Visual Motion Observer with Target Object Motion Models

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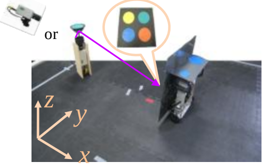
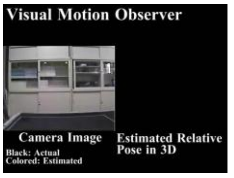
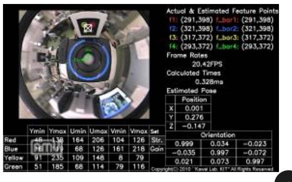
Experimental Result: Visual Motion Observer







Outline

- Basis on Rigid Body Motion on SE(3)
- Passivity-based Visual Motion Observer (VMO) [1]
- VMO with Target Object Motion Model
  - Convergence of Orientation Estimates
    Point 1: Velocity Feedback as Passivation
  - Convergence of Position Estimates
    Point 2: Two-stage-proof and Perturbation Theory
  - Class of Velocity Models
    Point 3: Developments of Software
- Concluding Remarks

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE TCST*, Vol. 15, No. 1, pp. 40-52, 2007.

Relative Pose and Body Velocity

$p_{wi} \in \mathcal{R}^3$: Position

$e^{\hat\xi\theta_{wi}} \in SO(3)$: Orientation

Pose of Vision Camera

$$g_{wc} = (p_{wc}, e^{\hat\xi\theta_{wc}}) \quad (37)$$

Pose of Object

$$g_{wo} = (p_{wo}, e^{\hat\xi\theta_{wo}}) \quad (38)$$

Pose of Object relative to Vision Camera

$$g_{co} = (p_{co}, e^{\hat\xi\theta_{co}}) \quad (39)$$

$$g_{co} = g_{wc}^{-1} g_{wo} \quad (40) \quad (g_{wo} = g_{wc} g_{co})$$

$$= \begin{bmatrix} e^{-\hat\xi\theta_{wc}} & -e^{-\hat\xi\theta_{wc}} p_{wc} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e^{\hat\xi\theta_{wo}} & p_{wo} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-\hat\xi\theta_{wc}} e^{\hat\xi\theta_{wo}} & e^{-\hat\xi\theta_{wc}}(p_{wo} - p_{wc}) \\ 0 & 1 \end{bmatrix}$$

$e^{\hat\xi\theta_{co}}$ p_{co}

Body Velocity of Vision Camera

$$V_{wc}^b = \begin{bmatrix} v_{wc}^b \\ \omega_{wc}^b \end{bmatrix} \quad (41) \qquad \hat V_{wc}^b = g_{wc}^{-1}\dot g_{wc} \quad (42)$$

Body Velocity of Object

$$V_{wo}^b = \begin{bmatrix} v_{wo}^b \\ \omega_{wo}^b \end{bmatrix} \quad (43) \qquad \hat V_{wo}^b = g_{wo}^{-1}\dot g_{wo} \quad (44)$$

Body Velocity of Object relative to Vision Camera

$$V_{co}^b = \begin{bmatrix} v_{co}^b \\ \omega_{co}^b \end{bmatrix} \quad (45) \qquad \hat V_{co}^b = g_{co}^{-1}\dot g_{co} \quad (46)$$

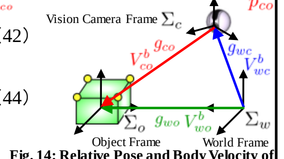


Fig. 14: Relative Pose and Body Velocity of Object relative to Vision Camera in *SE*(3)

Relative Rigid Body Motion

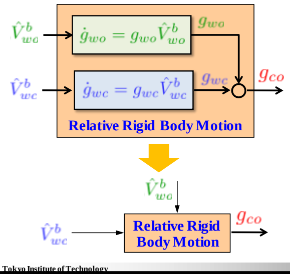
Relative Rigid Body Motion

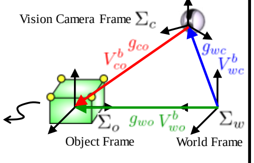
$$\dot g_{co} = -\hat V_{wc}^b g_{co} + g_{co}\hat V_{wo}^b$$

Body Velocity of Vision Camera — Body Velocity of Object

$\hat V_{wc}^b = g_{wc}^{-1}\dot g_{wc}$ ➡ $\dot g_{wc} = g_{wc}\hat V_{wc}^b$

$\hat V_{wo}^b = g_{wo}^{-1}\dot g_{wo}$ ➡ $\dot g_{wo} = g_{wo}\hat V_{wo}^b$





Feature Points

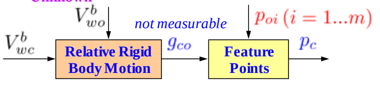
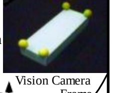


Fig. 18: Block Diagram of Relative Rigid Body Motion with Vision Camera



Object's *i*-th Feature Point

$$p_{oi}\ (i = 1, \cdots, m), m \geq 4$$

i-th Feature Point

$$p_{ci} = g_{co} p_{oi} = \begin{bmatrix} x_{ci} \\ y_{ci} \\ z_{ci} \end{bmatrix} \quad (62)$$

$$p_c = \begin{bmatrix} p_{c1} \\ \vdots \\ p_{cm} \end{bmatrix} = \begin{bmatrix} g_{co} p_{o1} \\ \vdots \\ g_{co} p_{om} \end{bmatrix} \quad (63)$$

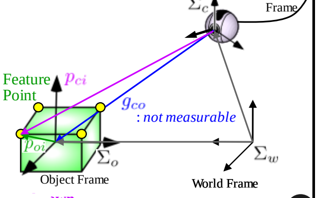
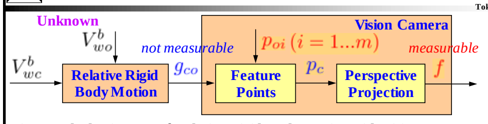


Fig. 19 : Pinhole Camera Model

Perspective Projection



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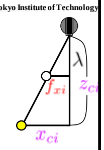

Perspective Projection (Pinhole Camera)

$$f_i = \begin{bmatrix} f_{xi} \\ f_{yi} \end{bmatrix} = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix} \quad (64)$$

Image Information (m Points)

$$f(g_{co}) = \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix} = \begin{bmatrix} \frac{\lambda}{z_{c1}} x_{c1} \\ \frac{\lambda}{z_{c1}} y_{c1} \\ \vdots \\ \frac{\lambda}{z_{cm}} x_{cm} \\ \frac{\lambda}{z_{cm}} y_{cm} \end{bmatrix} \quad (65)$$

