Cooperative Pose Localization and Estimation in Visual Sensor Networks

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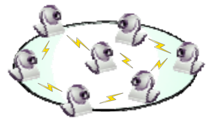
Introduction

Visual Sensor Networks [1]

Networks consisting of spatially distributed smart cameras



- Applications
  - Environmental Monitoring - Surveillance
- Tasks and Problems
  - Target Pose Estimation
  - Camera Localization

Distributed Algorithms [3,4]

It is essential to develop the tools for automatically analyzing and integrating the data.
Distributed algorithms have been developed for the following reasons

- Network capacity - Resource constraints
- Fault tolerance

Introduction

Distributed Algorithms in Visual Sensor Networks [3,4]

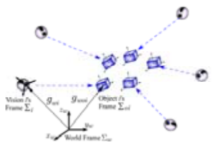
Multiagent systems [5,6] and Cooperative control [7,8] provide useful methodologies

Cooperative Target Pose Estimation [9,10]

- All Localized Cameras
- Averaging of pose estimates

Consensus-based Camera Localization [11]

- Gradient descent optimization in SE(3)
- There is no meaning of the camera pose which minimizes the cost function



Objective of Our Work

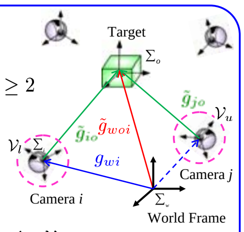
To present a simultaneous target estimation and localization algorithm based on distributed optimization [6] in SE(3)

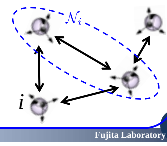
Problem Settings (Summary)

Visual Sensor Networks

- **Camera Set** $\mathcal{V} := \{1, \cdots, N\}$
  - Localized Cameras $\mathcal{V}_l \subset \mathcal{V}$ $|\mathcal{V}_l| \geq 2$
  - Unlocalized Cameras $\mathcal{V}_u = \mathcal{V} \backslash \mathcal{V}_l$
- **Measurements**
  - All Cameras $\tilde{g}_{io}$ $i \in \mathcal{V}$
  - Localized Cameras $\tilde{g}_{woi} = g_{wi}\tilde{g}_{io}$ $i \in \mathcal{V}_l$
- **Communication**
  - Communication Graph $G = (\mathcal{V}, \mathcal{E})$
  - Neighbor set $\mathcal{N}_i := \{j \in \mathcal{V} | (j,i) \in \mathcal{E}\}$

Camera i can get j's info.





Objective

Objective

Achieve the following requirements simultaneously

- Target Estimation of All Cameras
- Localization of Unlocalized Cameras

Target Estimation Problem (Averaging)

$$\bar{g}_{woi} = g^* \quad i \in \mathcal{V} \quad (1)$$

$\bar{g}_{woi}$: Estimates of target pose

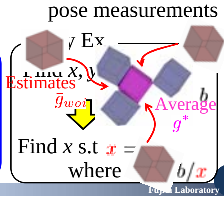
g^* : Average of target pose measurements

Localization Problem

$$\bar{g}_{wi}^{-1}\bar{g}_{woi} = \tilde{g}_{io} \quad i \in \mathcal{V}_u \quad (2)$$

Find $(\bar{g}_{woi})_{i\in\mathcal{V}_l}$ $(\bar{g}_{woi}, \bar{g}_{wi})_{i\in\mathcal{V}_u}$ which satisfy (1) and (2)

Given $\bar{g}_{wi} = \bar{g}_{woi}\tilde{g}_{io}^{-1}$, find $(\bar{g}_{woi})_{i\in\mathcal{V}}$ which satisfy (1)



Estimation and Localization Algorithm

Estimation and Localization Algorithm

$$\dot{\bar{g}}_{woi} = \bar{g}_{woi}\hat{u}_i$$

Gradient descent of $\psi(\bar{g}_{woi}^{-1}\tilde{g}_{woi})$

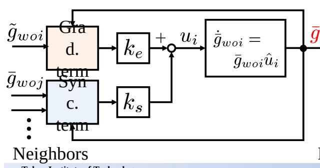
$$u_i = \begin{cases} k_e E_R(\bar{g}_{woi}^{-1}\tilde{g}_{woi}) + k_s \sum_{j\in\mathcal{N}_i} E_R(\bar{g}_{woi}^{-1}\bar{g}_{woj}) & i \in \mathcal{V}_l \\ k_s \sum_{j\in\mathcal{N}_i} E_R(\bar{g}_{woi}^{-1}\bar{g}_{woj}) & i \in \mathcal{V}_u \end{cases}$$

