



The Monitoring of Cloud Movements with Robotic Camera Networks



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Introduction

Photovoltaic (PV) power generation [1]

The installation of PV generation systems is rapidly growing due to concerns related to environment.

Merit

- A **clean** and **environmentally-friendly** source of energy

Demerit

- Depending on weather condition and **cloud-cover**
- Low conversion efficiency

Monitoring [2]

Satellite and ground-based sky imaging tools can be used to monitor and forecast cloud movements.

Forecast of PV energy production is available in advance.



Introduction

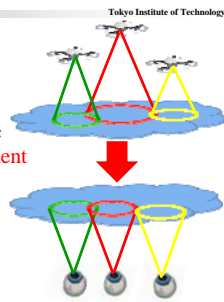
Monitoring cloud movements

Robotic camera networks

- Control for a group of aerial robots [3]
Positioning and orienting multiple robotic cameras to **collectively monitor environment**
Maximize aerial robots coverage
- The learning algorithm for potential game
Payoff-based Inhomogeneous Partially Irrational Play (PIPIP) [4]

All-sky imaging [5]

- The system includes a camera and wide-angle lens or hemispheric mirror.
- Analysis of image is based on **the red-to-blue ratio**.



Control for a group of aerial robots [3]

Camera networks

- Surveillance
- Tracking
- Catching speeding drivers
- **Gathering scientific data**

Approach

Minimum information per pixel

Get a **cost function** that represents how well a group of cameras covers an environment

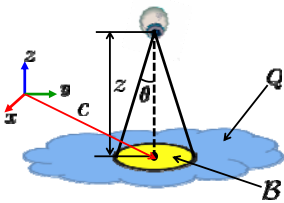
Obtain a control law by taking the negative gradient of the cost function

The controller is adaptive to the deletion or addition of cameras and to a changing environment



Control for a group of aerial robots [3]

Optimal camera placement



The state of camera i $p_i \in \mathcal{P}$
 $i = 1, 2, \dots, n$

$$p = [c^T, z]^T \quad c \in \mathbb{R}^2 \quad z \in \mathbb{R}$$

Bounded environment $Q \supset \mathbb{R}^2$

The field of view \mathcal{B}

$$\mathcal{B} = \{q \mid \frac{\|q-c\|}{z} \leq \tan\theta\}$$

We want to control n cameras such that their placement **minimizes the information per camera pixel** over the environment.

$$\min_{(p_1, \dots, p_n) \in \mathcal{P}^n} \int_Q \frac{\text{info}}{\text{pixel}} dq$$



Control for a group of aerial robots [3]

Single camera

The area per pixel $f(p, q)$

$$\min_p \int_Q \frac{f(p, q) \phi(q) dq}{\text{area pixel} \times \text{info area}}$$

Derived from **the optics of the camera** and **geometry of the environment**

Inside \mathcal{B}

$f(p, q)$ is equal to the inverse of the area magnification factor (defined from classical optics) times the area of one pixel.

$$f(p, q) = \frac{(b-z)^2}{b^2} \times (\text{the area of one pixel})$$

$a = \frac{(\text{the area of one pixel})}{b^2}$
 b : The focal length

The information per area $\phi(q)$

A positive weighting of importance over environment

Specified beforehand

Require more resolution, the value of $\phi(q)$ should be **large**.

Outside \mathcal{B}

There are no pixel.

$$f(p, q) = \begin{cases} a(b-z)^2 & \text{for } q \in \mathcal{B} \\ \infty & \text{otherwise} \end{cases}$$



Control for a group of aerial robots [3]

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Multiple cameras

An area that is observed by two cameras is better covered than by one camera.

It is **not twice** as well covered

The area per pixel at q

$$\frac{\text{area}}{\text{pixel}} = \left(\sum_{i=1}^n f(p_i, q)^{-1} \right)^{-1}$$

It is not the area per pixel, but the **pixel per area**.

We assume that the robots have **knowledge of the geometry of Q** , and **some notion of information content over it $\phi(q)$** .