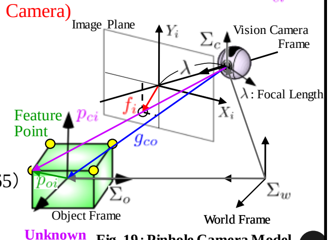

Fig. 19 : Pinhole Camera Model

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Luenberger Observer

not measurable
Actual Pose g_{co} ➡ Estimated Pose $\bar{g}_{co}$

Relative Rigid Body Motion (RRBM) | Relative Rigid Body Motion (RRBM) Model

$\dot{g}_{co} = -\hat{V}^b_{wc} g_{co} + g_{co}\hat{V}^b_{wo}$ | $\dot{\bar{g}}_{co} = -\hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co} u_e \quad (66)$

u_e : Input for Estimation Error

measurable
Image Information | Estimated Image Information

$f_i = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}$ (64) $\quad f(g_{co}) = [f_1^T, \cdots, f_m^T]^T$ (65)

$\bar{f}_i = \frac{\lambda}{\bar{z}_{ci}} \begin{bmatrix} \bar{x}_{ci} \\ \bar{y}_{ci} \end{bmatrix}$ (67) $\quad \bar{f}(\bar{g}_{co}) = [\bar{f}_1^T, \cdots, \bar{f}_m^T]^T$ (68)

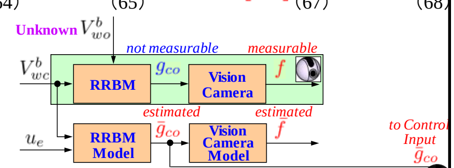

Fig. 20: Block Diagram of Estimation Error Vector

Estimation Error

Estimation Error

$$g_{ee} = \bar{g}_{co}^{-1} g_{co} \quad (69)$$

(Error between Estimated State and Actual One)

$g_{ee} = I$ ➡ $\bar{g}_{co} = g_{co}$

Estimation Error Vector

$$e_e = \begin{bmatrix} p_{ee} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ee}})^\vee \end{bmatrix} \begin{matrix} \text{(Position)} \\ \text{(Orientation)} \end{matrix}$$

$e_e = 0$ ➡ $g_{ee} = I \quad (70)$

Image Information Error

$$f_e = f - \bar{f} \quad (f_{ei} = f_i - \bar{f}_i) \quad (71)$$

(Actual) Image Information | **Estimated Image Information**

Relation between Image Information Error and Estimation Error Vector

$$f_{ei} = \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}^2_{ci}} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}^2_{ci}} \end{bmatrix} e^{\hat{\xi}\bar{\theta}_{co}} \begin{bmatrix} I & -\hat{p}_{oi} \end{bmatrix} e_e \quad (72)$$

$J_i(\bar{g}_{co})$: ***i*-th Image Jacobian**

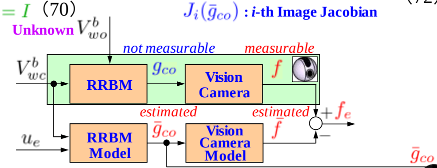

Fig. 20: Block Diagram of Estimation Error Vector

Estimation Error System

Estimation Error

$$g_{ee} = \bar{g}_{co}^{-1} g_{co} \quad (69)$$

Estimation Error Vector

$$e_e = \begin{bmatrix} p_{ee} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ee}})^\vee \end{bmatrix} \quad (70)$$

➡ Estimation error e_e can be calculated using image information f !!

$$e_e = J^\dagger(\bar{g}_{co}) f_e \quad (73) \quad (f_e = f - \bar{f})$$

Pose Information | Image Jacobian | Image Information

$$J(\bar{g}_{co}) = [J_1^T(\bar{g}_{co}), \cdots, J_m^T(\bar{g}_{co})]^T$$

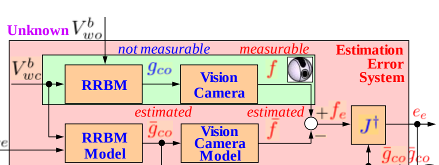

Fig. 20: Block Diagram of Estimation Error Vector

Passivity of Estimation Error System

Passivity Lemma 1
If the object is static ($V^b_{wo} = 0$), then the estimation error system satisfies

$$\int_0^T u_e^T \nu_e dt \geq -\beta_e, \ {}^\forall T > 0 \quad (76)$$

where $\nu_e = -e_e$ and β_e is a positive scalar.

Passive u_e ➡ ν_e

$$u_e = \begin{bmatrix} v^b_{ue} \\ \omega^b_{ue} \end{bmatrix}$$

Storage Function

$$V_e = \frac{1}{2}\|p_{ee}\|^2 + \phi(e^{\hat{\xi}\theta_{ee}}) = E(g_{ee}) \quad (77)$$

(Sketch of Proof)

$$\dot{V}_e = -e_e^T u_e - p_{ee}^T \hat{\omega}^b_{ue} p_{ee}$$

Skew-symmetric Matrix

$$= u_e^T \nu_e \quad (78)$$

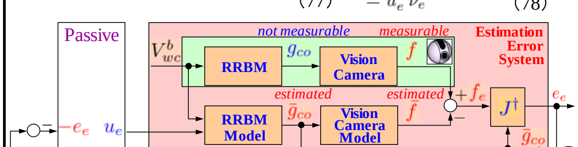

Fig. 21: Block Diagram of Passivity of Estimation Error System

Visual Motion Observer

Control Law for Visual Motion Observer : Passivity Approach

$u_e = -K_e \nu_e = -K_e(-e_e)$ (79) $K_e > 0$

Theorem 1
If $V^b_{wo} = 0$, then the equilibrium point $e_e = 0$ for the closed-loop system (75) and (79) is asymptotic stable.

Lyapunov Function Candidate

$V_e = E(g_{ee})$ (77)

$\dot{V}_e = u_e^T \nu_e$

$= -(-e_e)^T K_e (-e_e) V^b_{wo}$

< 0 (80)

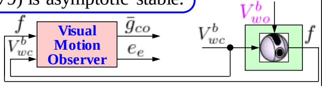

Fig. 23: Block Diagram of Visual Motion Observer

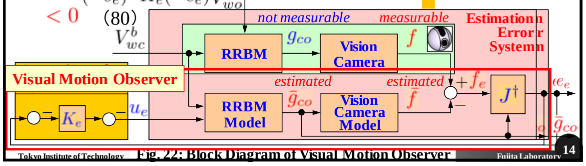

Fig. 22: Block Diagram of Visual Motion Observer

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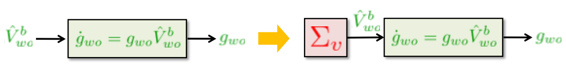
Target Object Motion

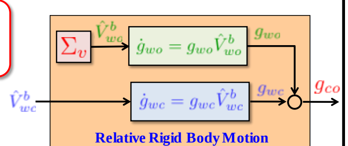


In this talk, we assume that target object has a constant velocity

$$\Sigma_v \ \dot{V}^b_{wo} = 0 \in \mathcal{R}^6$$

Then, the relative rigid body motion is reformulated by

$$\dot{V}^b_{wo} = 0$$
$$\dot{g}_{co} = -\hat{V}^b_{wc} g_{co} + g_{co} \hat{V}^b_{wo}$$