Pose sync. [12]

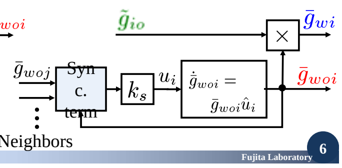
$$\bar{g}_{wi} = \bar{g}_{woi}\tilde{g}_{io}^{-1}$$

$$\left(\begin{array}{l} E_R(g) := \left[\begin{array}{cc} p^T & \mathrm{sk}(e^{\hat{\xi}\theta})^\vee \end{array} \right]^T \\ \mathrm{sk}(e^{\hat{\xi}\theta}) = \frac{1}{2}(e^{\hat{\xi}\theta} - e^{-\hat{\xi}\theta}) \quad k_e, k_s > 0 \end{array} \right)$$

Localized Cameras $i \in \mathcal{V}_l$



Unlocalized Cameras $i \in \mathcal{V}_u$



Assumptions (Averaging Performance)

Assumption 1 (Communication Graph)

The communication graph $G = (\mathcal{V}, \mathcal{E})$ is fixed, undirected and connected.

Assumption 2 (Target Pose)

- The Target is static. ($\dot{\bar{g}}_{woi} = 0 \ \forall i \in \mathcal{V}_l$)
- There exists a pair $(i, j) \in \mathcal{V}_l \times \mathcal{V}_l$ such that $\bar{g}_{woi} \neq \bar{g}_{woj}$
- $e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}} > 0 \quad \forall i \in \mathcal{V}_l$

  The relative angle between average and target pose is smaller than $\pi/2$

Lemma1

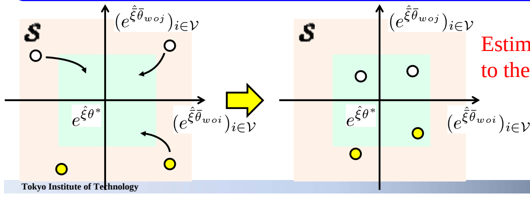
Lemma 1

Suppose the estimates $\bar{g}_{woi}$ are updated according to the present algorithm. Under assumption 1 and 2, if $e^{-\hat{\xi}\bar{\theta}_{woi}} e^{\hat{\xi}\theta^*} > 0 \ \forall t \geq 0$ holds, then for any positive scalar c, there exists a finite time $\tau(c)$ such that

$$\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}}) < \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woh}}) + c \quad \forall t \geq \tau(c) \quad i \in \mathcal{V}_l$$

Error between average and estimates — Maximum error between average and measurements

where $h := \arg\max_{j \in \mathcal{V}_l} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woj}})$



Estimates get closer to the average

○ $i, j \in \mathcal{V}_l$
● $i, j \in \mathcal{V}_u$

Sketch of Proof of Lemma1

○ $i, j \in \mathcal{V}_l$ ● $i, j \in \mathcal{V}_u$

Energy function

$U := \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wol}})$ Estimates farthest from the mean

$l(t) := \arg\max_{i \in \mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}})$

Case 1: $l \in \mathcal{V}_l$

Suppose that $\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wol}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woh}}) > c$

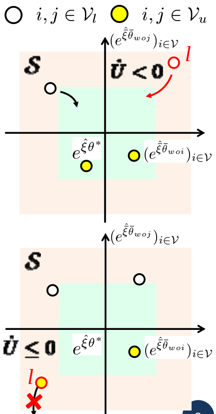
$\dot{U} \leq -ck_e < 0$

From theorem on Ultimate Boundedness, the trajectories of estimates $(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_l}$ ultimately converge to the set satisfying

$$\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}}) < \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woh}}) + c \quad i \in \mathcal{V}_l \quad (3)$$

Case 2: $l \in \mathcal{V}_u$ $\quad \dot{U} \leq 0$

It isn't sure the estimates $(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_l}$ ultimately converge to the set satisfying (3)



