Survey of Camera Network Localization

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Overview of Localization

Mobile Robot Localization
Determining the pose of a robot relative to a given map of the environment
Knowledge of the location is necessary in robotic tasks
- Navigation
- Surveillance
- Mapping

Sensors for detect the location
- Computer-vision
- GPS
- Encoder
- Range finder etc.

Localization Problems

Consideration
- Pose can usually not sensed directly
- No noise-free sensor
- Dynamic environment
- Integrate data to determine the pose

Taxonomy of localization problems
- Local localization (position tracking)
  Initial robot pose is known
- Global localization
  Initial robot pose is unknown
  Robot is initially placed
- Kidnapped robot problem
  During operation, the robot gets teleported

Multi-agent localization

Multi-agent localization (cooperation)
Each robot localize itself individually
Robots can detect each other (Relative location of robots)

Localization in sensor networks (Network localization)
- GPS
  Not work well in buildings or obstruction
  High cost
- Special nodes (beacon)
  Know position
  Relative pose estimation for pair of robots only distance measurements [2]

Heuristic-based (lack of theoretical foundation)

Heuristic-based (lack of theoretical foundation)

Theory of Network Localization

Theory of network localization [3]
1. What are the condition for unique network localizability?
2. What is the computational complexity of network localization?
3. What is the complexity of network localization in typical network deployment scenarios?

Localization theory in angle-of-arrival sensing [4]
- Formulation of frame localizability
- Distributed algorithm for planar orientation localization
- Conditions for orientation localizability in noiseless 3-D networks

Localization in Camera Networks

Distributed estimation of the poses of the cameras
- Estimated neighboring poses by standard computer vision techniques
- Minimizing a cost function on SE(3) in a distributed fashion

Related work
- Pose Averaging [6], [7]

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Comparison of Related Work

Visual sensor networks

- Static object and scene
  - R. Vidal
  - [6] [5]

Pose averaging
- Localization
  - Fujita Lab.
  - [7]
- My work??

- Dynamic Estimation
- Simultaneous Estimation & Optimization

Outline

- Introduction
- Survey of [5]
- Application to our Research

Preliminaries

Undirected connected graph $G = (V, E)$

Set of nodes: $V = \{1, \ldots, N\}$

Edge $(i, j) \in E : j$ communicate with $i$

Pose of each node: $g_i = (R_i, T_i) \in SE(3)$

Relative Pose: $g_{ij} = g_i^{-1} \circ g_j$

Path $l$ : sequences of nodes

$L = \{w_1, \ldots, w_m\}, w_j \in \mathcal{F}, (w_m - w_m) \in E, m \in [1, \ldots, n - 1]$

Relative pose along a path $l$: $g_i = g_{w_1w_{n-1}} \circ \cdots \circ g_{w_kw_j}$

Cycle: Path from node $i$ to itself without repeated nodes

**Definition 1 (Localized network)** [5]

A network is said to be localized if there is a set of relative transformations $\{g_{ij}\}$ such that, when the reference frame of the first node is fixed to $g_1$, the other absolute poses are uniquely determined.

Method to Get Noisy Estimates

Epipolar Constraint

Extract feature points $x_i, x_j$

$x_i, x_j$ : homogeneous coordinates

$x = [a \ b \ 1]$

Vector $x_i, T_j, R, x_j$ lie in an epipolar plane

$x_i^T R x_j = 0$

Use more than 8 feature points

Measurements are corrupted by noise

Unknown scales $\lambda, \tilde{T} = \lambda \tilde{T}$

Noisy relative pose $\tilde{g}_{ij} = (\tilde{R}_{ij}, \tilde{T}_{ij})$

Problem Settings

Goal: Find the relative transformations $g_{ij}$

1. Close as the relative measurements
2. Satisfy the constraints given in Definition 1

① Minimize the cost function

$\varphi = \frac{1}{2} \sum_{i, j} d_i^2(g_{ij}, \tilde{g}_{ij}) + \sum_{i, j} d_{SO(3)}(R_{ij}, R_k) + ||T_i - T_j||^2$

