Optimal Power Dispatch of Power Networks with Potential Games

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FL11-7-1
3th, June, 2011

Backgrounds

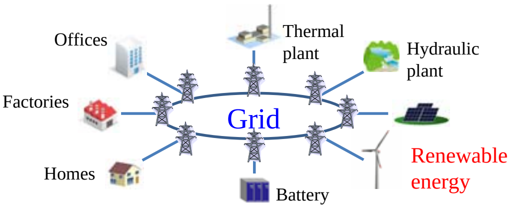
Smart Grid

- Two-way energy management system between supply and demand side
- Use of renewable energy (ex. solar, wind)

Objective: accomplishment of the following three points

- Environmental conservation
  Reducing CO_2 emissions
- Energy security
  Diversification of energy sources
  Central power plants (ex. Thermal)
  Distributed generations (ex. Renewable, battery)
  ⇨ Risk management for energy
- Economic growth — Building new energy industry

Objective (Optimal Power Dispatch)



Outputs of renewable energy depend largely on weather

⇨ It is more difficult to satisfy supply-demand balance

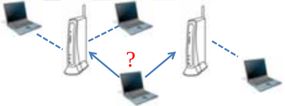
⇨ Optimal power dispatch for customers

Centralized and Distributed generations (renewable, battery) need to allocate electricity to customers in a coordinated way

No design policy of optimal power dispatch

⇨ Game theoretic approach

Objective (Game-theoretic Control)



Game-theoretic approach

- Agents are self-interested
  ⇨ Non-cooperative game
- The solution to the problem = the equilibrium of the game

Applications : resource allocation, sensor coverage

Advantages

- robustness to failures and environmental disturbances
- guarantee global convergence
- improved scalability

⇨ Potential game has
  Design policies of objective function
  Learning algorithms

Objective of this work

To apply potential game to optimal power dispatch problem of power network

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Definition of Game

- Player set $N = \{1, \cdots, n\}$
- Collection of action sets $\mathcal{A} = \mathcal{A}_1 \times \ldots \times \mathcal{A}_n$
  Agent *i*'s action set : $\mathcal{A}_i$
  Agent *i*'s action : $a_i \in \mathcal{A}_i$
- Collection of objective function $U = \{U_1, \cdots, U_n\}$
  Agent *i*'s objective function $U_i : \mathcal{A} \to \mathbb{R}$
  Every agent chooses a_i to maximize U_i

⇨ Game $G = < N, \mathcal{A}, U >$

Nash equilibrium

A pure Nash equilibrium is an action $a^* \in \mathcal{A}$ such that $\forall i \in N$

$$U_i(a_i^*, a_{-i}^*) = \max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i}^*)$$

$$(a_{-i} = (a_1, a_2, \cdots, a_{i-1}, a_{i+1}, \cdots, a_n))$$

(Ex.) Payoff Matrix

Player 2

Player 1	A	B
A	(2,2)	(1,0)
B	(0,1)	(4,4)

NE

NE

Potential Game

- Potential function (global objective function) $\phi : \mathcal{A} \to \mathbb{R}$

ϕ : max ⇨ Objective of a group is achieved

Potential game

A game $G = < N, \mathcal{A}, U >$ is a potential game if there is a potential function ϕ such that $\forall i \in N$, $\forall a_{-i} \in \mathcal{A}_{-i}$ and $\forall a_i', a_i'' \in \mathcal{A}_i$,

$$U_i(a_i', a_{-i}) - U_i(a_i'', a_{-i}) = \phi(a_i', a_{-i}) - \phi(a_i'', a_{-i})$$

Key property

- the guaranteed existence of a Nash equilibrium
- Local maxima of ϕ are Nash equlibra

(Ex.) Payoff

	A	B
A	(2,2)	(1,0)
B	(0,1)	(4,4)

Potential

	A	B
A	2	0
B	0	3

Global objective

⇨ Application of appropriate learning algorithms

maximize ϕ

Outline

- Introduction
- Optimal Power Dispatch Problem
- Simulation
- Applying to real-time
- Conclusions

Power Networks

◆ Node: $\mathcal{V} = \mathcal{G} \cup \mathcal{R} \cup \mathcal{B} \cup \mathcal{D}$

Generator: $\mathcal{G} = \{G_i | i = 1, \cdots, n_G\}$

Battery: $\mathcal{B} = \{B_i | i = 1, \cdots, n_B\}$

Renewable energy:

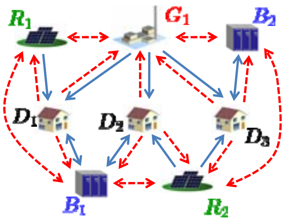
$\mathcal{R} = \{R_i | i = 1, \cdots, n_R\}$

Demand: $\mathcal{D} = \{D_i | i = 1, \cdots, n_D\}$

◆ Link: $\mathcal{E} = \mathcal{V} \times \mathcal{V}$

Energy flow: $\mathcal{E}_P$ ⟶

Information flow: $\mathcal{E}_I$ ---->



⇨ Graph: $G = (\mathcal{V}, \mathcal{E})$

Battery dynamics:

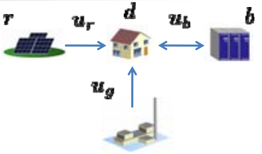
$$\Sigma_{\boldsymbol{B}}: \underline{b_i(k)} = b_i(k-1) - \sum_{j \in \mathcal{N}_{DB,i}} u_{bij}(k), i \in \mathcal{B}$$

battery level

$u_{bij} < 0$: charge

$\mathcal{N}_{DB,i}$: set of D_j can be supplied by B_i

Optimal Power Dispatch Problem



Assessment functions

- Demand = Supply:

  $J_D^1 = |d - (u_g + u_r + u_b)|$

- Minimize electricity prices (fuel cost):

  $J_D^2 = |u_g|$

- Maximize supply from renewable energy:

  $J_R = |r - u_r|$ *R*'s objective

- Remaining battery level = desired level:

  $J_B = |b_{ref} - b|$ *B*'s objective

d : demand[kW]

r : renewable's output[kW]

b : battery level[kW]

b_{ref} : desired battery level[kW]

Agent : Renewable energy, Battery Action: u_r , u_b

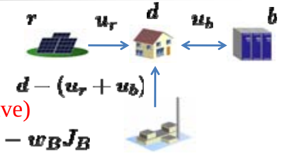
Central power plants adjust supply-demand balance:

$$\underline{u_g = d - (u_r + u_b)} \Rightarrow \begin{cases} J_D^1 = 0 \\ J_D = J_D^2 = |d - (u_r + u_b)| \end{cases}$$

D's objective

Optimal Power Dispatch Problem

$$\begin{cases} J_D = |d - (u_r + u_b)| \\ J_R = |r - u_r| \\ J_B = |b_{ref} - b| \end{cases}$$



⇨ Potential function (global objective)

$$\phi(u_r, u_b) = -w_D J_D - w_R J_R - w_B J_B$$

w_D, w_R, w_B : weight

Each agent's objective function

Renewable: $U_R(u_r, u_b) = -w_D J_D - w_R J_R$

Battery: $U_B(u_r, u_b) = -w_D J_D - w_B J_B$

Potential game

Sketch of Proof (Renewable)

$$\begin{aligned} & U_R(u_r', u_b) - U_R(u_r, u_b) \\ &= -w_D J_D(u_r') - w_R J_R(u_r') + w_D J_D(u_r) + w_R J_R(u_r) \\ &= \phi(u_r', u_b) - \phi(u_r, u_b) \end{aligned}$$

Same applies to battery

Learning algorithm

⇨ Payoff-based Inhomogeneous Partial Irrational Play (PIPIP) [6]

Payoff-based Inhomogeneous Partial Irrational Play (PIPIP)[6]

Each agent knows only the 2 last actions and utilities:

$a_i(t-1)$, $a_i(t-2)$, $U_i(a(t-1))$, $U_i(a(t-2))$

- Case 1: $U_i(a(t-1)) \geq U_i(a(t-2))$

  probability

  $1 - \varepsilon(t)$: $a_i(t) = a_i(t-1)$

  $\varepsilon(t)$: $a_i(t)$ is chosen randomly except $a_i(t-1)$ and $a_i(t-2)$

Exploration rate : $\varepsilon(t)$

Failure rate : k

- Case 2: $U_i(a(t-1)) < U_i(a(t-2))$

  $\Delta_i = U_i(a(t-2)) - U_i(a(t-1)) < 1$

  $\varepsilon(t)$: $a_i(t)$ is chosen randomly except $a_i(t-1)$ and $a_i(t-2)$

  $(1 - \varepsilon(t))k\varepsilon^{\Delta_i}$: $a_i(t) = a_i(t-1)$ ⟵ Mistake!