Sketch of Proof of Lemma1

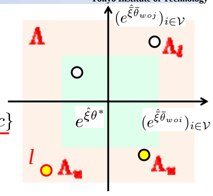
Consider new energy function

$$V := \sum_{i \in \Lambda} (\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woh}}) - c)$$

$$\Lambda := \{(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}} \mid \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woh}}) \geq c\}$$

Outside the set satisfying (3)

$\Lambda_l := \{(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_l} \in \Lambda\} \quad \Lambda_u := \{(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_u} \in \Lambda\}$



V is continuous but may not be differentiable at the boundary

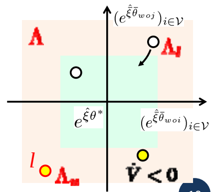
Case 2: $l \in \mathcal{V}_u$

Consider $i \in \Lambda_l$ exists $\quad \dot{V} \leq -k_e|\Lambda_l|c < 0$

The trajectories of estimates $(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_l}$ will enter the set satisfying (3).

But, it isn't sure the estimates converge

(They may leave the set satisfying (3))



Sketch of Proof of Lemma1

Consider the estimates $(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_l}$ enter the set at t_1, and leave the set at t_2

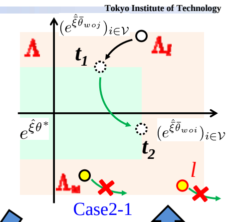
Case2-1: $\quad t_1 \leq t \leq t_2 \quad \Lambda_l = \emptyset \quad \dot{V} \leq 0$

$\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}}) \ \ i \in \Lambda_u$ don't increase

Case2-2: $\quad t_2 \leq t \leq t_2 + \epsilon$

$\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}}) \ \ i \in \Lambda_l$ increase, but $\dot{V} < 0$

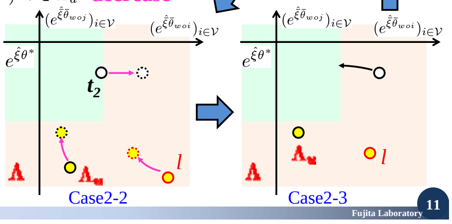
⇒ $\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{woi}}) \ \ i \in \Lambda_u$ **decrease**



Case2-3: $\quad t \geq t_2 + \epsilon$

Again, the estimates enter the set satisfying (3)

Back to Case2-1 (Cycle)



Sketch of Proof of Lemma1

After several repeat from Case2-1 to Case2-3, the trajectories of estimates $(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_u}$ will converge to the set satisfying (3)

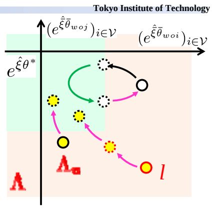
⇓

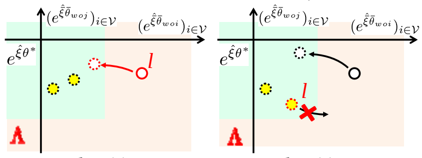
Because of $\dot{U} < 0 \ \ l \in \Lambda_l$ (Case1) and $\dot{U} \leq 0 \ \ l \in \mathcal{V}_u$, the estimates $(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_l}$ never leave the set satisfying (3).

(This cycle happens in a finite number)

⇓

The trajectories of estimates $(e^{\hat{\xi}\bar{\theta}_{woi}})_{i\in\mathcal{V}_l}$ finally converge to the set satisfying (3)





$l \in \mathcal{V}_l \qquad l \in \mathcal{V}_u$

Main Result

Theorem 1

Suppose the estimates $\bar{g}_{woi}$ are updated according to the present algorithm and that the initial estimates satisfy $e^{-\hat{\bar{\xi}}\bar{\theta}_{woi}}(0)e^{\hat{\xi}\theta^*} > 0$. Given any $\alpha > |\mathcal{V}_u|/|\mathcal{V}_l|$ and $\epsilon \in (0,1)$, under Assumptions 1 and 2, if the gain $k = k_e/k_s$ is sufficiently small, then for all sufficiently large times *T*,

$$\frac{1}{N}\sum_{i\in\mathcal{V}} \|\bar{p}_{woi} - p^*\|^2 < \frac{\epsilon+\alpha}{|\mathcal{V}_l|}\sum_{i\in\mathcal{V}_l} \|\tilde{p}_{woi} - p^*\|^2$$

$$\frac{1}{N}\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*}e^{\hat{\bar{\xi}}\bar{\theta}_{woi}}) < \frac{1-(1-\epsilon)\beta+\alpha}{|\mathcal{V}_l|}\sum_{i\in\mathcal{V}_l} \phi(e^{-\hat{\xi}\theta^*}e^{\hat{\tilde{\xi}}\tilde{\theta}_{woi}})$$