Target Object Motion Model

Actual

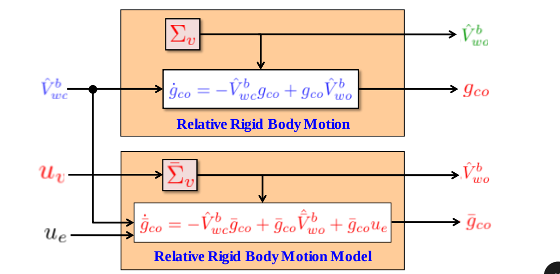
$\dot{V}^b_{wo} = 0$

$\dot{g}_{co} = -\hat{V}^b_{wc} g_{co} + g_{co} \hat{V}^b_{wo}$

Estimated

$\dot{\bar{V}}^b_{wo} = -u_v$

$\dot{\bar{g}}_{co} = -\hat{V}^b_{wc} \bar{g}_{co} + \bar{g}_{co} \hat{\bar{V}}^b_{wo} + \bar{g}_{co} u_e$



Computation of Estimation Error

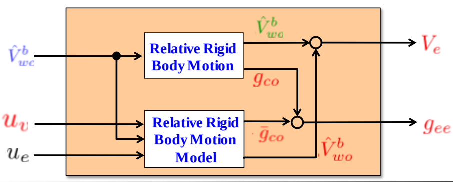
$$\dot{g}_{ee} = \frac{d}{dt}(\bar{g}_{co}^{-1} g_{co}) = -(\bar{g}_{co}^{-1})\dot{\bar{g}}_{co}(\bar{g}_{co}^{-1}) g_{co} + \bar{g}_{co}^{-1}\dot{g}_{co}$$

$$\left[\dot{g}_{co} = -\hat{V}^b_{wc} g_{co} + g_{co}\hat{V}^b_{wo} \quad \dot{\bar{g}}_{co} = -\hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co}\hat{\bar{V}}^b_{wo} + \bar{g}_{co}u_e \right]$$

$$= -(\bar{g}_{co}^{-1})\{\bar{g}_{co}\hat{\bar{V}}^b_{wo} - \hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co}\hat{u}_e\} g_e + \bar{g}_{co}^{-1}\{g_{co}\hat{V}^b_{wo} - \hat{V}^b_{wc} g_{co}\}$$

$$= -\hat{\bar{V}}^b_{wo} g_{ee} - \hat{u}_e g_{ee} + g_{ee}\hat{V}^b_{wo}$$

$$V_e = V^b_{wo} - \bar{V}^b_{wo} \quad \Rightarrow \quad \dot{V}_e = \dot{V}^b_{wo} - \dot{\bar{V}}^b_{wo} = 0 - (-u_v) = u_v$$



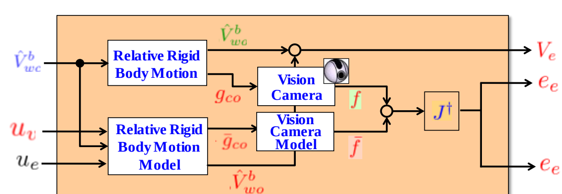
Estimation Error System (Augmented)

Estimation Error System

$\dot{V}_e = u_v$ $\Rightarrow$ Passive with Storage function $S(V_e) = \|V_e\|^2/2$

$\dot{g}_{ee} = -\hat{\bar{V}}^b_{wo} g_{ee} - \hat{u}_e g_{ee} + g_{ee}\hat{V}^b_{wo}$

Controlled Output: (e_e, V_e) Measured Output: $f_e \rightarrow e_e$



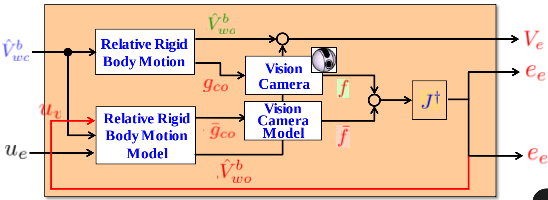
Closing An Inner Loop

Estimation Error System with Inner Loop $u_v = -Ke_e$

$$\dot V_e = -Ke_e$$

$$\dot g_{ee} = -\hat{\bar V}^b_{wo} g_{ee} - \hat u_e g_{ee} + g_{ee}\hat V^b_{wo}$$

Controller Output: (e_e, V_e) Measured Output: $f_e \rightarrow e_e$



Division into Position and Orientation Parts

$$\dot g_{ee} = -\hat{\bar V}^b_{wo} g_{ee} - \hat u_e g_{ee} + g_{ee}\hat V^b_{wo} \qquad [u_e = (v_{ue}, \omega_{ue})]$$

$$\frac{d}{dt}\begin{bmatrix} e^{\hat\xi\theta_{ee}} & p_{ee} \\ 0 & 1\end{bmatrix} = -\begin{bmatrix} \hat{\bar\omega}_{wo} + \hat\omega_{ue} & \bar v^b_{wo} + v_{ue} \\ 0 & 0\end{bmatrix}\begin{bmatrix} e^{\hat\xi\theta_{ee}} & p_{ee} \\ 0 & 1\end{bmatrix} + \begin{bmatrix} e^{\hat\xi\theta_{ee}} & p_{ee} \\ 0 & 1\end{bmatrix}\begin{bmatrix} \hat\omega_{wo} & v^b_{wo} \\ 0 & 0\end{bmatrix}$$

$$= -\begin{bmatrix} (\hat{\bar\omega}_{wo} + \hat\omega_{ue})e^{\hat\xi\theta_{ee}} & (\hat{\bar\omega}_{wo} + \hat\omega_{ue})p_{ee} + \bar v^b_{wo} + v_{ue} \\ 0 & 0\end{bmatrix} + \begin{bmatrix} e^{\hat\xi\theta_{ee}}\hat\omega_{wo} & e^{\hat\xi\theta_{ee}} v_{wo} \\ 0 & 0\end{bmatrix}$$

$$\dot e^{\hat\xi\theta_{ee}} = e^{\hat\xi\theta_{ee}}\hat\omega_{wo} - (\hat{\bar\omega}_{wo} + \hat\omega_{ue})e^{\hat\xi\theta_{ee}}$$

$$\dot p_{ee} = e^{\hat\xi\theta_{ee}} v^b_{wo} - (\hat{\bar\omega}_{wo} + \hat\omega_{ue})p_{ee} - \bar v^b_{wo} - v_{ue}$$

Inner Loop $u_v = -Ke_e$ ➡ $\begin{bmatrix} v_{uv} \\ \omega_{uv}\end{bmatrix} = -K\begin{bmatrix} e_{ep} \\ e_{eR}\end{bmatrix}$ ➡ $\begin{bmatrix} \dot v_e \\ \dot\omega_e\end{bmatrix} = -K\begin{bmatrix} e_{ep} \\ e_{eR}\end{bmatrix}$

Evolution of Orientation Error

$$\dot e^{\hat\xi\theta_{ee}} = e^{\hat\xi\theta_{ee}}\hat\omega_{wo} - (\hat{\bar\omega}_{wo} + \hat\omega_{ue})e^{\hat\xi\theta_{ee}}$$

$$\dot p_{ee} = e^{\hat\xi\theta_{ee}} v^b_{wo} - (\hat{\bar\omega}_{wo} + \hat\omega_{ue})p_{ee} - \bar v^b_{wo} - v_{ue}$$