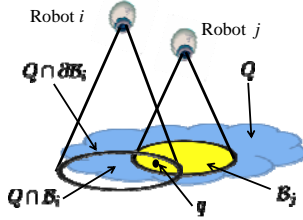
$$\frac{\text{area}}{\text{pixel}} = \left(\sum_{i=1}^n f(p_i, q)^{-1} + w^{-1} \right)^{-1} \quad \text{A prior area per pixel } w \in (0, \infty)$$

Arbitrarily vague

$$h_{N_q}(p_1, \dots, p_n, q) = \left(\sum_{i \in N_q} f(p_i, q)^{-1} + w^{-1} \right)^{-1} \quad N_q = \{i | q \in B_i\}$$

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Control for a group of aerial robots [3]

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Multiple cameras

The cost function

$$\mathcal{H}(p_1, \dots, p_n) = \int_Q h_{N_q}(p_1, \dots, p_n, q) \phi(q) dq$$

The cost function is **valid for any area per pixel function $f(p_i, q)$** , and for any camera state space \mathcal{P} .

The multiple camera optimization problem

$$\min_{(p_1, \dots, p_n) \in \mathcal{P}^n} \mathcal{H}$$

Notice that $\mathcal{H} > 0$ for all (p_1, \dots, p_n)

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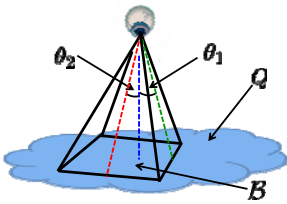


Control for a group of aerial robots [3]

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Rectangular field of view

Actual cameras have a rectangular charge coupled device (CCD) array, and therefore a **rectangular field of view**.



The state of camera i $p_i = [c_i^T \ z_i \ \psi_i^T]^T$
 ψ_i : The yaw angle

The rotation matrix

$$R(\psi_i) = \begin{bmatrix} \cos \psi_i & \sin \psi_i \\ -\sin \psi_i & \cos \psi_i \end{bmatrix}$$

The field of view B

$$B_i = \{q | |R(\psi_i)(q - c_i)| \leq z_i \tan \theta\}$$
$$\theta = [\theta_1 \ \theta_2]^T$$

The cost function $\mathcal{H}(p_1, \dots, p_n)$ is the **same as for the circular case**, as is the area per pixel function $f(p_i, q)$.

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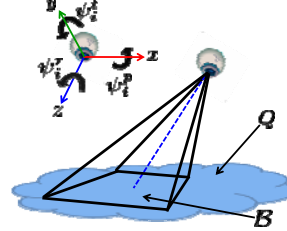
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Control for a group of aerial robots [3]

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Incorporating pan and tilt angles



The state of i th camera

$$p_i = [x_i \ y_i \ z_i \ \psi_i^t \ \psi_i^p \ \psi_i^r]^T$$

The position of the focal point

$$\rho_i = [x_i \ y_i \ z_i]^T$$

The rotation (or yaw) angle ψ_i^r

(+) The field of view **spins clockwise**

The pan (or roll) angle ψ_i^p

(+) The field of view **sweeps to the left**

The tilt (or pitch) angle ψ_i^t

(+) The field of view **sweeps to upward**

Consider two coordinate frames

The camera fixed frame of i th robot (**CF_i**)

Fixed to the camera, centered at focal point

The global fixed frame (**GF**)

Centered at a fixed origin on the ground

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Control for a group of aerial robots [3]

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To express vectors in the **CF_i**

Rotation matrices

$$R_c^i = \begin{bmatrix} \cos \psi_i^t & \sin \psi_i^t & 0 \\ -\sin \psi_i^t & \cos \psi_i^t & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_r^i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi_i^p & \sin \psi_i^p \\ 0 & -\sin \psi_i^p & \cos \psi_i^p \end{bmatrix}$$

$$R_t^i = \begin{bmatrix} \cos \psi_i^r & 0 & -\sin \psi_i^r \\ 0 & 1 & 0 \\ \sin \psi_i^r & 0 & \cos \psi_i^r \end{bmatrix}$$