  $(1 - \varepsilon(t))(1 - k\varepsilon^{\Delta_i})$: $a_i(t) = a_i(t-2)$

⇨ Determine $a_i(t)$ and calculate $U_i(a(t))$

- All agents can take their action at the same time
- Convergence to optimal Nash equilibrium

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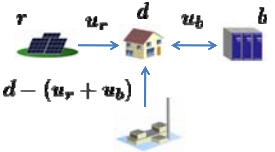
Simulation



Simulation setting

- Sampling time: $\tau = 10[\mathrm{min}]$
- Battery: $b(0) = 2[\mathrm{kW}]$

  $B_{max} = 5[\mathrm{kW}]$

  $b_{ref} = B_{max}/2 = 2.5[\mathrm{kW}]$
- Weight: $w_D = w_R = 0.6$, $w_B = 0.01$
- Actions:

  R: $u_r = \{0, 0.1, 0.2, \cdots, r\}$ in increment of 0.1

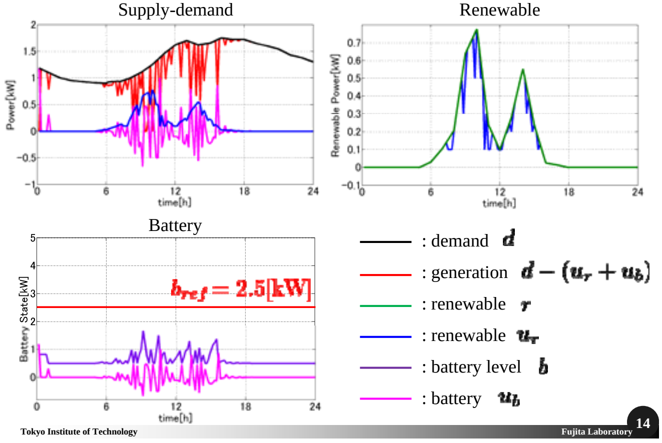
  B: $u_b = \{b - B_{max}, b - B_{max} + 0.5, b - B_{max} + 1, \cdots, b\}$ in increment of 0.5
- PIPIP: 2000step ➡ distribution
- Constraints

  No reverse power flow $u_r \geq 0$, $d-(u_r+u_b) \geq 0$

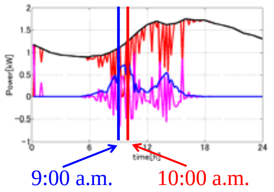
  Battery capacity $0 \leq b \leq B_{max}$

  Amount of charge $\underline{u_b \geq -u_r}$

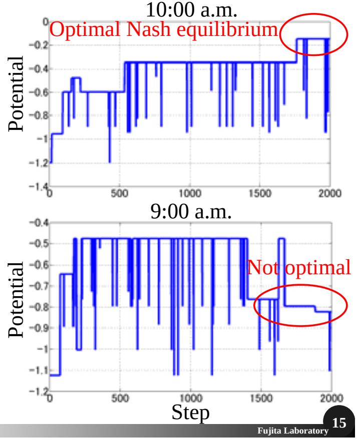
Simulation ($\varepsilon = 0.01$, $k = 0.5$)



Simulation ($\varepsilon = 0.01$, $k = 0.5$)



- ➢ can reduce fuel cost by renewable energy and battery
- ➢ not always converge to optimal Nash equilibrium



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Applying to Real-time

At first, Actually, Battery level

$$\left\{\begin{array}{l}\phi(u_r,u_b)\\U_B(u_r,u_b)\\U_R(u_r,u_b)\end{array}\right. \Longrightarrow \left\{\begin{array}{l}\phi(u_r,u_b,b)\\U_B(u_r,u_b,b)\\U_R(u_r,u_b)\end{array}\right.$$

Ex.

$$\phi(u_r,u_b,b) = -|3-u_r-u_b| - |2-u_r| - |1-b|$$

At time *t* - 1:

$$u_r(t-1) = 2,\ u_b(t-1) = 1,\ b(t-1) = 2$$

$$\phi(u_r(t-1), u_b(t-1), b(t-1)) = -1$$

At time *t* :

$$u_r(t) = 2,\ u_b(t) = 1,\ b(t) = b(t-1) - u_b(t) = 1$$

$$\underline{\phi(u_r(t), u_b(t), b(t))} = 0 \neq \underline{\phi(u_r(t-1), u_b(t-1), b(t-1))}$$