$\delta_{SO(3)}(R_1, R_2) = \frac{1}{2} \text{trace}(\log(R_1^T R_2))$

Neighbor set: $\mathcal{N} = \{j \in V | (j, i) \in E\}$

② Constraints

For any path from node 1 to node $i$, we have $g_1 = g_i$, regardless of chosen path

Consideration of the Consistent Constraints

Reparametrize relative pose $g_{ij}$

Using absolute pose $g_i, g_j$

**Proposition 1**:

The following are equivalent

1. The network is localized
2. For any cycle $l = \{w_m, \ldots, w_1\}, w_m \in \mathcal{F}, n > 1$

   The transformation along the cycle is $g_1 = g_1 g_2^{-1} \cdots g_{n-1}^{-1}$

Compute the absolute poses $g_1, \ldots, g_N$, relative pose $g_{ij}$ will be uniquely determined

The consistent constraints will be automatically satisfied
Problem Settings

Rewrite using absolute poses

\[ \varphi = \frac{1}{2} \sum_{j \in \mathcal{N}} d^2_j(y_j^i, y_j) = \frac{1}{2} \sum_{j \in \mathcal{N}} d_j^2(y_j^i \cdot y_j) \]

\[ = \frac{1}{2} \sum_{j \in \mathcal{N}} (d^2_k(R_j^i) + \|\{R_j^i \} - T_j\|) \]

Only Rotation

Consider \( \lambda = 0 \)

\( \lambda \geq 0 \) Global minimum but trivial

\( \lambda \) must be positive Proposed idea: Minimum scale \( \lambda \geq 1 \)

Summary (localization problem)

\[ \begin{align*}
\min_{(R_k^i, R_j^j, \lambda)} & \varphi((R_k^i, R_j^j, \lambda)) \\
\text{subject to} \quad & \lambda \geq 1 \quad (i, j) \in E
\end{align*} \]

Procedure of Minimization

Procedure of minimization (gradient descent)

1. Find an initial set of rotations by optimizing only \( \varphi \)

\[ \text{grad}_{R_k^i} \varphi_R = -R_k \sum_{i \in \mathcal{N}} \log(R_i R_k) + \log(R_i^T R_k R_i) \]

\[ R_k(l + 1) = \exp_{R_k}(\epsilon \text{grad}_{R_k} \varphi) \quad \text{Bk} \text{ (l) : Rotation at iteration } l \quad \epsilon \text{: Step size} \]

Initialization of the Rotations \( \varphi \) has multiple local minima

Set new cost function

\[ \psi_R = \frac{1}{2} \sum_{i \in \mathcal{N}} \|R_i - R_i R_k\|_2^2 \]

\( \psi_R \) doesn’t have local minima

Find \( R_k(0) \) by minimizing new cost function

\[ \frac{\partial \psi_R}{\partial R_k} = \sum_{i \in \mathcal{N}} (R_i - R_i R_k) + \sum_{i \in \mathcal{N}} (R_i - R_i R_k) \]

Reason of multi-step optimization

\( \varphi \) has multiple local minima Good initialization is needed

Choice of step size

In gradient descent, it is important to select

Outline

• Introduction

• Survey of [5]

• Application to our Research

Preliminaries

Pose of camera \( i \) relative to world frame \( g_{wi} = (p_{wi}, R_{wi}) \).

Pose of object relative to camera \( i \)

\( g_{oi} = (p_{oi}, R_{oi}) \).

Relative pose of the camera \( i \) and \( j \)

\( g_{ij} = (p_{ij}, R_{ij}) \).

Estimated by Visual Motion Observer [8]

\( g_{ij} \): Estimated pose of object by camera \( i \)

Definition of localization

Given a relative sensing network with reference node1. The reference frame transformation \( R_k \) for all \( i \in \{2, \cdots, n\} \) are uniquely determined.
Measurement Methods

Relative sensing by cameras
- One camera sense relative pose of another camera (visibility graph)

Attach feature points to camera

Directed graph $G=(V,E)$ Set of nodes: $V=\{1,\ldots,N\}$

Edge $(i,j)\in E$ : i measure j’s relative pose

- Use object to estimate the pose

$\tilde{g}_{ij} = g_{ij}\tilde{p}_0$ : Estimated relative pose

$\tilde{g}_{ij} : i$ frame $\rightarrow j$ frame

Problem Settings

Consider 3 cameras (for simplicity)

