Autonomous Pose Synchronization on *SE*(3): Convergence Analysis

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Introduction

Today's Objective

- Discussion about Technical Results for the 51st CDC
  (We would like to submit 2 papers for CDC: the other one is about Visual Feedback Pose Control with a Target's Velocity Model (Namba's Bachelor Thesis))

Introduction

- Autonomous Pose Synchronization on *SE*(3)
- Relative Information-based
- Flocking Algorithm?

(Next work)

Necessity of survey about recent publications about PS, Flocking: by middle of Feb.

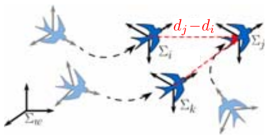
Contribution

- Autonomous approach: Completely relative information based
  Each agent does Not need global information such as defined in the world frame
- Wider class information topologies: Strongly connected groups
  Most of works assume bidirectional or balanced graphs in spite of unnaturalness in nature (Flocking [1], schooling, etc…)

[1] M. Ballerini, et al., "Interaction Ruling Animal Collective Behavior Depends on Topological rather than Metric Distance: Evidence from a Field Study," PNAC, Vol. 105, No. 4, pp. 1232-1237, 2008.

Outline

- Introduction
- Problem Settings
- Summary of Previous Results
- Main Results
  - Pose Synchronization with Desired Body Velocities
  - Pose Synchronization in Wider Class Information Topologies
- Discussions



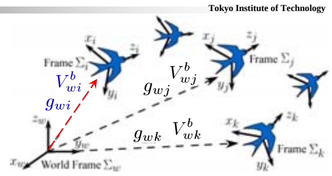
Rigid Body Motion

Kinematics of Rigid Bodies

Pose $(p_{wi}, e^{\hat{\xi}\theta_{wi}}) \in SE(3)$

$i = \{1, \cdots, n\}$

Exponential Coordinate for Rotation
- $\xi_{wi} \in \mathcal{R}^3$: rotation axis
- $\theta_{wi} \in \mathcal{R}$: rotation angle



Homogeneous Representation

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{4\times 4}$$

$$\wedge : \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$\vee$: Inverse Operator of $\wedge$

Body Velocity

$$V_{wi}^b = \begin{bmatrix} v_{wi}^b \\ \omega_{wi}^b \end{bmatrix} \in \mathcal{R}^6 \qquad \hat{V}_{wi}^b := g_{wi}^{-1}\dot{g}_{wi} = \begin{bmatrix} \hat{\omega}_{wi}^b & v_{wi}^b \\ 0 & 0 \end{bmatrix} \in \mathcal{R}^{4\times 4}$$

- $v_{wi}^b \in \mathcal{R}^3$: linear velocity
- $\omega_{wi}^b \in \mathcal{R}^3$: angular velocity

Rigid Body Motion

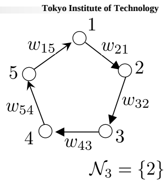
$$\dot{g}_{wi} = g_{wi}\hat{V}_{wi}^b \quad (1)$$

Rigid Body Motion: $V_{wi}^b \rightarrow \boxed{\dot{g}_{wi} = g_{wi}\hat{V}_{wi}^b} \rightarrow g_{wi} = (p_{wi}, e^{\hat{\xi}\theta_{wi}})$

Graph Topology

Graph to Represent the Interconnection Topology

Graph G: Graph consists of a triple $(\mathcal{V}, \mathcal{E}, \mathcal{W})$, where $\mathcal{V}$ is a finite nonempty set of nodes, $\mathcal{E} \subset \mathcal{V}\times\mathcal{V}$ is a set of pairs of nodes, called edges and $\mathcal{W}$ is a set of weights over the set of edges.



$\mathcal{N}_3 = \{2\}$

- $G := (\mathcal{V}, \mathcal{E}, \mathcal{W})$: Graph
- $\mathcal{V} := \{1, \cdots, n\}$: A set of vertices indexed by set of rigid-bodies
- $\mathcal{E} \subset \mathcal{V}\times\mathcal{V}$: A set of edges the represent the neighboring relations
- $e_{ij} \in \mathcal{E}$: An edge from node i to node j
- neighborhood $\mathcal{N}_i$: A set of rigid bodies whose information is available to rigid body i
- $w_{ij}, \mathcal{W}$: A weight on an edge e_{ij} and a set of weights over the set of edges