Orientation evolution is independent of the position evolution while position evolution depends on orientation evolution

We first consider the orientation part $\dot e^{\hat\xi\theta_{ee}} = e^{\hat\xi\theta_{ee}}\hat\omega_{wo} - (\hat{\bar\omega}_{wo} + \hat\omega_{ue})e^{\hat\xi\theta_{ee}}$

Energy Function: $\phi(e^{\hat\xi\theta_{ee}}) = \frac{1}{2}\mathrm{tr}(I_3 - e^{\hat\xi\theta_{ee}})$

Time Derivative of $\phi(e^{\hat\xi\theta_{ee}})$ along with (1):

$$\dot\phi(e^{\hat\xi\theta_{ee}}) = (\mathrm{sk}(e^{\hat\xi\theta_{ee}})^\vee)^T(\omega_{wo} - e^{\hat\xi\theta_{ee}}(\bar\omega_{wo} + \omega_{ue}))$$

$$= (\mathrm{sk}(e^{\hat\xi\theta_{ee}})^\vee)^T(\omega_{wo} - (\bar\omega_{wo} + \omega_{ue})) \quad [(\mathrm{sk}(R)^\vee)^T R = (\mathrm{sk}(R)^\vee)^T]$$

$$= e^T_{eR}\omega_e - e_{eR}\omega_{ue} \quad [e_e = (e_{ep}, e_{eR}) = (p_e, \mathrm{sk}(e^{\hat\xi\theta_{ee}})^\vee)]$$

Time Derivative of Energy and Lemma 1

$\dot\phi(e^{\hat\xi\theta_{ee}}) = e^T_{eR}\omega_e - e_{eR}\omega_{ue}$ The system would be passive from ω_{ue} to $-e_{eR}$ without the first term $e^T_{eR}\omega_e$

Inner Loop $u_v = -Ke_e$ ➡ $\begin{bmatrix} \dot v_e \\ \dot\omega_e\end{bmatrix} = -K\begin{bmatrix} e_{ep} \\ e_{eR}\end{bmatrix}$

$\dot V_e = -Ke_e$

$$S_\omega = \frac{1}{2K}\|\omega_e\|^2 \Rightarrow \dot S_\omega = \frac{1}{K}\omega_e^T(-Ke_{eR}) = -\omega_e^T e_{eR}$$

Estimation Error System with Inner Loop $u_v = -Ke_e$

$$(1)\begin{cases} \dot\omega_e = -Ke_{eR} \\ \dot e^{\hat\xi\theta_{ee}} = e^{\hat\xi\theta_{ee}}\hat\omega_{wo} - (\hat{\bar\omega}_{wo} + \hat\omega_{ue})e^{\hat\xi\theta_{ee}}\end{cases}$$

$$U_R = \phi(e^{\hat\xi\theta_{ee}}) + S_\omega \Rightarrow \dot U_R = (-e_{eR})^T\omega_{ue}$$

Lemma 1: The total system (1) is passive from ω_{ue} to $-e_{eR}$

Interpretation of Inner Loop (Point 1)

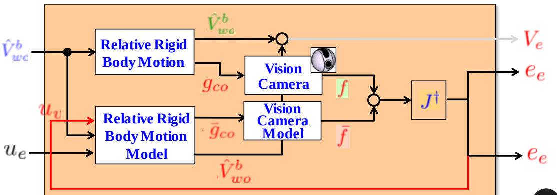
Estimation Error System with Inner Loop $u_v = -Ke_e$

$$\dot V_e = -Ke_e$$

$$\dot g_{ee} = -\hat{\bar V}^b_{wo} g_{ee} - \hat u_e g_{ee} + g_{ee}\hat V^b_{wo}$$

Controller Output: (e_e, V_e) Measured Output: $f_e \rightarrow e_e$

The inner loop is viewed as "*Passivation*" at least for orientation



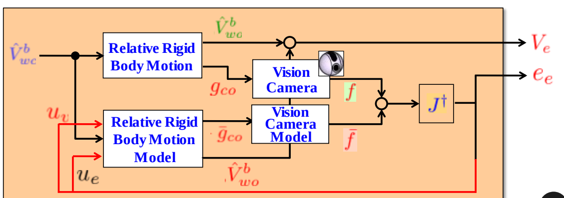
Lemma 2

Lemma 2: The trajectory of U_R along with

$$\begin{cases} \dot\omega_e = -Ke_{eR} \\ \dot e^{\hat\xi\theta_{ee}} = e^{\hat\xi\theta_{ee}}\hat\omega_{wo} - (\hat{\bar\omega}_{wo} + \hat\omega_{ue})e^{\hat\xi\theta_{ee}}\end{cases}$$

with $\omega_{ue} = (-K)(-e_{eR}) = Ke_{eR}$ is non-increasing

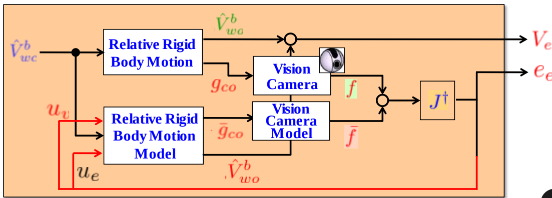
(Proof) $\dot U_R = (-e_{eR})^T\omega_{ue} = -K\|e_{eR}\|^2 \le 0$



LaSalle Invariance Principle

LaSalle Invariance Principle

Suppose that a system $\dot{x} = f(x)$ has a positively invariant set Ω. If a function V satisfies $\dot{V}(x) \le 0 \ \ \forall x \in \Omega$, then all the state trajectories converge to the largest invariant set contained in the set $E = \{x \in \Omega |\ \dot{V}(x) = 0\}$



Application of LaSalle Invariance Principle

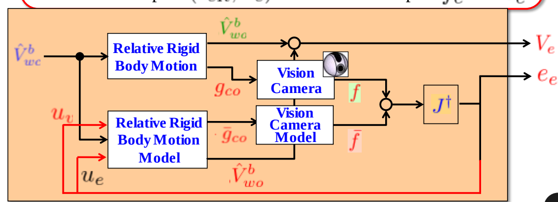
What is the state equation $\dot{x} = f(x)$ of the estimation error system?

Estimation Error System with Inner Loop $u_v = -Ke_e$

$$\begin{cases} \dot{\omega}_e = -Ke_{eR} \\ \dot{e}^{\hat{\xi}\theta_{ee}} = (e^{\hat{\xi}\theta_{ee}}\hat{\omega}_{wo} - \hat{\bar{\omega}}_{wo}e^{\hat{\xi}\theta_{ee}}) - K\mathrm{sk}(e^{\hat{\xi}\theta_{ee}})e^{\hat{\xi}\theta_{ee}} \\ \dot{\omega}_{wo} = 0 \end{cases}$$

not a function of $\omega_e = \omega_{wo} - \bar{\omega}_{wo}$

Controller Output: (e_{eR}, ω_e) Measured Output: $f_e \rightarrow e_e$



Application of LaSalle Invariance Principle

What is the set Ω for the estimation error system?