New estimate variables: $\tilde{g}_{ij}$

Measurements (estimated by VMO): $\tilde{g}_{ij}$

What we want to do is minimize the overall cost function

$\epsilon(\tilde{g}_{ij}, \tilde{g}_{ik}) + \epsilon(\tilde{g}_{ik}, \tilde{g}_{ij}) + \epsilon(\tilde{g}_{ji}, \tilde{g}_{ki})$

in a distributed way

Close as the measurements

with a constraints $\tilde{g}_{ij}\tilde{g}_{ij} = (I, 0)$

Determine the pose

Cost function $d^2(T_1, T_2) = \frac{1}{2} \text{trace}(R_1^T R_2)$

From [5] (Geodesic distance)

$\Phi = \frac{1}{2}\|\tilde{g}_{ij}\|_2 + \frac{1}{2}\|\tilde{g}_{ik}\|_2 + \frac{1}{2}\|\tilde{g}_{ji}\|_2$

Gradient Method

First we consider only orientation

Minimize the cost function

$\Phi = \min_{\tilde{R}_{ij}, \tilde{R}_{ik}, \tilde{R}_{ji}} \frac{1}{2}\|\tilde{R}_{ij}\|_2 + \frac{1}{2}\|\tilde{R}_{ik}\|_2 + \frac{1}{2}\|\tilde{R}_{ji}\|_2$

Gradient descent

Calculate the gradient

$\frac{\partial \Phi}{\partial \tilde{R}_{ij}}, \frac{\partial \Phi}{\partial \tilde{R}_{ik}}, \frac{\partial \Phi}{\partial \tilde{R}_{ji}}$

Update the estimates $\tilde{R}_{ij}(l+1) = \exp_\Psi \left( -\epsilon \frac{\partial \Phi}{\partial \tilde{R}_{ij}} \right)$

References


References


Appendix

Contributions of [4]

Contributions
1. Formulation of frame localizability
2. Characterization of frame localizability for planar networks
3. Compute least-squares estimate of the orientations in a 2-D ring network
4. Distributed algorithm for planar orientation localization
5. Conditions for orientation localizability in noiseless 3-D networks

Elements of Graph Theory[4]

Directed or undirected graph $G=(V,E)$
- Set of nodes: $V=\{1,\cdots,N\}$
- Edge: $(i,j)\in E$
- Path $P$: sequences of nodes $P=[w_1,\cdots,w_m], w_i \in V, (w_i-w_{i+1}) \in E, m \in \{1,\cdots,n-1\}$
- Cycle $l \in \mathcal{L}(G_e)$. $G_e$ : directed graph
- Path from node $i$ to itself without repeated nodes
- Cycle vector $1_i \in \{-1,0,1\}^n \subset \mathbb{R}^n$
  - 1: orientation is consistent with the orientation of $l$
  - -1: orientation is opposite with the orientation of $l$
  - 0: otherwise

Ex.) $1_4 = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}^T$

Localization Problem[4]

Angle-of-arrival sensing
Measurement of node $i$ for node $j$
- $\text{vers}(p_i) \in \mathbb{R}^4\ (\text{vers}(\cdot) = \frac{V}{\|V\|}, V \neq 0)$
- Sensing range: $r$

Remark 8 (Data referencing motivation)
Measurements are taken in their respective reference frames

Definition of localization

Problem 6 (Frame localizability)
Given a relative sensing network with reference node 1. The reference frame transformation $[R_i^0, \mathbf{a}_i]$ for all $i \in \{2,\cdots,N\}$ are uniquely determined by the relative measurements $R_i^0$'s orientation relative to node 1 $p_{i^*}$'s position
Orientation Localizability[4]

Relationships \( \text{vers}(p^i) = -R_i^r \text{vers}(p_i^r) \) (7)

Lemma 10 (Feasible orientations)

Compute \( u_i' = \text{vers}(p_i^r) \), \( H = \exp(\alpha_i' \theta_i') \), \( \alpha_i' \in [0, \pi] \)

\[
\theta_i' = \begin{cases} 
\text{vers}(u \times u') & \text{if } u \times u' \neq \mathbf{0} \\
\mathbf{0} & \text{otherwise}
\end{cases}
\]