Weighted Graph Laplacian

$$L_w = \{L_{wij}\} := \begin{cases} \sum_{j\in\mathcal{N}_i} w_{ij} & \text{if } i = j \\ -w_{ij} & \text{if } j \in \mathcal{N}_i \\ 0 & \text{if } j \notin \mathcal{N}_i \end{cases}$$

Pose Synchronization

Goal: Pose Synchronization

A group of rigid-bodies are said to achieve pose synchronization if

$$\lim_{t\to\infty} \psi(y_{wi}^{-1}y_{wj}) = 0 \ \forall i, j \in \mathcal{V} \quad (2)$$

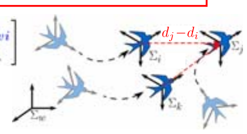
Controlled Output

$$y_{wi} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & p_{wi}+d_i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & q_{wi} \\ 0 & 1 \end{bmatrix}$$

$d_i \in \mathcal{R}^3$: position biases

$$\psi(y_{wi}) := \frac{1}{2}\|q_{wi}\|_2^2 + \phi(e^{\hat{\xi}\theta_{wi}}) \geq 0$$

$$\left(\phi(e^{\hat{\xi}\theta_{wi}}) := \frac{1}{2}\mathrm{tr}(I_3 - e^{\hat{\xi}\theta_{wi}}) \geq 0\right)$$



Pose Synchronization

$\boxed{\psi(y_{wi}) = 0 \Leftrightarrow y_{wi} = I_4}$ Eq. (2) ⇒ $y_{wi} \rightarrow y^* \ \forall i \in \mathcal{V}$

From the definition of the output $y_{wi} \in SE(3)$, pose synchronization means both virtual positions and orientations of all the rigid bodies converge to common values

Previous Result 1 [2]

Proposed Velocity Input $K_i = \mathrm{diag}(k_{pi}I_3, k_{ei}I_3) > 0$

$$V^b_{wi} = -K_i \sum_{j\in\mathcal{N}_i}\left(w_{ij}\begin{bmatrix} e^{-\hat\xi\theta_{wi}} & 0\\ 0 & I_3\end{bmatrix}\begin{bmatrix} q_{wi}-q_{wj}\\ \mathrm{sk}(e^{-\hat\xi\theta_{wj}}e^{\hat\xi\theta_{wi}})^\vee\end{bmatrix}\right) + \begin{bmatrix} e^{-\hat\xi\theta_{wi}} & 0\\ 0 & e^{-\hat\xi\theta_{wi}}\end{bmatrix}\begin{bmatrix} v_d\\ \omega_d\end{bmatrix} \quad (3)$$

relative poses

$V_d = \begin{bmatrix} v_d\\ \omega_d\end{bmatrix} \in \mathcal{R}^6$: common desired linear and angular velocity ($\omega_d := (e^{-\hat\xi\theta_d}\dot{e}^{\hat\xi\theta_d})^\vee$)

Fact 1 [2]: Pose Synchronization with Desired Velocities

Consider the n rigid bodies represented by (1). Then, under the assumptions that there exists $e^{\hat\xi\theta_a}$ such that $e^{-\hat\xi\theta_a}e^{-\hat\xi\theta_d}e^{\hat\xi\theta_{wi}}\ {}^\forall i \in \mathcal{V}$ are positive definite at the initial time and the interconnection graph G is fixed and strongly connected, velocity input (3) achieves Pose Synchronization in the sense of (2).

Technical Problem: V_d is defined in the inertial frame Σ_w

i.e. NOT completely autonomous

[2] T. Hatanaka, Y. Igarashi, M. Fujita and M. W. Spong, "Passivity-based Pose Synchronization in Three Dimensions," IEEE TAC, 2011. (accepted as a regular paper)

Previous Result 2 [3]

Leader: Rigid Body 0

Assumptions: $v^b_{w0} = const.,\ \omega^b_{w0} = 0\ (e^{\hat\xi\theta_{w0}} = const.)$

Proposed Velocity Input $c_i = 1$ if $0 \in \mathcal{N}_i$, $c_i = 0$ otherwise

$$v^b_{wi} = v^b_{w0}\ {}^\forall i \in \mathcal{V}$$
$$\omega^b_{wi} = k_{ei}\left(\sum_{j\in\mathcal{N}_i} w_{ij}\mathrm{sk}(e^{-\hat\xi\theta_{wi}}e^{\hat\xi\theta_{wj}})^\vee + c_i w_{i0}\mathrm{sk}(e^{-\hat\xi\theta_{wi}}e^{\hat\xi\theta_{w0}})^\vee\right) \quad (4)$$