State: $(e^{\hat{\xi}\theta_{ee}}, \omega_e, \omega_{wo})$

$$\begin{cases} \dot{\omega}_e = -Ke_{eR} \\ \dot{e}^{\hat{\xi}\theta_{ee}} = (e^{\hat{\xi}\theta_{ee}}\hat{\omega}_{wo} - \hat{\bar{\omega}}_{wo}e^{\hat{\xi}\theta_{ee}}) - K\mathrm{sk}(e^{\hat{\xi}\theta_{ee}})e^{\hat{\xi}\theta_{ee}} \end{cases}$$

$\dot{U}_R = -K\|e_{eR}\|^2 \le 0 \qquad U_R = \phi(e^{\hat{\xi}\theta_{ee}}) + S_\omega$

$(e^{\hat{\xi}\theta_{ee}}, \omega_e)$ never gets out of the set $\Omega_1 = \{(e^{\hat{\xi}\theta_{ee}}, \omega_e)|\ U_R \le \text{const.}\}$

$\dot{\omega}_{wo} = 0$ never gets out of the set $\Omega_2 = \{\omega|\ \omega = \text{const.}\}$

$$\Omega = \Omega_1 \times \Omega_2$$

$\dot{V} = \dot{U}_R = -K\|e_{eR}\|^2 \le 0$ holds for all $(e^{\hat{\xi}\theta_{ee}}, \omega_e, \omega_{wo}) \in \Omega$

⇨ $E = \{(e^{\hat{\xi}\theta_{ee}}, \omega_e, \omega_{wo})|\ \dot{U}_R = \|e_{eR}\| = 0\} = \{(e^{\hat{\xi}\theta_{ee}}, \omega_e, \omega_{wo})|\ e^{\hat{\xi}\theta_{ee}} = I\}$

Application of LaSalle Invariance Principle

In the set $E = \{(e^{\hat{\xi}\theta_{ee}}, \omega_e, \omega_{wo})|\ e^{\hat{\xi}\theta_{ee}} = I\}$, we have

$$\dot{e}^{\hat{\xi}\theta_{ee}} = (e^{\hat{\xi}\theta_{ee}}\hat{\omega}_{wo} - \hat{\bar{\omega}}_{wo}e^{\hat{\xi}\theta_{ee}}) - K\mathrm{sk}(e^{\hat{\xi}\theta_{ee}})e^{\hat{\xi}\theta_{ee}}$$

⇨ $\dot{e}^{\hat{\xi}\theta_{ee}} = \hat{\omega}_{wo} - \hat{\bar{\omega}}_{wo} = 0$ The velocity estimate is equal to the actual one

From LaSalle invariance principle, all the trajectories of $(e^{\hat{\xi}\theta_{ee}}, \omega_e, \omega_{wo})$ asymptotically converge to the set

$$E = \{(e^{\hat{\xi}\theta_{ee}}, \omega_e, \omega_{wo})|\ e^{\hat{\xi}\theta_{ee}} = I,\ \bar{\omega}_{wo} = \omega_{wo}\}$$

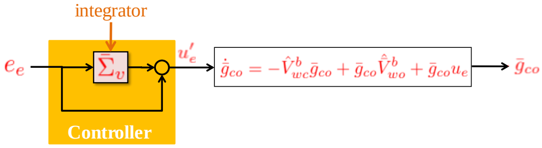
Theorem 1: for the constant velocity motion $\dot{V}^b_{wo} = 0 \in \mathcal{R}^6$, the observer input $u_v = -Ke_e$ $\omega_{ue} = (-K)(-e_{eR}) = Ke_{eR}$ leads the estimates $(e^{\hat{\xi}\bar{\theta}_{wo}}, \bar{\omega}_{wo})$ to the actual values $(e^{\hat{\xi}\theta_{wo}}, \omega_{wo})$

Internal Model Principle

Relative Rigid Body Motion Model

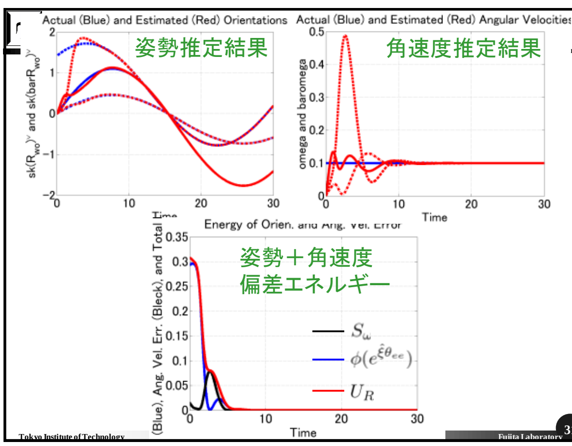
$$\dot{\bar{g}}_{co} = -\hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co}\hat{\bar{V}}^b_{wo} + \bar{g}_{co}u_e$$

$$= -\hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co}(\hat{\bar{V}}^b_{wo} + u_e) = -\hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co}u'_e$$



"P control" → "PI control"

Internal Model Principle for a class of nonlinear systems



General Motion

Actual

$\dot{x}_v = f(x_v)$

$V^b_{wo} = h(x_v)$

$\dot{g}_{co} = -\hat{V}^b_{wc} g_{co} + g_{co}\hat{V}^b_{wo}$

Estimated

$\dot{\bar{x}}_v = f(\bar{x}_v) + g(\bar{x}_v)u_v$

$\bar{V}^b_{wo} = h(\bar{x}_v)$

$\dot{\bar{g}}_{co} = -\hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co}\hat{\bar{V}}^b_{wo} + \bar{g}_{co}u_e$

Velocity Error System

$\dot{x}_\omega = f_\omega(x_\omega)$

$\dot{\bar{x}}_\omega = f_\omega(\bar{x}_\omega) + g_\omega(\bar{x}_\omega)\omega_{uv}$

$\omega_e = h_\omega(x_\omega) - h_\omega(\bar{x}_\omega)$

Orientation Error System

$\dot{e}^{\hat{\xi}\theta_{ee}} = e^{\hat{\xi}\theta_{ee}}\hat{\omega}_{wo} - (\hat{\bar{\omega}}_{wo} + \hat{\omega}_{ue})e^{\hat{\xi}\theta_{ee}}$

Assumption 1.1: the velocity error system is passive from ω_{uv} to ω_e with a storage function S_ω ⇒ $\dot{S}_\omega \le \omega_e^T\omega_{uv}$

Assumption 1.2: the state x_ω is bounded → positively invariance

Theorem 1

$\dot{\phi}(e^{\hat{\xi}\theta_{ee}}) = e_{eR}^T\omega_e - K\|e_{eR}\|^2 \quad \dot{S}_\omega/K \le \frac{1}{K}\omega_e^T(-Ke_{eR}) = -\omega_e^T e_{eR}$