⇨ This game is affected significantly by battery state

Dynamic Game: Example1

Assessment functions

$$J_D = |3-u_r-u_b| \quad (d=3)$$

$$J_R = |2-u_r| \quad (r=2)$$

$$J_B = |1-(b-u_b)| \quad (b_{ref}=1)$$

Potential function

$$\phi = -10J_D - J_R - J_B$$

Utility

R : $U_R = -10J_D - J_R$

B : $U_B = -10J_D - J_B$

Actions

R : $u_r = \{0,1,2\}$

B : $u_b = \{-1,0,1\}$

Payoff matrix

Renewable energy

Battery

$b=0$	0	1	2
1			
0	(-31,-32)	(-21,-21)	(-11,-10)
-1		(-30,-31)	(-20,-20)

$b=1$	0	1	2
1	(-21,-22)	(-11,-11)	(-1,0)
0	(-30,-32)	(-20,-21)	(-10,-10)
-1		(-31,-31)	(-21,-20)

B_{max} → $b=2$

$b=2$	0	1	2
1	(-20,-22)	(-10,-11)	(0,0)
0	(-31,-32)	(-21,-21)	(-11,-10)
-1			

Dynamic Game: Example1

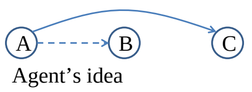
Best response

Select a_i such that

$$U_i(a_i, a_{-i}) \geq U_i(a_i', a_{-i})$$

Ex.

Battery



Battery is charged or discharged

⇨ Game transit to another state

Renewable energy

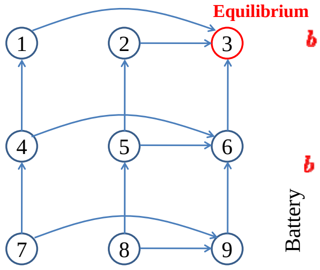
Battery

$b=0$	0	1	2
1	🟧	🟧	🟧
0	(-31,-32)	(-21,-21)	Ⓒ (-11,-10)
-1	🟧	(-30,-31)	(-20,-20)

$b=1$	0	1	2
1	(-21,-22)	(-11,-11)	Ⓑ (-1,0)
0	(-30,-32)	(-20,-21)	Ⓐ (-10,-10)
-1	🟧	(-31,-31)	(-21,-20)

$b=2$	0	1	2
1	(-20,-22)	(-10,-11)	(0,0)
0	(-31,-32)	(-21,-21)	(-11,-10)
-1	🟧	🟧	🟧

Dynamic Game: Example1



Equilibrium of dynamic game

$[u_r^*, u_b^*, b^*]$ ⇨ $\underline{[u_r^*, 0, b^*]}$

Renewable energy

Battery

$b=0$	0	1	2
1	🟧	🟧	🟧
0	① (-31,-32)	② (-21,-21)	③ (-11,-10)
-1	🟧	(-30,-31)	(-20,-20)

$b=1$	0	1	2
1	(-21,-22)	(-11,-11)	(-1,0)
0	④ (-30,-32)	⑤ (-20,-21)	⑥ (-10,-10)
-1	🟧	(-31,-31)	(-21,-20)

$b=2$	0	1	2
1	(-20,-22)	(-10,-11)	(0,0)
0	⑦ (-31,-32)	⑧ (-21,-21)	⑨ (-11,-10)
-1	🟧	🟧	🟧

Dynamic Game: Example2

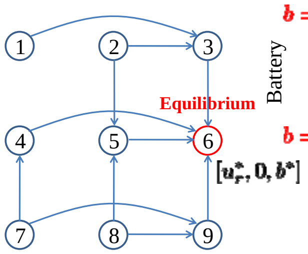
Potential function

$$\phi = -\underline{0.1}J_D - J_R - J_B$$

Utility

$$U_R = -\underline{0.1}J_D - J_R$$

$$U_B = -\underline{0.1}J_D - J_B$$



Renewable energy

Battery

$b=0$	0	1	2
1	🟧	🟧	🟧
0	① (-1.3,-2.3)	② (-1.2,-1.2)	③ (-1.1,-0.1)
-1	🟧	(-0.3,-1.3)	(-0.2,-0.2)

$b=1$	0	1	2
1	(-1.2,-2.2)	(-1.1,-1.1)	(-1,0)
0	④ (-0.3,-2.3)	⑤ (-0.2,-1.2)	⑥ (-0.1,-0.1)
-1	🟧	(-1.3,-1.3)	(-1.2,-0.2)

$b=2$	0	1	2
1	(-0.2,-2.2)	(-0.1,-1.1)	(0,0)
0	⑦ (-1.3,-2.3)	⑧ (-1.2,-1.2)	⑨ (-1.1,-0.1)
-1	🟧	🟧	🟧