Fact 2 [3]: Leader-following Attitude Synchronization

Consider the n rigid bodies represented by (1). Then, under the assumptions that $e^{-\hat\xi\theta_{wi}}e^{\hat\xi\theta_{w0}}\ {}^\forall i \in \mathcal{V}$ are positive definite at the initial time, the interconnection graph excluding the leader G_l is fixed and strongly connected and there exists at least one i satisfying $c_i = 1$, velocity input (4) achieves Leader-following Attitude Synchronization defined by $v^b_{wi} = v^b_{w0},\ \lim_{t\to\infty}\phi(e^{-\hat\xi\theta_{wi}}e^{\hat\xi\theta_{w0}}) = 0\ {}^\forall i \in \mathcal{V}$.

Technical Problem: $\omega^b_{w0} = 0$ and NOT consider position coordination

[3] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in $SE(3)$," IEEE TCST, Vol. 17, No. 5, pp. 1119-1134, 2011.

Discussions

Difficulty of Pose Synchronization with Desired Body Velocities or Leader-following Pose Synchronization

From (1), $\dot p_{wi} = e^{\hat\xi\theta_{wi}}v^b_{wi}$ holds. Thus, if each body inputs v_d instead of $e^{-\hat\xi\theta_{wi}}v_d$ in (3), then the following equations hold.

$$\dot p_{wi} = e^{\hat\xi\theta_{wi}}v_d + k_{pi}\sum_{j\in\mathcal{N}_i} w_{ij}(q_{wj} - q_{wi}) \quad \text{: depends on } e^{\hat\xi\theta_{wi}}$$

$$\dot p_{wj} - \dot p_{wi} = (e^{\hat\xi\theta_{wj}} - e^{\hat\xi\theta_{wi}})v_d + k_{pj}\sum_{l\in\mathcal{N}_j} w_{jl}(q_{wl} - q_{wj}) - k_{pi}\sum_{l\in\mathcal{N}_i} w_{il}(q_{wl} - q_{wi})$$

cannot be canceled

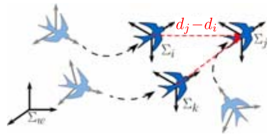
Namely, the effects of orientations appear in position dynamics. This causes that we can NOT use the strictly nonpositive properties of $\dot U$ where

$$U := \sum_{i=1}^{n}\gamma_i\left(\frac{1}{2k_{pi}}\|q_{wi}\|_2^2 + \frac{1}{k_{ei}}\phi(e^{\hat\xi\theta_{wi}})\right).$$

The same fact holds for Leader-following Pose Synchronization.

× $\sum_{i=1}^{n}\frac{\gamma_i}{2k_{pi}}\|q_{w0} - q_{wi}\|_2^2$ (In [3], $U_l := \sum_{i=1}^{n}\frac{\gamma_i}{k_{ei}}\phi(e^{-\hat\xi\theta_{wi}}e^{\hat\xi\theta_{w0}})$)

Outline



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Approach for PS with Desired Body Velocities

Main Idea

Key Property: $e^{\hat\xi\theta_{wi}}\mathrm{sk}(e^{\hat\xi\theta_{wi}})^\vee = \mathrm{sk}(e^{\hat\xi\theta_{wi}})^\vee$ (5)

If we set $\omega^b_{wi} = \omega'_{wi} + \omega_d$ (ω_d is defined in the body frame), then

$$\dot\phi(e^{\hat\xi\theta_{wi}}) = (\mathrm{sk}(e^{\hat\xi\theta_{wi}})^\vee)^T\omega^b_{wi} = (\mathrm{sk}(e^{\hat\xi\theta_{wi}})^\vee)^T(\omega'_{wi} + \omega_d) = (\mathrm{sk}(e^{\hat\xi\theta_{wi}})^\vee)^T(\omega'_{wi} + e^{-\hat\xi\theta_{wi}}\omega_d)$$

the same as (3)

Namely, the frame does NOT influence the time derivative of $\phi(e^{\hat\xi\theta_{wi}})$

➡ Attitude synchronization with a desired body angular velocity can be proved by the same approach as Fact 1

Approach for Position Synchronization

$$\dot p_{wj} - \dot p_{wi} = (e^{\hat\xi\theta_{wj}} - e^{\hat\xi\theta_{wi}})v_d + k_{pj}\sum_{l\in\mathcal{N}_j} w_{jl}(q_{wl} - q_{wj}) - k_{pi}\sum_{l\in\mathcal{N}_i} w_{il}(q_{wl} - q_{wi})$$

converge to 0 by AS

➡ We consider the additional term emerging by the desired body velocity as the perturbation term for the position error system and use Lemma 9.4 in [4]

[4] H. K. Khalil, Nonlinear Systems, Third Edition, Prentice Hall, 2002.

