Survey of MPC Traffic Control and Analyze Vehicle Dynamics

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Background

Highway congestion

Highway congestion is imposing an intolerable burden on urban residents

Congestion occurs when vehicle's velocity variation propagates to following vehicles

It is difficult for human drivers to recognize tiny changing of the precede vehicle's velocity

Approaches

There are various approaches to improve congestion
They can be classified as
macro perspective and micro perspective

Macro perspective: On-ramp control, Transportation Network
Micro perspective: Vehicle Platoon Control

P. Varaiya, "Smart Cars on Smart Roads: Problems of Control", *IEEE Transactions on Automatic Control*, Vol. 38, No. 2, Feb. 1993

B. D. Schutter, H. Hellendoorn

[1]L. D. Baskar, B. D. Schutter and H. Hellendoorn, "Model-based Predictive Traffic Control for Intelligent Vehicle: Dynamic Speed Limits and Dynamic Lane Allocation," *Proc. of 2008 IEEE Intelligent Vehicles Sympsium(IV08)*, pp. 174-179, 2008
[2]L. D. Baskar, B. D. Schutter and H. Hellendoorn, "Dynamic Speed Limits and On-ramp Metering for IVHS Using Model Predictive Control," *Proc. of the 11th IEEE Conference on Intelligent Transportation Systems*, pp. 821-826, 2008

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Intelligent Vehicle

Intelligent vehicle(IV)

A vehicle equipped with control systems that can sense the environment around the vehicles and that result in a more efficient vehicle operation by assisting the driver or by taking complete control of the vehicle

IV technologies

Intelligent Speed Adaptation(ISA):
A speed limiter that adjust the maximum driving speed to the speed limit specified by the roadside infrastructure

Adaptive Cruise Control(ACC):
An automation system that adjust speed to the preceding vehicle and maintain a safe intervehicle distance

Dynamic Route Planning and Guidance:
A route guidance system advises a driver about the best route he can take to reach his requested destination

IV-based traffic management

IV-based traffic management

A traffic control that support and improve the platooning concept by allowing vehicle-vehicle and vehicle-roadside coordination

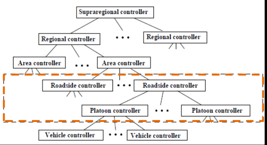
Hierarchical Controller for IV-based traffic management

○Higher-level controllers:
Providing network-wide coordination of the lower-level

○Roadside controllers:
Controlling a part of a highway, an entire highway collection of highways

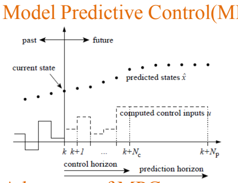
○Platoon controllers:
Receiving commands from roadside controller and controlling each IV inside the platoon

○Vehicle controllers:
Translating command into control signals for the vehicle actuators



Model Predictive Control

Model Predictive Control(MPC)



T_{ctrl} : Control sampling interval
k : Control step
N_c : Control horizon
N_p : Prediction horizon

Advantage of MPC

・Easily handle MIMO process, process with time-delay, unstable process etc..
・Can consider constraints on the inputs and the outputs of the process
・can handle changes in system parameters or system structure

Performance criteria and constraints

$$J_{tot}(k) = J_{perf}(k) + \alpha \sum_{j=0}^{N_c-1} \| u(k+j) - u(k+j-1) \|_2$$

Performance criteria
Input constraint

Prediction Model

Vehicle model

$$x_i(l+1) = x_i(l) + v_i(l)T_{sim} + \frac{1}{2}a_i(l)T_{sim}^2$$
$$v_i(l+1) = v_i(l) + a_i(l)T_{sim}$$