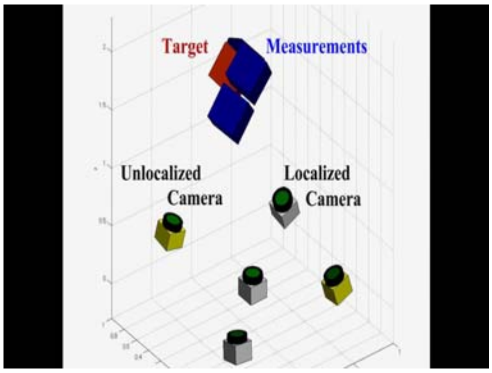
Error between average and estimates

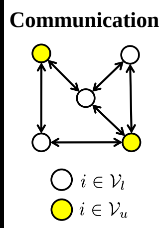
Error between average and measurements

$$\beta := 1 - \sqrt{2(\phi(e^{-\hat{\xi}\theta^*}e^{\hat{\tilde{\xi}}\tilde{\theta}_{woh}}) + c)}$$ holds.

We derive an upper bound of the ultimate error between estimates and the average

Simulation Movie





Conclusion

Summary

- We have defined the target estimation and localization problems for visual sensor networks
- We have presented a cooperative localization and estimation algorithm based on distributed optimization approach
- We have shown the averaging performance of the algorithm by evaluating the error between the estimates and the average pose of the target
- We have demonstrated the effectiveness of the algorithm through simulations

Future Works

- Tracking performance analysis

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Appendix

Problem Settings

Camera Set

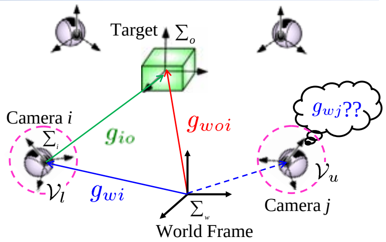
$i \in \mathcal{V} := \{1, \cdots, N\}$

Localized Cameras

$i \in \mathcal{V}_l \subset \mathcal{V} \quad |\mathcal{V}_l| \geq 2$

Unlocalized Cameras

$i \in \mathcal{V}_u = \mathcal{V} \backslash \mathcal{V}_l$



Pose Representation

Pose of camera i

$g_{wi} = (p_{wi}, e^{\hat{\xi}\theta_{wi}}) \in SE(3)$

Pose of target (object)

$g_{woi} = (p_{woi}, e^{\hat{\xi}\theta_{woi}})$

Pose of target relative to camera i

$g_{io} := g_{wi}^{-1} g_{woi}$

Exponential Coordinates

$\xi_{wi} \in \mathbb{R}^3$: axis

$\theta_{wi} \in \mathbb{R}$: angle

Homogeneous Representation

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix}$$

Problem Settings

Measurements

All Cameras $\bar{g}_{io} \quad i \in \mathcal{V}$

Localized Cameras

$\bar{g}_{woi} = g_{wi}\bar{g}_{io} \quad i \in \mathcal{V}_l$

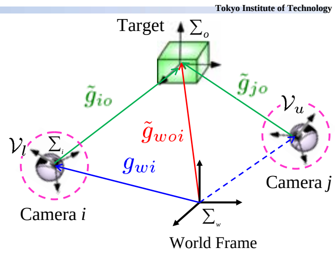
Pose Average

$g^* = (p^*, e^{\hat{\xi}\theta^*})$

$$g^* := \arg \min_{g \in SE(3)} \sum_{j \in \mathcal{V}_l} \psi(g^{-1}\bar{g}_{woj})$$

$$\psi(g) := \frac{1}{2}\|p\|^2 + \phi(e^{\hat{\xi}\theta})$$

$$\phi(e^{\hat{\xi}\theta}) := \mathrm{tr}(I_3 - e^{\hat{\xi}\theta})$$



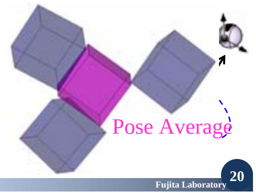
Communication Graph

$G = (\mathcal{V}, \mathcal{E})$

$(j, i) \in \mathcal{E} \subset \mathcal{V} \times \mathcal{V}$

(Camera i can get j's info.)