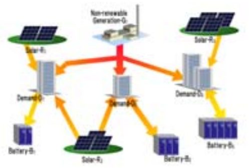
Conclusion

Summary

- Formulate Optimal Power Dispatch Problem
- Apply PIPIP for Optimal Power Dispatch Problem
- Consideration of Dynamic game

Future Works

- Design of potential game
  - Change assessment function
    Ex. absolute value → square
- Analysis of multi-agent case
- Applying to real-time
  - Check whether $[u_r^*, u_b^*, b^*]$ exist or not by simulation
  - Analysis of $[u_r^*, u_b^*, b^*]$ ⇨ $[u_r^*, 0, b^*]$



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[1] K. M. Candy, S. H. Low, U. Topcu and H. Xu, “A Simple Optimal Power Flow Model with Energy Storage,” Proc. of *the 49th IEEE Conference on Decision and Control*, pp.1051-1057, 2010

[2]星, “分散協調予測制御を用いたバッテリーを含むパワーネットワークの動的最適潮流計算,” 東京工業大学修士論文, 2011

[3] L. Chen, N. Li, S. H. Low, and J. C. Doyle, “On Two Market Model for Demand Response in Power Networks,” Proc. of *the first IEEE International Conference on Smart Grid Communications*, pp.1708-1713, 2010

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[5] R. Gopolakrishnan, “An Architectural View of Game Theoretic Control,” *California Institute of Technology Master's Thesis*, 2010

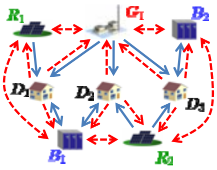
[6]後藤, “ポテンシャルゲーム理論的協調制御における学習アルゴリズムの提案,” 東京工業大学修士論文, 2011

[7] J. R. Marden and A. Wierman, “Overcoming Limitations of Game-Theoretic Distributed Control,” Proc. of *the 48th IEEE Conference of Decision and Control*, pp. 6466 - 6471, 2009

Appendix

Optimal Power Dispatch Game

$$\begin{cases} J_D = \sum_{j\in D} |d_j - (\sum_{i\in N_{RD,j}} u_{rij} + \sum_{i\in N_{BD,j}} u_{bij})| \\ J_R = \sum_{j\in R} |r_i - \sum_{j\in N_{DR,i}} u_{rij}| \\ J_B = \sum_{j\in B} |\bar{b}_i - \sum_{t=0}^{T} \sum_{j\in N_{DB,i}} u_{bij}| \end{cases}$$



Potential function

$$\phi(u_r, u_b) = -w_D J_D - w_R J_R - w_B J_B$$

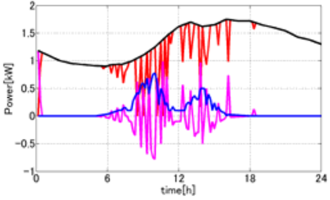
Each agent's objective function

$$U_{R,i} = -\sum_{j\in N_{DR,i}} w_{D,ij}|d_j - (\sum_{k\in N_{RD,j}} u_{rkj} + \sum_{k\in N_{BD,j}} u_{bkj})| - w_{R,i}|r_i - \sum_{j\in N_{DR,i}} u_{rij}|$$

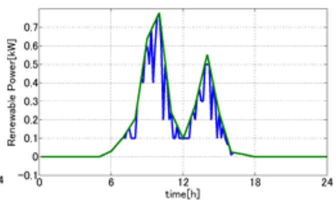
$$U_{B,i} = -\sum_{j\in N_{DB,i}} w_{D,ij}|d_j - (\sum_{k\in N_{RD,j}} u_{rkj} + \sum_{k\in N_{BD,j}} u_{bkj})| - w_{B,i}|\bar{b}_i - \sum_{t=0}^{T} \sum_{j\in N_{DB,i}} u_{bij}|$$

Simulation ($\varepsilon = 0.1,\ k = 0.5$)

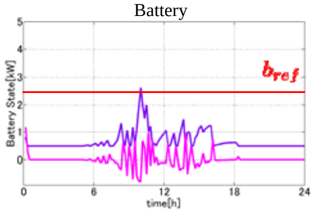
Supply-demand



Renewable



Battery



— : demand d

— : generation $d - (u_r + u_b)$

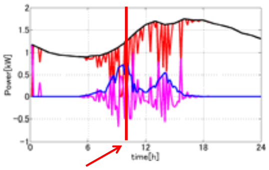
— : renewable r

— : renewable u_r

— : battery level b

— : battery u_b

Simulation ($\varepsilon = 0.1,\ k = 0.5$)

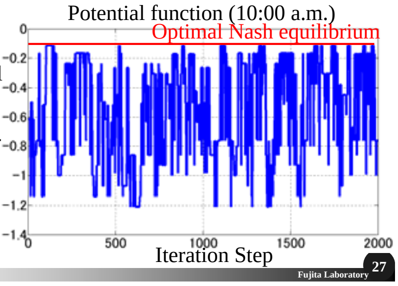


10:00 a.m.