Input

l : Simulation step
T_{sim} : Simulation time step

Input

Platoon leader model

$$a_i(l) = K_1(v_{ISA}(l) - v_i(l))$$

K : Gain
V_{ISA} : Reference ISA speed

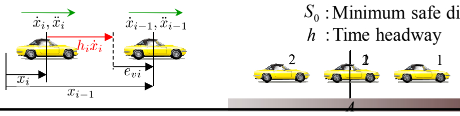
Follower vehicle model

$$a_i(l) = K_2(h_{ref,i}(l) - (x_{i-1}(l) - x_i(l)) + K_3(v_{i-1}(l) - v_i(l))$$

Position error　Velocity error

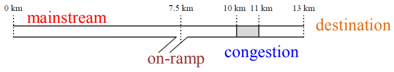
$$h_{ref,i}(l) = S_0 + v_i(l)h + L_i$$

L_i : Length of vehicle
S_0 : Minimum safe distance
h : Time headway



Case study

Simulation Setting



Using a basic set-up consisting of a 13km single lane highway stretch with one mainstream origin, one on-ramp and one destination
There is a congestion 10~11km

IV model in simulation

In this simulation,
・Vehicle prediction model in MPC and vehicle simulation model is same
・A platoon vehicles can be considered as a single entity
・Length of platoon is given by

$$L_{platoon,p} = (n_p - 1)(S_0 + hv_{n_p}(l)) + \sum_{i=1}^{n_p} L_i$$

n_p : number of vehicles in a platoon

Control Problem

Controller

○Roadside controllers:
MPC is applied for speed control and on-ramp control as roadside controller
State: position and velocity of the platoon leaders, platoons' lengths
Control input: ISA, on-ramp

○Platoon controllers:
Do nothing(Time headway, vehicle number of platoon is fixed)

Control problem

$$J_{perf}(k) = \sum_{l=kK+1}^{(k+N_p)K} (n_{veh}(l) + q_{main}(l) + q_{on}(l))T_{sim}$$

$n_{veh}(l)$: vehicles in the network
$q_{main(l)}$: vehicles in the queue at the mainstream
$q_{on(l)}$: vehicles in the queue in the on-ramp

$$K = \frac{T_{ctrl}}{T_{sim}}$$

The objective is minimization of the Total Time Spent(TTS) that show all vehicles' time spent in the highway

Discussion

Result

Case	TTS (veh.h)	Relative improvement
uncontrolled case	39.80	0 %
controlled (human drivers)	35.43	10.98 %
controlled (platoons)	29.39	26.16 %

Discussion

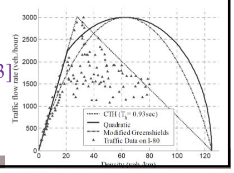
This thesis doesn't consider vehicle dynamics

$$\begin{bmatrix} \dot{x}_i \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_i \\ v_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a_i(l)$$

→If there is no regulation,
objective function simply increase ・making v_i larger
・making $h_{ref,i}$ shorter

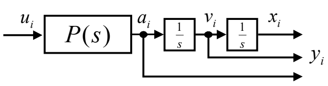
It sounds too simple!

In reality,
exceeded traffic density makes TTS worse[3]
→need to reflect in the model



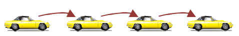
Trial

Controller



$$\dot{q}_i = Aq_i + Bu_i \qquad q_i = [x_i, v_i, a_i, p_i]^T$$
$$y_i = Cq_i + Du_i \qquad p_i : \text{State of } P(s)$$

Controller



Predecessor following

$$u_i = K_p((x_{i-1} - x_i) - S_i) + K_v(v_{i-1} - v_i)$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix} = \begin{bmatrix} A & & & \\ & A & & \\ & & \ddots & \\ & & & A \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} + \begin{bmatrix} B & & & \\ & B & & \\ & & \ddots & \\ & & & B \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

$$\dot{q} = A'q + B'u$$

Trial

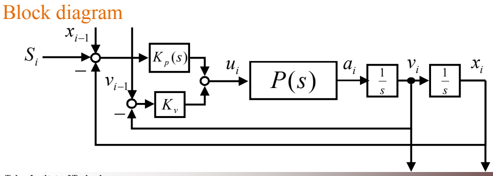
$$u_i = K_p(x_{i-1} - x_i) + K_v(v_{i-1} - v_i) - K_pS_i$$