⇒ $\dot{U}_R \le -K\|e_{eR}\|^2 \le 0$

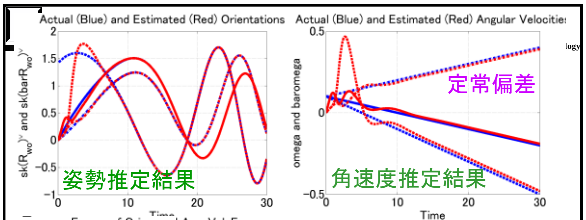
Corollary 1: Suppose that the target object motion satisfies Assumption 1.1 and 1.2, then the observer input

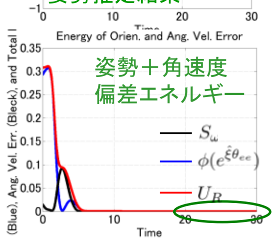
$u_v = -Ke_e \quad \omega_{ue} = (-K)(-e_{eR}) = Ke_{eR}$

leads the estimates $(e^{\hat{\bar{\xi}}\bar{\theta}_{wo}}, \bar{\omega}_{wo})$ to the actual $(e^{\hat{\xi}\theta_{wo}}, \omega_{wo})$

Conjecture : Under only assumption 1.1, at least the orientation estimate converges to the actual value.

$$\dot{U}_R = -K\|e_{eR}\|^2 \le 0$$





Constant Acceleration Model
Not satisfies Assumption 1.2

Outline

- Basis on Rigid Body Motion on SE(3)
- Passivity-based Visual Motion Observer (VMO) [1]
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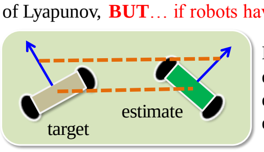
    Point 1: Velocity Feedback as Passivation
  - Convergence of Position Estimates
    Point 2: Two-stage-proof and Perturbation Theory
  - Class of Velocity Models
    Point 3: Developments of Software
- Concluding Remarks

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE TCST*, Vol. 15, No. 1, pp. 40-52, 2007.

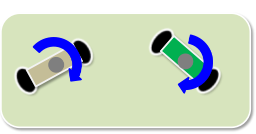
Body? World? Spatial?

In all of the research works at Fujita Lab. including pose sync. and flocking, it is proved that both of the position and orientation errors monotonically decrease simultaneously in order to use a framework of Lyapunov, **BUT**… if robots have constant **BODY** velocity…



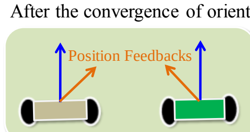
In the presence of the orientation error, the position error can increase even in the presence of the position error feedbacks

In case of orientation, the body angular velocity is independent of the orientation error. The orientation evolution is independent of the position
→ Orientations get equal



Body? World? Spatial?

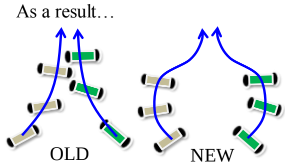
After the convergence of orientations …



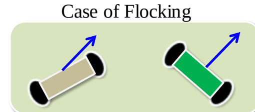
The velocities are equal from the inertial frame and hence the situation becomes equal to the case without body velocity

The energy of the position error can temporarily increase and hence we need a framework other than a simple Lyapunov Theorem

As a result…



Case of Flocking



never executed without a common knowledge on the reference frame

Evolution of Position Error

$\dot v_e = v_{uv}, \quad v_e = v_{wo} - \bar v_{wo}$ (Constant velocity)

$\dot p_{ee} = e^{\hat\xi\theta_{ee}} v^b_{wo} - (\hat{\bar\omega}_{wo} + \hat\omega_{ue}) p_{ee} - \bar v_{wo} - v_{ue}$

$u_v = -K e_e, \quad \omega_{ue} = e_{eR}$

$\dot v_e = -K p_{ee}$

$\dot p_{ee} = e^{\hat\xi\theta_{ee}} v_{wo} - (\hat{\bar\omega}_{wo} + Ksk(e^{\hat\xi\theta_{ee}}))p_{ee} - \bar v_{wo} - v_{ue}$

$= v_{wo} - \bar v_{wo} - \left(v_{ue} + \hat{\bar\omega}_{wo}p_{ee}\right) - \left((I - e^{\hat\xi\theta_{ee}})v_{wo} + Ksk(e^{\hat\xi\theta_{ee}})p_{ee}\right)$

$v_{ue} = v'_{ue} - \hat{\bar\omega}_{wo}p_{ee}$

$= v_e - v'_{ue} - \left((I - e^{\hat\xi\theta_{ee}})v_{wo} - Ksk(I - e^{\hat\xi\theta_{ee}})p_{ee}\right)$

depends on orientation

$v'_{ue} = K p_{ee} \ (v_{ue} = (KI - \hat{\bar\omega}_{wo})p_{ee})$

Evolution of Position Error

$$\begin{cases} \dot v_e = -K p_{ee} \\ \dot p_{ee} = v_e - K p_{ee} - \left((I - e^{\hat\xi\theta_{ee}})v^b_{wo} - Ksk(I - e^{\hat\xi\theta_{ee}})p_{ee}\right) \end{cases}$$

$x_e = \begin{bmatrix} p_{ee} \\ v_e \end{bmatrix}$ ⇒ $\dot x_e = \begin{bmatrix} -KI & I \\ -KI & 0 \end{bmatrix} x_e + G$ **(perturbation)**

nominal (exponential stable)

$G = \begin{bmatrix} Ksk(I - e^{\hat\xi\theta_{ee}}) & 0 \\ 0 & 0 \end{bmatrix} x_e - (I - e^{\hat\xi\theta_{ee}})v_{wo}$

$\|G\|_2 \le \left\| \begin{bmatrix} Ksk(I - e^{\hat\xi\theta_{ee}}) & 0 \\ 0 & 0 \end{bmatrix} \right\|_\infty \|x_e\|_2 + \|I - e^{\hat\xi\theta_{ee}}\|_\infty \|v_{wo}\|_2$

$\le K\|I - e^{\hat\xi\theta_{ee}}\|_\infty \|x_e\|_2 + \|I - e^{\hat\xi\theta_{ee}}\|_\infty \|v_{wo}\|_2$

$\le K\|I - e^{\hat\xi\theta_{ee}}\|_F \|x_e\|_2 + \|I - e^{\hat\xi\theta_{ee}}\|_F \|v_{wo}\|_2$

$\le 2\sqrt{\phi(e^{\hat\xi\theta_{ee}})}\|x_e\|_2 + 2\sqrt{\phi(e^{\hat\xi\theta_{ee}})}\|v_{wo}\|_2$

Theory of Perturbed System (Point 2)

$\|G\|_2 \le \underbrace{2\sqrt{\phi(e^{\hat\xi\theta_{ee}})}}_{\gamma(t)}\|x_e\|_2 + \underbrace{2\sqrt{\phi(e^{\hat\xi\theta_{ee}})}\|v_{wo}\|_2}_{\delta(t)}$ $\quad \gamma(t) \to 0$ (Theorem 1)

System with exponentially stable equilibrium and perturbation
(Nonlinear Systems, Sec. 9.3)

$\dot x_e = \begin{bmatrix} -KI & I \\ -KI & 0 \end{bmatrix} x_e + G \quad \|G\| \le \gamma(t)\|x_e\| + \delta(t)$