Then all solutions to (7) are \( R = \exp(\beta \theta_i')H \) \( \beta \in [-\pi, \pi] \); arbitrary angle

Theorem 11 (Orientation Localizability for 2-D)

Consider a relative sensing network with 2-D and with noiseless angle-of-arrival sensing. The following statements are equivalent:

(i) The sensing graph is connected
(ii) The network is orientation localizable

Problem Settings[4]

Let \( \phi_e : \text{estimate of } \theta_e = \theta \)
\( \phi \in \mathbb{R}^n : \text{vector of angle estimates for all edges} \)
\( y : \text{measurement vector} \) with \( y = y' \)

Cycle error \( \epsilon_i = \text{proj}(1_i \cdot \phi) \) \( \epsilon_i = \text{proj}(\sum_{i \in G_e} 1_i \cdot \phi) = 0 \)

\( \hat{\xi} \): whether direction of the edge \( f \) is concordant with the direction of cycle \( l \)

Least-squares estimation problem

\[
\min_x \|y - \phi\|^2 \\
\text{subject to } \text{proj}(1_i \cdot \phi) = 0 \text{ for all } i \in \mathcal{L}(G_e)
\]

Proof of the Theorem[4]

\( \dim(L) = r \)

Fundamental cycles: \( l_1, \ldots, l_r \)

Cycle error vector \( \epsilon = [\epsilon_1, \ldots, \epsilon_r]^T \)

\[
\psi(t+1) = \psi(t) - \kappa \sum_{i \in \mathcal{E}(G_e)} 1_i \epsilon_i(t)
\]

For every loop \( \alpha \in \mathcal{L}(G_e) \)

\[
\epsilon_\alpha(t+1) = \epsilon_\alpha(t) - \kappa \sum_{i \in \mathcal{E}(G_e)} (1_i \cdot 1_\alpha) \epsilon_i(t) \quad \epsilon_\alpha(t) = (1_\alpha \cdot \psi(t))
\]

For all loop \( i(t+1) = \hat{i}(t) - \kappa C_e C_f^T \hat{i}(t) \)

\[
\epsilon(t+1) = \text{proj}(i_i - \kappa F \psi(t)) \quad F = C_f C_e^T
\]

Orientation Localization with Noise[4]

Consider planer network

\( y = \text{proj}((\mathcal{A}p_i + n) - (\mathcal{A}p_i + n) + \pi) \)

\( n, n' \): Noise (Gaussian with zero mean and variance \( \sigma^2 \))

\( \text{proj}(\theta = (\theta + \pi) \mod 2\pi - \pi) \)

Enforce cycle constraint and mitigate the noise

Assign a direction to each edge

If \( i > j \)

Get directed graph \( G = (V, E) \) \( \text{Different from sensing graph!} \)

Oriented edge \( e = (j,i) \in E, \text{with } (i > j) \)

Estimation Algorithm[4]

Arbitrary network \( G \)

Subset of the cycle set: \( \mathcal{L} \subseteq \mathcal{L}(G_e) \)

\( \psi_e : \text{estimate of edge } e \)

Cycle-distributed system

\[
\psi_e(0) = y_e \\
\psi_e(t+1) = \psi_e(t) - \kappa \sum_{i \in \mathcal{E}(G_e) \cap \mathcal{L}} (1_i \cdot \epsilon_i) \psi_i(t) \quad 0 < \kappa \ll 1
\]

\( \epsilon : i \text{th entry is } 1, \text{all the others are } 0 \)

Theorem 18 (Exponential convergence of iterative estimation algorithm)

The solution of cycle-distributed system with \( \hat{\xi} = \xi \), converges exponentially fast with factor \( \rho = (1 - \kappa') \), with zero cycle error with \( \kappa < 2/(1 + \lambda_{min}(F)) \)

\[ F = C_f^T C_e \quad \lambda_{min}(F) : \text{Maximum eigenvalue of } F \]

Proof of the Theorem[4]

Consider linear system

\[
x(t+1) = (I - \kappa F)x(t)
\]

Lyapunov function candidate

\( V(x) = x^T P x \)

Define \( \bar{Q} = (2x - \kappa F) I, \quad \lambda_{min}(F) > 0, \quad A = I - \kappa F \)

Discrete-time Lyapunov inequality

\[
\kappa < 2/(1 + \lambda_{min}(F)) \quad P - \bar{Q} x \leq 0, \quad \rho = (1 - \kappa')
\]
Lemma 19
Consider a network composed by three nodes in 3-D space with angle-of-arrival sensing. Pick any one of the three nodes as reference. If the sensing graph is the complete graph and the nodes are in generic position with generic orientations, then there are precisely two feasible configurations for the three nodes and, therefore, the network is not orientation localizable.