$$\dot{q} = A'q + B'u$$
$$= A'q + B'T[K_pL_g, K_vL_g, 0, 0]T^{-1}q - B'TD_gK_pS$$
$$= A''q + B''S$$

$$L_g = \begin{bmatrix} 0 & & & \\ 1 & -1 & & \\ & \ddots & \ddots & \\ & & 1 & -1 \end{bmatrix} \quad D_g = [D_g', 0, 0, 0]^T \quad D_g' = [0, 1, \cdots, 1]^T$$

$$S = [S_1, S_2, \cdots, S_n]^T \quad q = T[x, v, a, p]^T$$

Block diagram



LQR Approach

Linear quadratic regulation

$\dot{q} = A''q + B''S$

$J = \int_0^\infty (q^T Q q + S^t R S) dt$

LQR find a controller law $u = -Kq$ that minimize the cost function

→velocity converge to 0!

Considering Reference signal

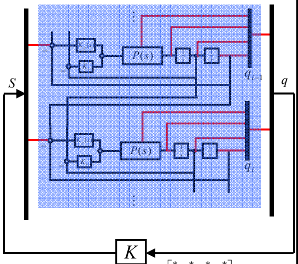
$\dot{q} = A''q + B''S$

$\tilde{q} = q + q_e$ q_e:Reference

$\dot{\tilde{q}} = A''\tilde{q} - A''q_e + B''S$ $\dot{q} = A''q + B''S$

$0 = A''q_e + B''S_e$

$\dot{\tilde{q}} = A''\tilde{q} + B''(S - S_e)$ $q_e = -A''^{-1}B''S_e$



Additional term come in input

A,B matrix are static

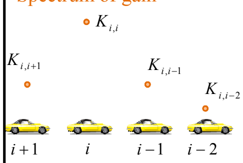
Gain K plot

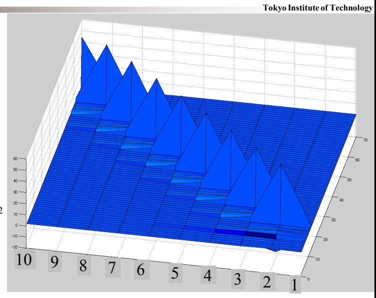
Result

3D plot of Gain－Kmatrix

$n = 10, kp = 0.1, kv = 0.1$

Spectrum of gain



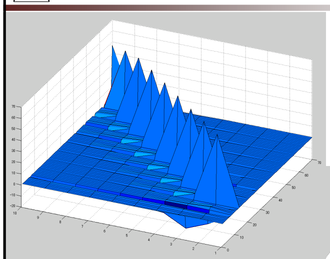


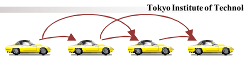
・The largest gain is self feedback gain

・The second largest gains are predecessor and successor

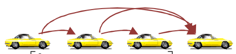
→If we can get all vehicles' information, only 3 vehicles' information are important

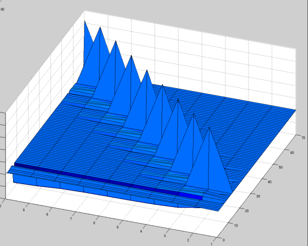
Other Communication Graph





$L_g = \begin{bmatrix} 0 & & & \\ 1 & -1 & & \\ 1 & 1 & -2 & \\ & 1 & 1 & -2 \\ & & \ddots & \ddots & \ddots \end{bmatrix}$





$L_g = \begin{bmatrix} 0 & & & \\ 1 & -1 & & \\ 1 & 1 & -2 & \\ 1 & 0 & 1 & -2 \\ \vdots & & & \ddots & \ddots \end{bmatrix}$

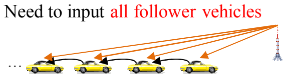
New Approach

L2 norm controller

$\dot{q} = A''q + B''S$

$S = Kq$ $K = \begin{bmatrix} 0 & * & * & * & * \\ 0 & * & * & * & * \\ 0 & * & * & * & * \\ 0 & * & * & * & * \\ \vdots & \vdots & \ddots & & \ddots \end{bmatrix}$