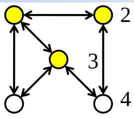
Neighbor set: $\mathcal{N}_i := \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$



Simulation Settings

Camera Settings $i \in \mathcal{V} := \{1, \cdots, 5\}$

○ $i \in \mathcal{V}_l = \{1, 2, 3\}$ ● $i \in \mathcal{V}_u = \{4, 5\}$



$\mathrm{diam}(G) = 2$

$W = 1$

Target pose measurements

$$\bar{p}_{wo1} = \begin{bmatrix} -0.3 \\ 0.6 \\ 1.9 \end{bmatrix}, \bar{p}_{wo2} = \begin{bmatrix} -0.2 \\ 0.5 \\ 1.6 \end{bmatrix}, \bar{p}_{wo3} = \begin{bmatrix} -0.6 \\ 0.4 \\ 1.8 \end{bmatrix}$$

$$\bar{\xi}\bar{\theta}_{wo1} = \begin{bmatrix} 0.3 \\ 0.2 \\ 0.3 \end{bmatrix}, \bar{\xi}\bar{\theta}_{wo2} = \begin{bmatrix} 0.4 \\ 0.15 \\ 1.2 \end{bmatrix}, \bar{\xi}\bar{\theta}_{wo3} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.15 \end{bmatrix}$$

Average of target pose

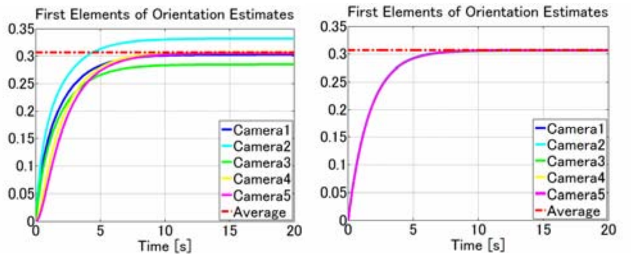
$$\bar{p}^* = \begin{bmatrix} -0.3667 \\ 0.5000 \\ 1.7667 \end{bmatrix}, \bar{\xi}\bar{\theta}^* = \begin{bmatrix} 0.3168 \\ 0.2002 \\ 0.2168 \end{bmatrix} \Rightarrow \beta = 0.7937$$

Initial Estimates $\bar{p}_{woi}(0) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \quad e^{\hat{\bar{\xi}}\bar{\theta}_{woi}}(0) = I_3$

Simulation Results (Target Estimates)

Target Orientation Estimates $e^{\hat{\bar{\xi}}\bar{\theta}_{woi}}$

$k_e = 1, k_s = 1 (k = 1)$ $\qquad$ $k_e = 1, k_s = 50 (k = 0.02)$



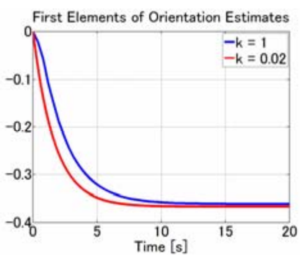
If the gain k is sufficiently small, the estimates are getting close to the average.

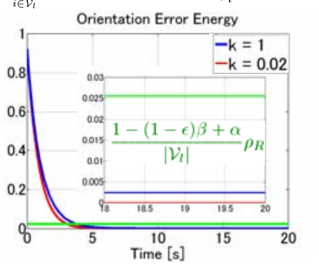
Simulation Results

Camera Orientation Estimates $e^{\hat{\bar{\xi}}\bar{\theta}_{w4}}$

Energy Function $\frac{U_R}{N}$

$\rho_R = \sum_{i \in \mathcal{V}_l} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{woi}}) \quad \alpha = 0.7 > \frac{\mathcal{V}_u}{\mathcal{V}_l} \quad \epsilon = 0.01$





The Camera estimate depends on the gain k.

Theorem1 is satisfied.
Estimates are close to the average.