To take a point x in the **GF** and express it in the **CF_i**

Translate the vector by ρ_i

→ Rotate the vector about z -axis by $\pi/2$

→ Rotate the vector about x -axis by π

→ Rotate the vector through $\psi_i^t, \psi_i^p,$ and ψ_i^r

$$R^i = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_i(\psi_i^t, \psi_i^p, \psi_i^r)(x - \rho_i)$$

$$R_i(\psi_i^t, \psi_i^p, \psi_i^r) = R_t^i R_r^i R_c^i R^i$$

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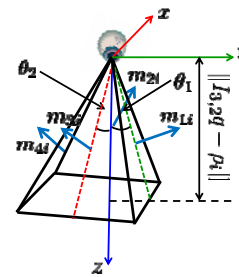
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The four outward facing unit normal vectors of faces of the pyramid

$$m_{1E} = [0 \ \cos \theta_1 \ -\sin \theta_1]^T \quad m_{2E} = [\cos \theta_2 \ 0 \ -\sin \theta_2]^T$$
$$m_{3E} = [0 \ -\cos \theta_1 \ -\sin \theta_1]^T \quad m_{4E} = [-\cos \theta_2 \ 0 \ -\sin \theta_2]^T$$

Denote them in **CF_i** frame



The vector from the focal point ρ_i to a point in the leg q is **perpendicular** to the normal of the k th pyramid face.

$$m_{kE}^T R_i(I_{3,2}q - \rho_i) = 0 \quad I_{3,2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

The field of view

$$B_i = \{q | m_{kE}^T R_i(I_{3,2}q - \rho_i) \leq 0, \ k = 1, 2, 3, 4\}$$

The area per pixel function

$$f(p_i, q) = \begin{cases} a(b - \|I_{3,2}q - \rho_i\|)^2 & \text{for } q \in B_i \\ \infty & \text{otherwise} \end{cases}$$

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Application of PIPIP

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$$\min_p \int_Q f(p, q) \phi(q) dq \xrightarrow{=} \max_p \left(- \int_Q f(p, q) \phi(q) dq \right)$$

$= \mathcal{H} > 0$ $= U_i$ The utility function

↓

Potential game
Actions
Pan and Tilt angles

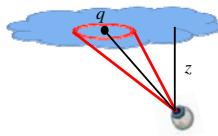
$f(p, q)$: Derived from the optics of the camera

$$f(p, q) = \begin{cases} a(b - \|I_{3,2}q - p\|)^2 & \text{for } q \in B_i \\ \infty & \text{otherwise} \end{cases}$$

Given z , $f(p, q)$ is acquired

$\phi(q)$: The separation of cloudy from clear sky

→ Cloud : high
Clear sky : low



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N. Gans et al [7]

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Shannon's entropy [7, 8]

The entropy has found wide spread use in information, communication estimation theory

Maximum entropy can indicate an information rich data set

The entropy H

$$H(X) = - \sum_{x \in X} p(x) \log_2 p(x)$$

The pixel value X, x
Discrete random variable
The occurrence probability $p(x)$

The entropy gives a measurement of the uncertainty associated with X

The random variable x is **uniformly distributed**

→ The entropy takes its **maximum**

The random variable x is **completely determined**

→ The entropy takes its **minimum**

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N. Gans et al [7]

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Shannon's entropy [7, 8]

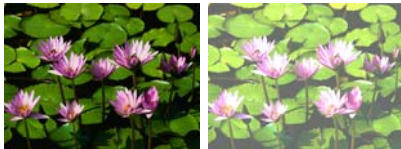
In the case of images

The image with a **single shade or color** has **zero entropy**

A blank image such as a camera facing a wall

The image with **many shades and colors** has **high entropy**

Example of similar images with different entropy levels



Poor lighting condition

Entropy **High**

Low

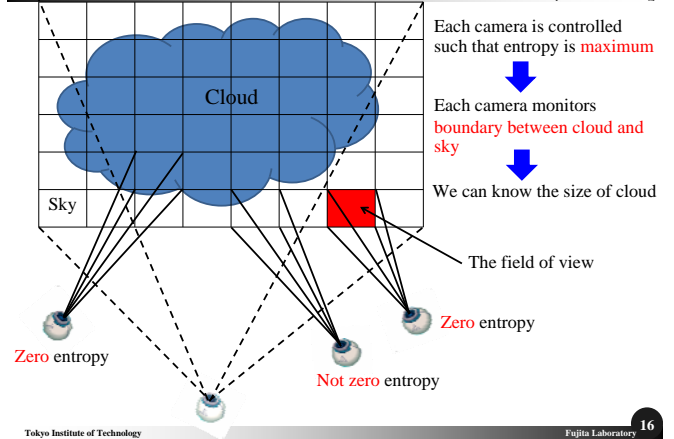
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N. Gans et al [7]