Pose Synchronization with Desired Body Velocities

Velocity Input $K_i = \mathrm{diag}(k_{pi}I_3, k_{ei}I_3) > 0$

$$V^b_{wi} = -K_i \sum_{j\in\mathcal{N}_i}\left(w_{ij}\begin{bmatrix} e^{-\hat\xi\theta_{wi}} & 0\\ 0 & I_3\end{bmatrix}\begin{bmatrix} q_{wi}-q_{wj}\\ \mathrm{sk}(e^{-\hat\xi\theta_{wj}}e^{\hat\xi\theta_{wi}})^\vee\end{bmatrix}\right) + \begin{bmatrix} v_d\\ \omega_d\end{bmatrix} \quad (6)$$

relative poses common in body frames

Velocity input (6) is completely based on relative information.

Theorem 1: Pose Synchronization with Desired Body Velocities

Consider the n rigid bodies represented by (1). Then, under the assumptions that there exists $e^{\hat\xi\theta_a}$ such that $e^{-\hat\xi\theta_a}e^{-\hat\xi\theta_d}e^{\hat\xi\theta_{wi}}\ {}^\forall i \in \mathcal{V}$ are positive definite at the initial time and the interconnection graph G is fixed and strongly connected, velocity input (6) achieves Pose Synchronization in the sense of (2).

Proof

$\lim_{t\to\infty}\phi(e^{-\hat\xi\theta_{wi}}e^{\hat\xi\theta_{wj}}) = 0\ {}^\forall i,j \in \mathcal{V}$ can be proved by the same approach as Fact 1 and the property (5).

Pose Synchronization with Desired Body Velocities

We next consider the position error dynamics written by

$$\dot{q}_{wj} - \dot{q}_{wi} = \underline{(e^{\hat{\xi}\theta_{wj}} - e^{\hat{\xi}\theta_{wi}})}v_d + k_{pj}\sum_{l\in\mathcal{N}_j} w_{jl}(q_{wl} - q_{wj}) - k_{pi}\sum_{l\in\mathcal{N}_i} w_{il}(q_{wl} - q_{wi})$$

We define $q \in \mathcal{R}^{3(n-1)}$ as the stuck vector of $q_{w(i+1)} - q_{wi}$ $\forall i \in \mathcal{V}\setminus\{n\}$.
Then, the dynamics of q is given by

$$\dot{q} = Aq + Bv_d, \quad (7)$$

where $A \in \mathcal{R}^{3(n-1)\times 3(n-1)}$ is a linear constant matrix and $B \in \mathcal{R}^{3(n-1)\times 3}$ the matrix whose vertical block elements are $e^{\hat{\xi}\theta_{w(i+1)}} - e^{\hat{\xi}\theta_{wi}}$ $\forall i \in \mathcal{V}\setminus\{n\}$.
From the results of Fact 1 under the condition that $V_d \equiv 0$, we can conclude that when $B = 0$, the equilibrium point $q = 0$ is exponentially stable.
We finally define $g := Bv_d$ and regard g as the perturbation of system (7).
Then, we obtain the following inequality.

$$\|g\|_2 \le \|B\|_\infty \|v_d\|_2 \le \|B\|_F \|v_d\|_2 = \sum_{i=1}^{n-1} 2\sqrt{\phi(e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{w(i+1)}})}\,\|v_d\|_2$$

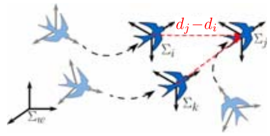
converge to 0 by AS

Thus, since $\lim_{t\to\infty} g = 0$ holds from AS, $\lim_{t\to\infty} q = 0$ holds by the same approach as [5].