Each agent explores too much

⇨ It is difficult to converge to optimal Nash equilibrium

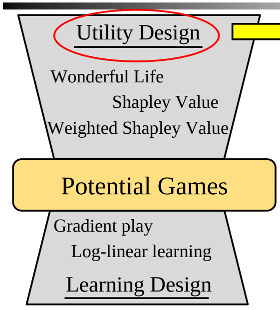
Potential function (10:00 a.m.)



➢ rate of converge to optimal Nash equilibrium

⇨ $\varepsilon = 0.01 > \varepsilon = 0.1$

Architecture for Potential Games and Utility Designs



Utility Design

Wonderful Life
Shapley Value
Weighted Shapley Value

Potential Games

Gradient play
Log-linear learning

Learning Design

⇨ Cost sharing methodologies

Player set $S \subseteq N,\ i \in S$

- Wonderful Life Utility (WLU)

$$f_r(i, S) = W_r(S) - W_r(S \setminus \{i\})$$

- The Shapley Value (SV)

$$f_r(i, S) = \sum_{T\subseteq S\setminus\{i\}} \omega_S(W_r(S) - W_r(S \setminus \{i\}))$$

$$\omega_S = \frac{|T|!(|S| - |T| - 1)!}{|S|!}$$

- The Weighted Shapley Value (WSV)

Distribution Rule	Existence of Equilibrium	Potential Game	Budget Balanced	Tractable	Informational Requirement
WLU	yes	yes	**no**	**yes**	Medium
SV, WSV	yes	yes	**yes**	**no**	**High**

State-Based Non-Cooperative Design

State-based games

state $x(t) \in X$

- Utility function $U_i(a_i, a_{-i}, x)$
- State transition function $P : \mathcal{A} \times X \to \Delta(X)$: the set of probability distribution over X
- Each player selects an action to maximize his expected utility

$$\therefore\ a_i(t) \in \arg\max_{a_i(t)\in\mathcal{A}_i} \mathbf{E}[U_i(a(t), x(t))]$$

- $x(t+1)$ is chosen randomly according to $P(a(t), x(t)) \in \Delta(X)$

State-based Nash Equilibrium $[a^*, x^*]$

for every x' in the support of $P(a^*, x^*)$,

$$U_i(a_i^*, a_{-i}^*, x') = \max_{a_i\in\mathcal{A}_i} U_i(a_i, a_{-i}^*, x')$$

State-based Potential games

$\exists\,\phi : \mathcal{A} \to \mathbb{R}$ such that $a_i' \in \mathcal{A}_i$,

$$U_i(a_i', a_{-i}, x) - U_i(a, x) > 0 \Rightarrow \phi(a_i', a_{-i}) - \phi(a) > 0$$

Priority-Based Distribution Rule

state = priority ⇨ Priority-based

x_i^r : the priority of player *i* at resource *r*

$$\begin{cases} x_i^r < x_j^r : i \text{ has higher priority than } j \\ x_i^r = 1 \ \ : \text{top priority} \end{cases}$$

State dynamics : first in first out (FIFO)

- Multiple players seek to join a resource simultaneously

  ⇨ The order of the entering players is randomly chosen
- $a(t) = a(t-1) \Rightarrow x(t+1) = x(t)$

Priority-based utility $U_i(a', x) = \mathbf{E}_{P(a',x)} V_i(a', x')$

Expectation which x' is chosen : $\mathbf{E}_{P(a',x)}$ $\quad \bar{x}_i^r := \{j \in N : x_j^r \le x_i^r\}$

Marginal contribution : $V_i(a, x) = \sum_{r\in a_i} (W^r(\bar{x}_i^r) - W^r(\bar{x}_i^r \setminus i))$

⇨

Distribution Rule	Budget Balanced	Tractable	PoS	PoA
Priority-Based	yes	yes	1	1/2