Lemma 20
Consider a network composed by four nodes in the 3-D space with angle-of-arrival sensing. If the sensing graph is connected and there are at least two independent loops, then the network is orientation localizable.

Lemma 21
A necessary condition for a network in the 3-D space with angle-of-arrival sensing to be orientation localizable is to have at least 4 nodes.

Definition of Localization [5]
Definition 1 (Localized network) [5]
A network is said to be localized if there is a set of relative transformations \( R \) such that, when the reference frame of the first node is fixed to \( R_1 \), the other absolute poses \( R_i \) are uniquely determined.

For any path \( l \) from node 1 to node \( i \), we have \( R_i = R_i R_1 \).

Estimation of the Rotations [5]
Gradient of \( \phi \) with respect to \( R \),
\[ \text{grad}_R \phi = -\sum_{i,j} \log(R_i R_j R_i^{-1}) + \log(R_i R_j R_i^{-1}) \]
Update procedure
\[ R(t+1) = \exp_\phi(-\varepsilon \text{grad}_R \phi) \]
\( \varepsilon \) : Step size
Initialization of the Rotations
1. Set \( R(0) = I \) where \( R(0) \) is not distributed.
2. New cost function \( \phi_i = \frac{1}{2} \sum_{j} R_i R_j R_i^{-1} + \sum_{k} (R_i R_j R_k) \)
Find \( R(0) \) by minimizing \( \phi_i \) : \( \phi_i \) doesn’t have local minima

Choice of the step size [5]
Minimization of quadratic form
\[ \phi(R) = \frac{1}{2} \| R - Q \|^2 \quad \gamma \in \mathbb{R}^n \]
• Quadratic cost function restricted to a line (direction of the grad.) is a parabola
• Maximum step-size is determined by maximum possible curvature of the parabola
Related to maximum eigenvalue of \( R^T R \)
Gersgorin Discs and Gersgorin Theorem
Maximum eigenvalue can be substituted with the maximum of the absolute row sum of \( R^T R \)
Apply for minimizing \( \phi \), computed in a distributed way.

Calculation of the Gradient
Consider
\[ \phi(R) = \frac{1}{2} \| R - Q \|^2 = \text{trace}(I - R^T Q) \quad \phi : SO(3) \rightarrow \mathbb{R} \]
SO(3) is submanifold of \( \mathbb{R}^{3 \times 3} \)
Define \( \phi : \mathbb{R}^{3 \times 3} \rightarrow \mathbb{R} \)
\[ \text{grad}_R \phi(R) = P_R \text{grad}_Q \phi(Q) \]
Projection: \( P_R : \mathbb{R}^{3 \times 3} \rightarrow T_R SO(3) \)
\[ P_R Z = Bsk(BR^T Z) \]
Tangent space: \( T_R SO(3) = \{ RX \in \mathbb{R}^{3 \times 3} | X \in so(3) \} \)
\[ < \text{grad}_R \phi(R), Z > = D\phi(R)[Z] \]
\( < , > : \text{Inner product} \quad < Z_1, Z_2 > = \text{trace}(Z_1^T Z_2) \quad Z_1, Z_2 \in \mathbb{R}^{3 \times 3} \)
Calculation of the Gradient

Directional derivative

\[ D\phi(R)[z] = \lim_{t \to 0} \frac{\phi(R + tz) - \phi(R)}{t} = -\text{tr}a_0(z^T Q) \]  \hspace{1cm} (3)

From (2) and (3)

\[ \text{grad}_\phi \phi(R) = -Q \]

\[ \text{grad}_\phi \phi(R) = P_k \text{grad}_\psi \phi(R) = P_k (-Q) = -\text{Re}b(R^T Q) \]