U_p: Lyanunov Function for Nominal System ⇒ $W = U_p^{1/2}$

$\dot W = -(c_1 - c_2\gamma(t))W + c_3\delta(t), \ c_1, c_2, c_3 > 0$

$\gamma(t) \to 0$

$\dot W = -c_1 W + c_3\delta(t), \ c_1, c_2, c_3 > 0$

Theorem 2

$\dot W = -c_1 W + c_3\delta(t), \ c_1, c_2, c_3 > 0$

Assumption 2: $\delta(t) = 2\sqrt{\phi(e^{\hat\xi\theta_{ee}})}\|v_{wo}\|_2 \to 0$ **(the velocity increases slower than the decay of the square root of the energy function),**

Then, both of the position and linear velocity estimates converge to actual values

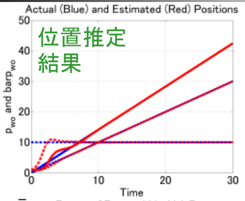
Assumption 3: the target model provides a nominal model with an exponential stable origin such as constant velocity.

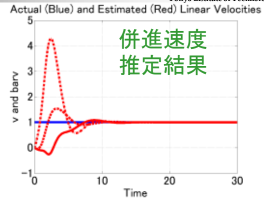
Theorem 2: Suppose that the target object motion satisfies Assumptions 1.1, 1.2, 2 and 3, then the observer input

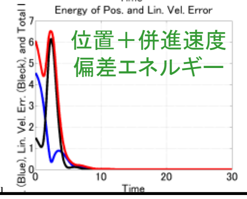
$u_v = -K e_e \quad u_e = \begin{bmatrix} KI - \hat{\bar\omega} & 0 \\ 0 & KI \end{bmatrix} e_e$

achieves all the estimates converge to their actual values

Constant Velocity

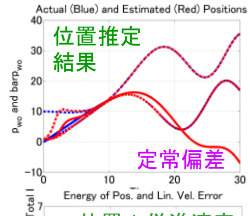


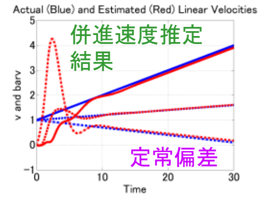


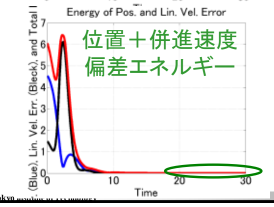


Assumption 2 is satisfied

Constant Acceleration







Assumption 2 is not satisfied

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On Assumptions

Target Object Motion Model

$\dot{x}_V = f(x_V)$, $V^b_{wo} = h(x_V)$ ⇒ $\dot{x}_v = f_v(x_v)$, $v_{wo} = h_v(x_v)$; $\dot{x}_\omega = f_\omega(x_\omega)$, $\omega_{wo} = h_\omega(x_\omega)$

Assumption 1.1: the velocity error system is passive from ω_{uv} to ω_e with a storage function S_ω ⇒ $\dot{S}_\omega \leq \omega_e^T \omega_{uv}$

Assumption 1.2: the state x_ω is bounded → positively invariance

Assumption 2: $\delta(t) = 2\sqrt{\phi(e^{\hat{\xi}\theta_{ee}})}\|v_{wo}\|_2 \to 0$ **(the velocity increases slower than the decay of the square root of the energy function),**

Assumption 3: the target model provides a nominal model with an exponential stable origin such as constant velocity.

A Variety of Target Motion

$$\omega_{wo}(t) = \sin(at) \to \ddot{\omega}_{wo}(t) = -a^2 \sin(at)$$

$$\frac{d}{dt}\begin{bmatrix}\omega_{wo}\\ \dot{\omega}_{wo}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -a^2 & 0\end{bmatrix}\begin{bmatrix}\omega_{wo}\\ \dot{\omega}_{wo}\end{bmatrix}$$

$\begin{bmatrix}\omega_{wo}\\ \dot{\omega}_{wo}\end{bmatrix}$: bounded (Assumption 1.2)

$$\frac{d}{dt}\begin{bmatrix}\bar{\omega}_{wo}\\ \dot{\bar{\omega}}_{wo}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -a^2 & 0\end{bmatrix}\begin{bmatrix}\bar{\omega}_{wo}\\ \dot{\bar{\omega}}_{wo}\end{bmatrix} - \begin{bmatrix}1\\ 0\end{bmatrix}\omega_{uv}$$

$$\frac{d}{dt}\begin{bmatrix}\omega_e\\ \dot{\omega}_e\end{bmatrix} = \begin{bmatrix}0 & 1\\ -a^2 & 0\end{bmatrix}\begin{bmatrix}\omega_e\\ \dot{\omega}_e\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\omega_{uv} \quad S_\omega = \begin{bmatrix}\omega_e\\ \dot{\omega}_e\end{bmatrix}^T\begin{bmatrix}1 & 0\\ 0 & 1/a^2\end{bmatrix}\begin{bmatrix}\omega_e\\ \dot{\omega}_e\end{bmatrix}$$

$$\dot{S}_\omega = \begin{bmatrix}\omega_e\\ \dot{\omega}_e\end{bmatrix}^T\begin{bmatrix}1 & 0\\ 0 & 1/a^2\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ -a^2 & 0\end{bmatrix}\begin{bmatrix}\omega_e\\ \dot{\omega}_e\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\omega_{uv}\right)$$
$$= \begin{bmatrix}\omega_e\\ \dot{\omega}_e\end{bmatrix}^T\left(\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}\omega_e\\ \dot{\omega}_e\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\omega_{uv}\right) = \omega_e\omega_{uv}$$

(Assumption 1.1)

A Variety of Target Motion

$$\omega_{wo}(t) = c_1\sin(at) + c_2\sin(bt)$$
$$\to \ddot{\omega}_{wo}(t) = -a^2 c_1\sin(at) - b^2 c_2 \sin(bt)$$

$$x_1(t) = c_1\sin(at),\ x_2(t) = c_2\sin(bt)$$

⇒ $\ddot{x}_1 = -a^2 x_1,\ \ddot{x}_2 = -b^2 x_2$, $\omega_{wo} = x_1 + x_2$

$$\frac{d}{dt}\begin{bmatrix}x_1\\ \dot{x}_1\\ x_2\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -a^2 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -b^2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ \dot{x}_1\\ x_2\\ \dot{x}_2\end{bmatrix} \qquad \frac{d}{dt}\begin{bmatrix}\bar{x}_1\\ \dot{\bar{x}}_1\\ \bar{x}_2\\ \dot{\bar{x}}_2\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -a^2 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -b^2 & 0\end{bmatrix}\begin{bmatrix}\bar{x}_1\\ \dot{\bar{x}}_1\\ \bar{x}_2\\ \dot{\bar{x}}_2\end{bmatrix} - \begin{bmatrix}1\\ 0\\ 1\\ 0\end{bmatrix}\omega_{uv}$$

$$\Rightarrow \frac{d}{dt}\begin{bmatrix}x_{e1}\\ \dot{x}_{e1}\\ x_{e2}\\ \dot{x}_{e2}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -a^2 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -b^2 & 0\end{bmatrix}\begin{bmatrix}x_{e1}\\ \dot{x}_{e1}\\ x_{e2}\\ \dot{x}_{e2}\end{bmatrix} + \begin{bmatrix}1\\ 0\\ 1\\ 0\end{bmatrix}\omega_{uv},\ \omega_e = x_{e1} + x_{e2}$$