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Application of PIPIP

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Shannon's entropy

$$H(X) = - \sum_{x \in X} p(x) \log_2 p(x)$$

↓ We want to **maximize the entropy**

$$\max \left(- \sum_{x \in X} p(x) \log_2 p(x) \right)$$

$= U_i$ The utility function

↓

Potential game

Actions

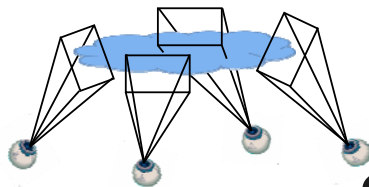
Pan and Tilt angles

$p(x)$: Acquired from the image

Image

0	0	0	0	0	0
0	2	2	1	1	0
0	2	4	4	3	0
0	2	5	2	3	0
0	1	1	1	3	0
0	0	0	0	0	0

Pixel value



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Summary

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- Formulate a cost function for **single camera which has a circle field of view**
- Formulate a cost function for **multiple cameras which have a circle field of view**
- Single camera which have a rectangular field of view
- Incorporating **pan and tilt angles**
- Study of Payoff based Inhomogeneous Partially Irrational Play (PIPI)
- Study of the monitoring of cloud with **Shannon's entropy**

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Future Works

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- The separation of cloudy from clear sky
Imaging processing
The threshold in the red-to-blue ratio [5]
- Modeling in the case of **single camera** (**six degrees of freedom**)
 $[x_i, y_i, z_i, \psi_i^r, \psi_i^p, \psi_i^z]$
- Modeling in the case of **multiple cameras** (**six degrees of freedom**)
- Compose experimental system

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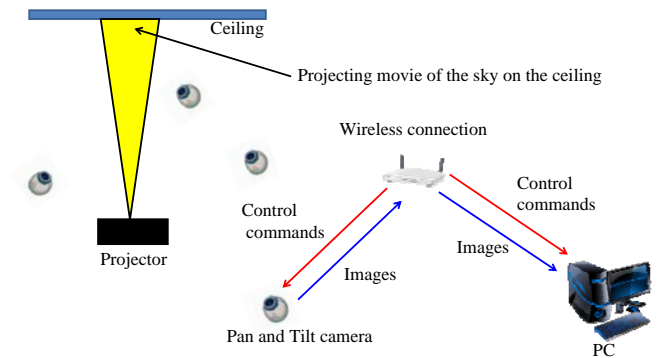
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Future Works

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Experimental system[6]



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Appendix

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Control for a group of aerial robots [3]

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Decentralize gradient based controller (Multiple cameras)

Theorem 1

- The lateral component

$$\frac{\partial \mathcal{H}}{\partial c_i} = \int_{Q_i \cap \Theta_i} (h_{N_i} - h_{N_i \setminus \{i\}}) \frac{q - c_i}{\|q - c_i\|} \phi(q) dq$$

The integral component causes the robot to **move to increase the amount of the environment in its field of view**, while also **moving away from other robots j** whose field of view overlaps with its own.

- The vertical component

$$\frac{\partial \mathcal{H}}{\partial z_i} = \int_{Q_i \cap \Theta_i} (h_{N_i} - h_{N_i \setminus \{i\}}) \phi(q) \tan \theta dq - \int_{Q_i \cap \Theta_i} \frac{2h_{N_i}^2}{a(b - z_i)^3} \phi(q) dq$$

The first integral causes the robot to move up to **bring more of the environment into its field of view**.

The second integral causes it to move down to **get a better look at the environment already in its field of view**.

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Control for a group of aerial robots [3]

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Decentralize gradient based controller (Multiple cameras)

Use a **gradient control law** in which every robot follows the negative of its own gradient component

$$w_i = -k \frac{\partial \mathcal{H}}{\partial p_i} \quad \text{The control input for robot } i \quad p_i$$

Control gain $k \in (0, \infty)$

Assuming **integrator dynamics** for the robots

$$\dot{p}_i = w_i$$

Theorem 2

$$\lim_{t \rightarrow \infty} \frac{\partial \mathcal{H}}{\partial p_i} = 0 \quad \forall i \in \{1, \dots, n\}$$

An equilibrium (p_1^*, \dots, p_n^*) defined by $\left. \frac{\partial \mathcal{H}}{\partial p_i} \right|_{p_i=p_i^*} = 0 \quad \forall i \in \{1, \dots, n\}$

is Lyapunov stable if and only if it is a local minimum of \mathcal{H} .