Need to input all follower vehicles

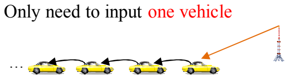


New approach

$\dot{q} = A''q + B''S$

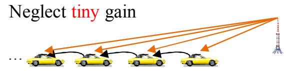
$S = Kq$ $K = \begin{bmatrix} * \\ * \\ * \\ * \\ \vdots \end{bmatrix}$

Only need to input one vehicle



$K = \begin{bmatrix} 0 & * & & & \\ 0 & * & * & & \\ 0 & * & * & * & \\ 0 & & * & * & * \\ \vdots & & & \ddots & \ddots \end{bmatrix}$

Neglect tiny gain



Orientation

[4]A. Rantzer, "Dynamic Dual Decomposition for Distributed Control," ACC '09, 2009

$K = \begin{bmatrix} 0 & * & & & \\ 0 & * & * & & \\ 0 & * & * & * & \\ 0 & & * & * & * \\ \vdots & & & \ddots & \ddots \end{bmatrix}$

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[5]G. C. Chasparis and J. S. Shamma, "Control of Preferences in Social Networks," *Proc. of the 49th IEEE Conference on Decision and Control* ,pp. 6651-6656, 2010

$K = \begin{bmatrix} * \\ * \\ * \\ * \\ \vdots \end{bmatrix}$

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Similarity between Study and [5]

Dynamics

$x_{k+1} = Ax_k + Bu_k$

$x_k \geq 0$ $u_k \geq 0$ $\sum_i^n u_{i,k}^i \leq M$

$x_k = [x_{1,k}, \cdots, x_{n,k}]$ $u_k = [u_{1,k}, \cdots u_{n,k}]$

x_k: Proclivity of n consumers

A: Network(chain)

u_k: Input(advertisement)

Want to derive optimal advertising strategies

Object function

$\max_u \left\{ J(x) = \lim_{N \to \infty} \sum_{k=0}^{N-1} \beta^k g(x_k, u_k(x_k)) \right\}$

$V(x_k) = v^t x_k$: Reward function

$C(u_k) = c^t u_k$: Cost function

$g(x_k, u_k) = V(x_k) - C(u_k)$ $v, c \in R_+^n$

$J_N(x) = v^T \tilde{A}_N x + \sum_{k=0}^{N-1} \beta^k h_{N-k}^T u_{N-k}$

$h_k^T = \beta v^t \tilde{A}_k B - c^t$

$\tilde{A}_k = \sum_{j=0}^k \beta^j A^j$

DP

$u_i^* = \begin{cases} M & i = \arg\max^+(h_\infty) \\ 0 & otherwise \end{cases}$

$\max^+(h_\infty) = \max(0, h_{1,\infty}, \cdots, h_{n,\infty})$

Similarity between Study and [5]

$$\max_u \left\{ \lim_{N\to\infty} \sum_{k=0}^{N-1} \beta^k g(x_k^l, u_k^l(x_k^l)) \right\} \leftarrow \quad g(x_k^l,u_k^l) = V(x_k^l) - C(u_k^l), \quad V(x_k) = v^t x_k \quad C(u_k) = c^t u_k$$

$$= \max_u \left\{ \sum_{k=0}^{\infty} \beta^k (v^T x_k - c^T u_k) \right\} \leftarrow \quad u'_k = -u_k$$

$$= \min_{u'} \left\{ \sum_{k=0}^{\infty} \beta^k (v^T x_k + c^T u'_k) \right\} \leftarrow \quad Q = diag(v^T) \; x_k, u_k \in R_+^n, \quad R = diag(c^T) \; v, c \in R_+^n \quad \|x\|_p = \left(\sum_{i=!}^{n} |x_i|^p \right)^{1/p}$$

$$= \min_{u'} \left\{ \sum_{k=0}^{\infty} \beta^k (\|Qx_k\|_1 + \|Ru'_k\|_1) \right\}$$

Optimization problem[5]

$$x_{k+1} = Ax_k + Bu_k$$

$$\min_{u'} \left\{ \sum_{k=0}^{\infty} \beta^k (\|Qx_k\|_1 + \|Ru'_k\|_1) \right\}$$