Namely, Pose Synchronization is achieved in the sense of (2). □

[5] T. Hatanaka and M. Fujita, "Passivity-based Visual Motion Observer Integrating Internal Representation of 3D Target Object Motion," Proc. of the 2012 ACC, 2012. (submitted)

Outline



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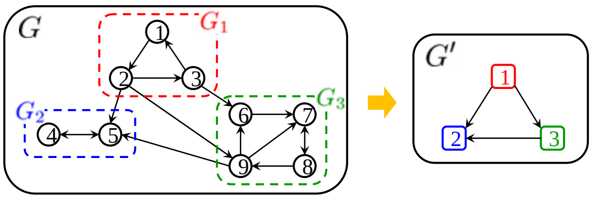
Wider Class Information Topologies

Strongly Connected Groups



We define $m\ (\le n)$ strongly connected digraphs $G_i \subseteq G,\ i \in \mathcal{V}' := \{1, \cdots, m\}$, where each node forms the largest strongly connected graph including itself.
We next define new digraph $G' := (\mathcal{V}', \mathcal{E}')$ where we consider G_i as new nodes $i \in \mathcal{V}'$ and set new edges $e'_{ij} \in \mathcal{E}' \subset \mathcal{V}' \times \mathcal{V}'$ if there exists at least one edge from one node in G_i to that in G_j.

Graph Assumption (GA)

(GA 1) Graph G is fixed and includes at least one strongly connected group
(GA 2) Graph G' has a directed spanning tree

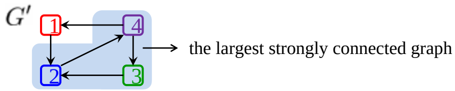
unnecessary

Wider Class Information Topologies

Strongly Connected Groups

Lemma 1
Under the assumption GA, graph G' is acyclic.

Proof
If there exists a cyclic path between node $i, j \in \mathcal{V}'$, G_i is not the largest strongly connected graph: contradiction to the definition of G_i



Corollary 1
Under GA, the root of the spanning tree in G' is uniquely determined.

Corollary 2
Under GA, there exists at least one node in G' which obtains information from only the root of the spanning tree.

PS in Wider Class Information Topologies

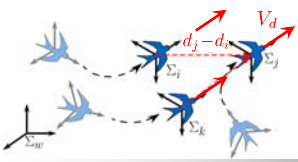
Velocity Input

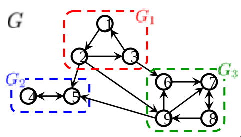
$K_i = \mathrm{diag}(k_{pi}I_3, k_{ei}I_3) > 0$

$$V_{wi}^b = -K_i \sum_{j\in\mathcal{N}_i}\left(w_{ij}\begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} & 0 \\ 0 & I_3 \end{bmatrix}\begin{bmatrix} q_{wi} - q_{wj} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wj}} e^{\hat{\xi}\theta_{wi}})^\vee \end{bmatrix}\right) + \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} \quad (6)$$

Theorem 2: PS in Wider Class Information Topologies
Consider the n rigid bodies represented by (1). Then, under GA and the assumption that $e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{wj}}$ $\forall i, j \in \mathcal{V}$ are positive definite for all $t \ge 0$, velocity input (6) achieves Pose Synchronization in the sense of (2).

Proof
Theorem 2 can be proved by using Lemma 1 and the following lemma.





Leader-following Pose Synchronization

Leader: Rigid Body 0

Assumptions: $v_{w0}^b = 0,\ \omega_{w0}^b = \omega_d$

Goal': Leader-following Pose Synchronization

$$\lim_{t\to\infty} \psi(y_{wi}^{-1} y_{w0}) = 0 \ \forall i \in \mathcal{V} \quad (7)$$

Velocity Input

$c_i = 1$ if $0 \in \mathcal{N}_i$, $c_i = 0$ otherwise

$$V_{wi}^b = K_i\left(\sum_{j\in\mathcal{N}_i} w_{ij}\begin{bmatrix} e^{-\hat{\xi}\theta_{wi}}(q_{wj} - q_{wi}) \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{wj}})^\vee \end{bmatrix} + c_i w_{i0}\begin{bmatrix} e^{-\hat{\xi}\theta_{wi}}(q_{w0} - q_{wi}) \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{w0}})^\vee \end{bmatrix}\right) + \begin{bmatrix} 0 \\ \omega_d \end{bmatrix} \quad (8)$$

Lemma 2: Leader-following Pose Synchronization
Consider the n rigid bodies represented by (1). Then, under the assumptions that $e^{-\hat{\xi}\theta_{wi}} e^{\hat{\xi}\theta_{w0}}$ $\forall i \in \mathcal{V}$ are positive definite at the initial time, the interconnection graph excluding the leader G_l is fixed and strongly connected and there exists at least one i satisfying $c_i = 1$, velocity input (8) achieves Leader-following Pose Synchronization in the sense of (7).