Control for a group of aerial robots [3]

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The gradient function of the cost function (Rectangular field of view)

- The lateral component

$$\frac{\partial \mathcal{H}}{\partial c_i} = \sum_{k=1}^4 \int_{Q_i \cap \mathcal{F}_k} (h_{N_c} - h_{N_c \setminus \{i\}}) R(\psi_i)^T n_{k,i} \phi(q) dq$$

- The vertical component

$$\frac{\partial \mathcal{H}}{\partial z_i} = \sum_{k=1}^4 \int_{Q_i \cap \mathcal{F}_k} (h_{N_c} - h_{N_c \setminus \{i\}}) \tan \theta^T n_{k,i} \phi(q) dq - \int_{Q_i \cap \mathcal{F}_i} \frac{2h_{N_c}^*}{a(b-z_i)^3} \phi(q) dq$$

- The angular component

The component rotates the robot to **get more of its field of view into the environment**, while also **rotating away from other robots** whose field of view intersects its own.

$$\frac{\partial \mathcal{H}}{\partial \psi_i} = \sum_{k=1}^4 \int_{Q_i \cap \mathcal{F}_k} (h_{N_c} - h_{N_c \setminus \{i\}}) \cdot (q - c_i)^T R(\psi_i + \pi/2)^T n_{k,i} \phi(q) dq$$



Control for a group of aerial robots [3]

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The gradient function of the cost function (Incorporating pan and tilt angles)

$$\frac{\partial \mathcal{H}}{\partial p_i} = \sum_{k=1}^4 \int_{Q_i \cap \mathcal{F}_k} (h_{N_c} - h_{N_c \setminus \{i\}}) \frac{R_i^T m_{k,i}}{\|I_{2,2} R_i^T m_{k,i}\|} \phi(q) dq + \int_{Q_i \cap \mathcal{F}_i} \frac{2h_{N_c}^*}{a(b - \|I_{2,2} q - p_i\|)^3} \frac{(I_{2,2} q - p_i)}{\|I_{2,2} q - p_i\|} \phi(q) dq$$

$$\frac{\partial \mathcal{H}}{\partial \psi_i^s} = \sum_{k=1}^4 \int_{Q_i \cap \mathcal{F}_k} (h_{N_c} - h_{N_c \setminus \{i\}}) \frac{w_{k,i}^T \frac{\partial R_i}{\partial \psi_i^s} (p_i - I_{2,2} q)}{\|I_{2,2} R_i^T m_{k,i}\|} \phi(q) dq \quad s \in \{r, p, t\}$$

Where

$$\frac{\partial R_i}{\partial \psi_i^r} = R_i^T R_i^r \begin{bmatrix} -\sin \psi_i^r & \cos \psi_i^r & 0 \\ -\cos \psi_i^r & -\sin \psi_i^r & 0 \\ 0 & 0 & 1 \end{bmatrix} R_i^r \quad \frac{\partial R_i}{\partial \psi_i^p} = R_i^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin \psi_i^p & \cos \psi_i^p \\ 0 & \cos \psi_i^p & \sin \psi_i^p \end{bmatrix} R_i^p R_i^r$$

$$\frac{\partial R_i}{\partial \psi_i^t} = \begin{bmatrix} -\sin \psi_i^t & 0 & -\cos \psi_i^t \\ 0 & 0 & 0 \\ \cos \psi_i^t & 0 & -\sin \psi_i^t \end{bmatrix} R_i^t R_i^r R_i^p$$



The learning algorithm for potential game [4]

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Payoff based Inhomogeneous Partially Irrational Play (PIPIP)

Features

- Finite and a little memory

- Payoff-based

- PIPIP allows agents to make **irrational decisions** with a certain probability

- PIPIP assures that the actions of the group **converge in probability to optimal Nash equilibria**, though only convergence to pure Nash equilibria



Optimal Nash equilibria is the Nash equilibria **maximizing** the potential function.



