = k_2\sqrt{(h_{21} - \sin\varphi)^2 + (h_{22} - \cos\varphi)^2}$$
$$\omega = k_1\sqrt{(h_{11} - \cos\varphi)^2 + (h_{12} + \sin\varphi)^2}$$
: Step 2

$$v = 0$$
$$\omega = -k_w h_{21}$$
: Step 3

$$k_1 = sign((h_{12} + \sin\varphi)/(h_{22} - \cos\varphi))$$
$$k_2 = sign((\cos\varphi - h_{22})n_y)$$

Title

$$\begin{pmatrix} h_{11} - \cos\varphi & h_{12} + \sin\varphi & 0 \\ h_{21} - \sin\varphi & h_{22} - \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -t_x\frac{n_x}{d} & -t_x\frac{n_x}{d} & 0 \\ -t_x\frac{n_x}{d} & -t_x\frac{n_x}{d} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Lyapunov function

Step1 : $V_1 = \frac{t_x^{\,2}}{2}$ $\quad \dot{V}_1 = t_x\dot{t}_x = \cdots = -k_w sign\left(\frac{t_x}{t_y}\right)|t_x|t_x t_y < 0$

Step2 : $V_2 = \frac{t_y^{\,2}}{2}$ $\quad \dot{V}_2 = t_y\dot{t}_y = \cdots = -k_v sign(t_y)|t_y|t_y < 0$

Step3 : $V_3 = \frac{\varphi^2}{2}$ $\quad \dot{V}_3 = \varphi\dot{\varphi} = \cdots = -k_w\varphi\sin\varphi < 0$

Previous Work [7]

Simulation : directional angle

- Classical homography method
  - Only one homography : most poor performance in [7]
- Two-view epipolar geometry estimated by means of the eight-point algorithm
- Two-view epipolar geometry estimated by means of the five-point algorithm

Problem:
- Few matches
- Short baseline

Experiment

- Classical homography method
  - It can extract only limited matching points

⇕

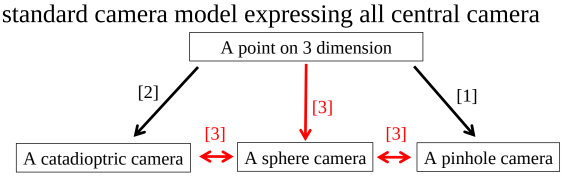
- It can extract more matching points : [7]

Recap : Sphere Camera Model

But…

Problem

A standard camera model expressing all central camera



× A standard camera model like

$$\phi(\boldsymbol{x}, a) : R^3 \to R^2 \quad (\boldsymbol{x} \in R^3)$$

$$a = \begin{cases} 1 & \to \text{A pinhole camera} \\ -1 & \to \text{A catadioptric camera} \end{cases}$$