A Variety of Target Motion

$$S_\omega = \begin{bmatrix}x_{e1}\\ \dot{x}_{e1}\end{bmatrix}^T\begin{bmatrix}1 & 0\\ 0 & 1/a^2\end{bmatrix}\begin{bmatrix}x_{e1}\\ \dot{x}_{e1}\end{bmatrix} + \begin{bmatrix}x_{e2}\\ \dot{x}_{e2}\end{bmatrix}^T\begin{bmatrix}1 & 0\\ 0 & 1/b^2\end{bmatrix}\begin{bmatrix}x_{e2}\\ \dot{x}_{e2}\end{bmatrix}$$

$$\dot{S}_\omega = \begin{bmatrix}x_{e1}\\ \dot{x}_{e1}\end{bmatrix}^T\begin{bmatrix}1 & 0\\ 0 & 1/a^2\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ -a^2 & 0\end{bmatrix}\begin{bmatrix}x_{e1}\\ \dot{x}_{e1}\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\omega_{uv}\right)$$
$$+ \begin{bmatrix}x_{e2}\\ \dot{x}_{e2}\end{bmatrix}^T\begin{bmatrix}1 & 0\\ 0 & 1/b^2\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ -b^2 & 0\end{bmatrix}\begin{bmatrix}x_{e2}\\ \dot{x}_{e2}\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\omega_{uv}\right)$$
$$= \begin{bmatrix}x_{e1}\\ \dot{x}_{e1}\end{bmatrix}^T\left(\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_{e1}\\ \dot{x}_{e1}\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\omega_{uv}\right) + \begin{bmatrix}x_{e2}\\ \dot{x}_{e2}\end{bmatrix}^T\left(\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_{e2}\\ \dot{x}_{e2}\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\omega_{uv}\right)$$
$$= (x_{e1} + x_{e2})\omega_{uv} = \omega_e\omega_{uv}$$

$\omega_{wo} = c_0 + \sum_{i=1}^{n} c_i \sin(a_i t)$ satisfies Assumptions 1.1 and 1.2

A Variety of Target Motion

$$v_{wo}(t) = \sin(at) \to \ddot{v}_{wo}(t) = -a^2\sin(at)$$

$$\frac{d}{dt}\begin{bmatrix}v_e\\ \dot{v}_e\end{bmatrix} = \begin{bmatrix}0 & 1\\ -a^2 & 0\end{bmatrix}\begin{bmatrix}v_e\\ \dot{v}_e\end{bmatrix} - \begin{bmatrix}1\\ 0\end{bmatrix}Kp_{ee} + \dot{p}_{ee} = v_e - Kp_{ee}$$

$$\frac{d}{dt}\begin{bmatrix}p_{ee}\\ v_e\\ \dot{v}_e\end{bmatrix} = \begin{bmatrix}-K & 1 & 0\\ -K & 0 & 1\\ 0 & -a^2 & 0\end{bmatrix}\begin{bmatrix}p_{ee}\\ v_e\\ \dot{v}_e\end{bmatrix}$$

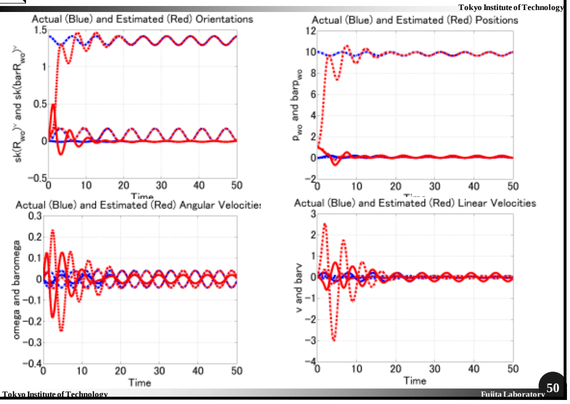
⇒ (Perhaps) stable just checked by MATLAB

$v_{wo} = \sum_{i=1}^{n} c_i\sin(a_i t)$ ⇒ $\begin{bmatrix}-K & 1 & 0 & \cdots & 0\\ & & A_1 & & \\ & & A_2 & & \\ & & \cdots & & \end{bmatrix}$ ⇒ (Perhaps) stable

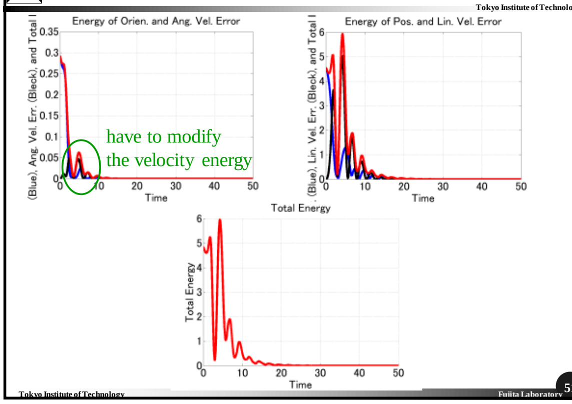
$$A_i = \begin{bmatrix}-K & \overbrace{0 & \cdots & 0}^{2(i-1)} & 0 & 1 & \overbrace{0 & \cdots & 0}^{2(n-i)}\\ 0 & 0 & \cdots & 0 & -a_i^2 & 0 & 0 & \cdots & 0\end{bmatrix}$$

$v_{wo} = c_0 + \sum_{i=1}^{n} c_i\sin(a_i t)$ (perhaps) satisfies Assumptions 2 and 3

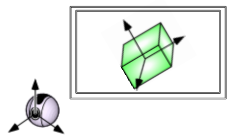
Simulation Results

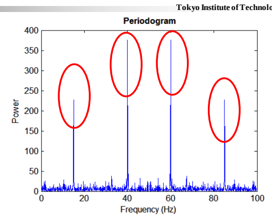


Simulation Results



Future Developments (Point 3)





MATLAB function: "fft"
for image data

Approximate !

The frequency can be flexibly adjusted according to the change of measurements

$$v_{wo} = c_0 + \sum_{i=1}^{n} c_i \sin(a_i t)$$

This adaptation can be implemented automatically

$$\omega_{wo} = c_0 + \sum_{i=1}^{n} c_i \sin(a_i t)$$

Developments of intelligent vision systems with adaptation and learning

Conclusion

In this talk, we have presented a visual motion observer with target object motion models.

- **The velocity loop is closed as a passivation of the pose error loop**
- **We have presented a two-stage procedure (position convergence after orientation convergence) in order to allow temporary position energy increase caused by the orientation error**

Roughly speaking, if the velocity error system is passive and the velocity is bounded, then we can correctly estimate both of the target object pose and body velocities