The learning algorithm for potential game [4]

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The set of agents $\mathcal{V} = \{1, \dots, n\}$ The iteration $t = \{0, 1, 2, \dots\}$

The collective action set \mathcal{A}, a

$$\mathcal{A} = \mathcal{A}_1 \times \dots \times \mathcal{A}_n \quad \mathcal{A}_i \quad i \in \mathcal{V}: \text{The set of agent } i\text{'s actions}$$

$$a = (a_i, a_{-i}) \quad a_i: \text{Agent } i\text{'s action}$$

$$a_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$$

The set of actions $\mathcal{R}_i(a_i)$: The set of actions which agent i will be take in case he takes an action a_i

The utility function of agent i $U_i: \mathcal{A} \rightarrow \mathbb{R}$

The potential function $\phi: \mathcal{A} \rightarrow \mathbb{R}$

Constrained potential game $\Gamma = (\mathcal{V}, \mathcal{A}, \{U_i\}_{i \in \mathcal{V}}, \{\mathcal{R}_i\}_{i \in \mathcal{V}})$



The learning algorithm for potential game [4]

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Definition (Constrained Potential games)

A constrained strategic game Γ is a **constrained potential game**

if $\exists \phi \quad \forall i \in \mathcal{V} \quad \forall a_i \in \mathcal{A}_i \quad \forall a_i' \in \mathcal{R}_i(a_i) \quad \forall a_{-i} \in \prod_{j \neq i} \mathcal{A}_j$

$$U_i(a_i', a_{-i}) - U_i(a_i, a_{-i}) = \phi(a_i', a_{-i}) - \phi(a_i, a_{-i})$$

If an changes his action,

(The change of the utility function)

= (The change of the potential function)

Definition (Constrained Nash Equilibria)

For constrained strategic game Γ , a collection of actions is a **constrained pure Nash equilibrium** if the following equation

$$U_i(a_i^*, a_{-i}^*) = \max_{a_i \in \mathcal{R}_i(a_{-i}^*)} U_i(a_i, a_{-i}^*) \quad i \in \mathcal{V}$$

Any constrained potential game has **at least one** pure Nash equilibrium

A potential function maximizer is an **optimal Nash equilibrium**

There may **exist undesirable pure Nash equilibria** not maximizing the potential function



The learning algorithm for potential game [4]

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Initialization Action a is chosen randomly from \mathcal{A} ($a = \text{rnd}(\mathcal{A})$)

$$t = 0 : a_i(0) \quad U_i(0)$$

$$t = 1 : a_i(1) = a_i(0) \quad U_i(1) = U_i(0) \quad \forall i \in \mathcal{V} \quad \kappa \in \left(\frac{1}{C-1}, \frac{1}{2} \right]$$

Step 1 Update $\varepsilon = t^{-\frac{1}{n(D+1)}}$ (Exploration rate) $C = \max_{i \in \mathcal{V}} \max_{a_i \in \mathcal{A}_i} |\mathcal{R}_i(a_i)|$

$D = \max_{i \in \mathcal{V}} D_i$ D_i : The minimal number of steps

Step 2 • $U_i(a(t-1)) \geq U_i(a(t-2))$

$$a_i^{tmp} \leftarrow \begin{cases} \text{rnd}(\mathcal{R}_i(a_i(t-1)) \setminus \{a_i(t-1)\}) & \text{w.p. } \varepsilon \\ a_i(t-1) & \text{w.p. } 1 - \varepsilon \end{cases}$$

• **Otherwise** $U_i(a(t-1)) < U_i(a(t-2))$

$$a_i^{tmp} \leftarrow \begin{cases} \text{rnd}(\mathcal{R}_i(a_i(t-1)) \setminus \{a_i(t-1), a_i(t-2)\}) & \text{w.p. } \varepsilon \\ a_i(t-1) & \text{w.p. } (1-\varepsilon)(\kappa \cdot \varepsilon^{\Delta_i}) \\ a_i(t-2) & \text{w.p. } (1-\varepsilon)(1-\kappa \cdot \varepsilon^{\Delta_i}) \end{cases}$$

Step 3 Execute the action a_i^{tmp} and receive $U_i(a^{tmp})$

Step 4 $a_i(t-1) \leftarrow a_i^{tmp}$ $a_i(t-2) \leftarrow a_i(t-1)$ $U_i(a(t-1)) \leftarrow U_i(a^{tmp})$
 $U_i(a(t-2)) \leftarrow U_i(a(t-1))$ $\Delta_i \leftarrow U_i(a(t-2)) - U_i(a(t-1))$

Step 5 $t \leftarrow t + 1$ and go to **Step 1**