Leader-following Pose Synchronization

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Proof of Lemma 2

Key Property: $\dot{\phi}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}}) = (\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}})^\vee)^T(\omega_{wj}^b - e^{-\hat{\xi}\theta_{wj}}e^{\hat{\xi}\theta_{wi}}\omega_{wi}^b)$

$= (\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}})^\vee)^T(\omega_{wj}^b - \omega_{wi}^b)$

ω_d can be canceled

➡ $\dot{\phi}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}}) = (\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee)^T(\omega_{w0}^b - \omega_{wi}^b)$

$$= -(\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee)^T\left(\sum_{j\in\mathcal{N}_i} w_{ij}\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}})^\vee + c_i w_{i0}\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee\right)$$

Namely, ω_d does not influence the trajectory of $\phi(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})$. (the same as $\omega_d = 0$)
This property allows us to use the assumption that $e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}}\ \forall i \in \mathcal{V}$ are positive definite only at the initial time.
We next consider the following potential function.

$$U_l := \frac{1}{2}\sum_{i=1}^n \frac{\gamma_i}{k_{pi}}\|q_{w0} - q_{wi}\|_2^2 + \sum_{i=1}^n \frac{\gamma_i}{k_{ei}}\phi(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}}) \geq 0$$

Then, the time derivative of U_l is given by

Leader-following Pose Synchronization

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$$\dot{U}_l = \sum_{i=1}^n \gamma_i \begin{bmatrix} q_{w0} - q_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee \end{bmatrix}^T \begin{bmatrix} \frac{1}{k_{pi}}I_3 & 0 \\ 0 & \frac{1}{k_{ei}}I_3 \end{bmatrix} \begin{bmatrix} \dot{q}_{w0} - \dot{q}_{wi} \\ \omega_{w0}^b - \omega_{wi}^b \end{bmatrix} = -e^{\hat{\xi}\theta_{wi}}v_{wi}^b$$

ω_d can be canceled

$$= -\sum_{i=1}^n \gamma_i \begin{bmatrix} q_{w0} - q_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee \end{bmatrix}^T \left(\sum_{j\in\mathcal{N}_i} w_{ij} \begin{bmatrix} q_{wj} - q_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}})^\vee \end{bmatrix} + c_i w_{i0} \begin{bmatrix} q_{w0} - q_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee \end{bmatrix}\right)$$

$$= -\sum_{i=1}^n \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij} \begin{bmatrix} q_{w0} - q_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee \end{bmatrix}^T \begin{bmatrix} q_{wj} - q_{wi} \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}})^\vee \end{bmatrix} - \sum_{i=1}^n c_i w_{i0}\left(\|q_{w0} - q_{wi}\|_2^2 + \|\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee\|_2^2\right)$$

$$(q_{w0} - q_{wi})^T(q_{wj} - q_{wi}) = (q_{w0} - q_{wi})^T(q_{w0} - q_{wi} - (q_{w0} - q_{wj}))$$

$$= \frac{1}{2}\|q_{w0} - q_{wi}\|_2^2 - \frac{1}{2}\|q_{w0} - q_{wj}\|_2^2 + \frac{1}{2}\|q_{w0} - q_{wi} - (q_{w0} - q_{wj})\|_2^2$$

$= 0$ for the total energy; $q_{wj} - q_{wi}$

Leader-following Pose Synchronization

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$$(\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee)^T\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}})^\vee = (\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee)^T\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}}e^{-\hat{\xi}\theta_{w0}}e^{\hat{\xi}\theta_{wj}})^\vee$$

$$\geq \phi(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}}) - \phi(e^{-\hat{\xi}\theta_{wj}}e^{\hat{\xi}\theta_{w0}}) + \frac{1}{2}\lambda_{min}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}} + e^{-\hat{\xi}\theta_{w0}}e^{\hat{\xi}\theta_{wi}})\phi(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}})$$