Optimal control

$$q_{k+1} = A''_d q_k + B''_d S_k$$

$$\min_{s} \left\{ \sum_{k=0}^{\infty} (\|Qq_k\|_1 + \|RS_k\|_1) \right\}$$

Optimization problem is almost same

→Derive only need to input one vehicle control?

$$\min_s \left\{ \sum_{k=0}^{N} (\|Qq_k\|_1 + \|RS_k\|_1) \right\}$$

$$= \min_s \left\{ \sum_{k=0}^{N} \left(\left\| QA_d''^k q_0 + \sum_{j=0}^{k-1} A_d''^j B''_d S_{k-j-1} \right\|_1 + \|RS_k\|_1 \right) \right\}$$

$Q \geq 0 \;\; q_0 \geq 0$ $\qquad S_k \geq 0 \;\; R \geq 0$

⇩

S_1^*

⇩

DP

$J_{k+1} \geq J_k$?

⇩

S_k^*

$A, B \geq 0$?

$$= \min_s \left\{ \sum_{k=0}^{N} \left(q^T \left(A_d''^k q_0 + \sum_{j=0}^{k-1} A_d''^j B''_d S_{k-j-1} \right) + r^T S_k \right) \right\}$$

$diag(q) = Q$

$diag(r) = R$

S_1^*

DP

$$\sum_{i=1}^{n} S_{i,k} = \begin{cases} L_m - (x_1 - x_n) & (L_m - (x_1 - x_n) \geq 0) \\ 0 & (L_m - (x_1 - x_n) < 0) \end{cases} \quad ?$$

Reference

[1]L. D. Baskar, B. D. Schutter and H. Hellendoorn, "Model-based Predictive Traffic Control for Intelligent Vehicle: Dynamic Speed Limits and Dynamic Lane Allocation," *Proc. of 2008 IEEE Intelligent Vehicles Sympsium(IV08)*, pp. 174-179, 2008

[2]L. D. Baskar, B. D. Schutter and H. Hellendoorn, "Dynamic Speed Limits and On-ramp Metering for IVHS Using Model Predictive Control," *Proc. of the 11th IEEE Conference on Intelligent Transportation Systems*, pp. 821-826, 2008

[3]J. Zhou and H. Peng, "Range Policy of Adaptive Cruise Control Vehicles for Improved Flow Stability and String Stability," *IEEE Transactions on Intelligent Transportation Systems*, Vol. 6, No. 2, pp. 229-237, 2005

[4]A. Rantzer, "Dynamic Dual Decomposition for Distributed Control," *ACC '09*, 2009

[5]G. C. Chasparis and J. S. Shamma, "Control of Preferences in Social Networks," *Proc. of the 49th IEEE Conference on Decision and Control* ,pp. 6651-6656, 2010

Appendix

String Stability

Definition[1]

Consider a string of N dynamic systems .the error signals $e(t)$ *depends on the disturbances* $d(t)$ *in the following manner:*

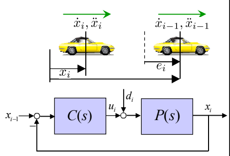
$$e(t) = H_{e,d}(s) d(t) \qquad e, d \in R^n \qquad H_{n,d}(s): R^N \to R^N \qquad (*)$$

The system (*) is L_2 *string stable if given any* $\varepsilon > 0$ *there exist a* $\delta > 0$ *such that*

$$\|d(\cdot)\|_2 < \delta \Rightarrow \|e(\cdot)\|_2 < \varepsilon$$

Assumption

- LTI SISO plant/controller
- Each loop has relative degree
- Homogeneous loop



Deformation

$$\|d(\cdot)\|_2 < \delta \Rightarrow \|e(\cdot)\|_2 < \varepsilon$$

⇕ $\quad \|G(s)\|_\infty = \sup_{d(t)} \frac{\|e(t)\|_2}{\|d(t)\|_2}$ [2]