$= 0$ for the total energy; > 0 if $e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}}$ is positive definite

$$\dot{U}_l \leq -\frac{1}{2}\sum_{i=1}^n \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij}\left(\|q_{wj} - q_{wi}\|_2^2 + \lambda_{min}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}} + e^{-\hat{\xi}\theta_{w0}}e^{\hat{\xi}\theta_{wi}})\phi(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{wj}})\right) - \sum_{i=1}^n c_i w_{i0}\left(k_{pi}\|q_{w0} - q_{wi}\|_2^2 + k_{ei}\|\mathrm{sk}(e^{-\hat{\xi}\theta_{wi}}e^{\hat{\xi}\theta_{w0}})^\vee\|_2^2\right)$$

≤ 0

➡ LaSalle's Invariance Principle

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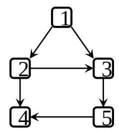
Proof of Theorem 2

From Lemma 1 and Corollary 1, the nodes of G' are divided in the following five groups.

- one root which does not get information from any node (1)
- nodes having the root as only the parent node (2)
- nodes having only one parent node other than the root (5)
- nodes having multiple parent nodes including the root (3)
- nodes having multiple parent nodes other than the root (4)



For the first strongly connected group, Pose Synchronization is proved by Theorem 1. Thus, all rigid bodies in the group achieve a common pose and move in V_d after sufficient time.

We next consider the second group.
From the above fact, we can regard that the second group has the same leader after enough time.
Therefore, from Lemma 2, pose synchronization is achieved if the relative orientation matrices between the leader and the bodies are positive definite for all $t \geq 0$.

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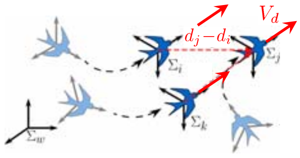
Note: In this situation, the body having multiple leader neighbors has to multiply the number of them by the second term of (8). But the analysis does not change.

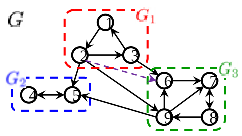
Then, we can regard the rigid bodies in the first and second groups as the same leader after sufficient time.
Thus, similarly, the group which has both the first and second groups as parent nodes or only second groups as the parent node achieves Pose Synchronization.
By conducting this analysis recursively, we can prove synchronization for all groups.

Need to analyze transient states in detail: $\dot{q} = Aq + Bv_d + h$

Leader's transient velocity which converge to 0

Present reports soon





Discussions

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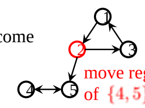
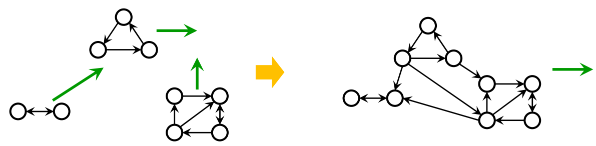
- Compared with Fact 1 or Fact 2, we need the assumption that the relative orientation matrices between all rigid bodies are positive definite.
  This is because that the node in the parent group will become the leader and moves regardless of the child group.



- Theorem 2 implies merging of some flocks.



Probably, we can extend the results to brief connectivity loss by using dwell time for strongly connected group or spanning tree structures as shown in the above figure.

Discussions

Flocking Algorithm

The proposed input (6) includes cohesion and alignment rules for flocking.
For the separation rule, we have presented the following velocity input.

$$V_{wi}^{b} = -K_i \left(\sum_{j \in \mathcal{N}_i} w_{ij} \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}}(q_{wi} - q_{wj}) \\ \mathrm{sk}(e^{-\hat{\xi}\theta_{wj}} e^{\hat{\xi}\theta_{wi}})^{\vee} \end{bmatrix} + \sum_{j \in \mathcal{N}_{Ci}} \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} \frac{\partial U_{ij}}{\partial p_{wi}} \\ 0 \end{bmatrix} \right) + \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} v_d \\ e^{-\hat{\xi}\theta_{wi}} \omega_d \end{bmatrix}$$

Probably, we can show the boundedness of position errors as follows.

$$\|p_e\|_2 \leq \max \left(c_1 N^{3/2} g, \sqrt{c_2 \beta(c_3 \|p_e(0)\|_2^2, t)} \right)$$

p_e : position error vector
$\beta(\cdot, \cdot) \in \mathcal{KL}$

However, we need the following strict assumptions for the above results.

- Graph is unweighted bidirectional connected.
- All position gains are the same, i.e. $k_{pi} = k_p \ {}^{\forall} i \in \mathcal{V}$.
- Desired velocities defined in the inertial frame.

Since these assumptions are not suited to the last results, we postpone presenting.