$$\|H_{e,d}(s)\|_\infty < \gamma \qquad \gamma = \frac{\varepsilon}{\delta}$$

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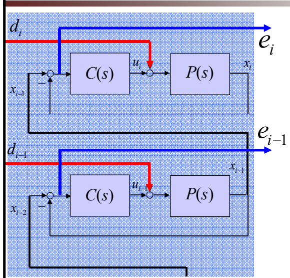
x_i : ith vehicle's position

u_i : input

d_i : disturbance

e_i : error

String Stability



From [3],

If $\left\| \frac{e_i(s)}{e_{i-1}(s)} \right\|_\infty < 1$, then $\exists \gamma > 0$ such that

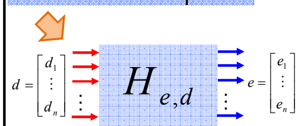
$$\|H_{e,d}(s)\|_\infty < \gamma, \forall N$$

sufficient condition:

$$\left\| \frac{e_i(s)}{e_{i-1}(s)} \right\|_\infty < 1$$

⇩

The perturbation doesn't propagate to following vehicles



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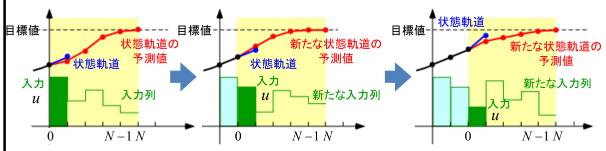
予測制御

制御対象（線形時不変システム）: $x(k+1) = Ax(k) + Bu(k)$



予測制御

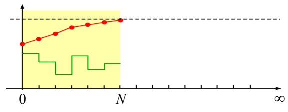
1) 各時刻 k において観測した状態を x_0 として有限時間最適制御問題を解く
2) 最適入力列の最初のステップのものを実際にシステムに入力する
3) 次の時間には1)に戻って同様の操作を繰り返す

予測制御則

最適制御則: $u(k) = K(k)x(k)$

$$U^{N,o} = \begin{bmatrix} u^o(0) \\ \vdots \\ u^o(N-1) \end{bmatrix} = \begin{bmatrix} K(0)x(0) \\ \vdots \\ K(N-1)x(N-1) \end{bmatrix}$$

最適（optimal）



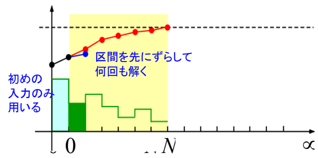
予測制御

時刻 k において観測した $x(k)$ を初期状態 $x_0 = x(k)$ として最適制御問題を解き，その最初のステップを入力

→ $u(k) = K(0)x(k)$

次のステップでは $x_0 = x(k+1)$ として同じ操作

→ $u(k+1) = K(0)x(k+1)$



（制約なし）予測制御則

$u(k) = K(0)x(k)$

（線形，時不変，状態フィードバック）

制約付き有限時間最適制御問題

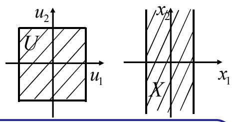
制御対象（線形時不変システム）: $x(k+1) = Ax(k) + Bu(k)$

制約条件（線形制約）

$u(k) \in U, x(k) \in X \quad \forall k$

例：バネマスダンパ系　力 −5N〜5N

位置 −1m〜1m



初期状態 $x(0) = x_0$ が与えられたとき，制約条件 $x(k) \in X$ $u(k) \in U$，$\forall k \in \{0, \cdots, N-1\}$ を満足した上で評価関数 J を最小化する入力列 $U^N = (u(0), \cdots, u(N-1))$ を求めよ

$$\min_{u(0),\cdots,u(N-1)} \sum_{k=0}^{N-1} \left(x^T(k)Qx(k) + u^T(k)Ru(k) \right) + x^T(N)P_f x(N)$$

制約条件: $x(k+1) = Ax(k) + Bu(k), \ x(0) = x_0$

$u(k) \in U, \ x(k) \in X \ \forall k \in \{0, \cdots, N-1\}$

$x(N) \in X_f$ 終端制約（最後の状態を縛る）