Safety Analysis of Vehicle Platoon under V2V2I Communication

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Introduction

Background

- Traffic congestion[1]
- Adaptive cruise control[2]
- Intelligent road transportation system[8]
  - Vehicle-to-Vehicle communication(V2V)
  - Vehicle-to-Infrastructure communication(V2I)

Approaches

- Macro perspective： On-ramp control[3,4], Transportation network[5]
- Micro perspective： Vehicle platoon control[6,7]

Purpose of Research

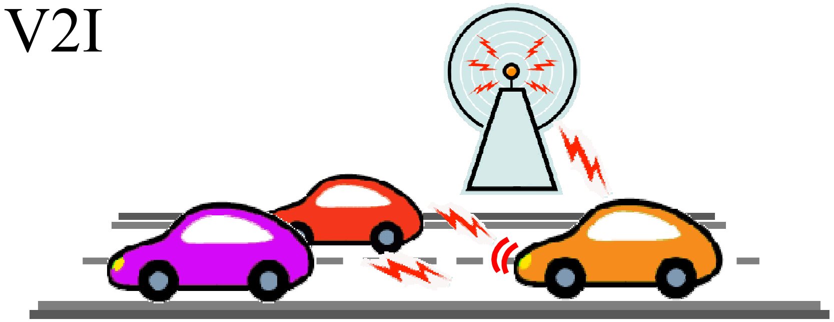
New approach

Middle perspective

- Considering vehicular strings and infrastructure
- Under Vehicle-to-Vehicle-to-Infrastructure communication

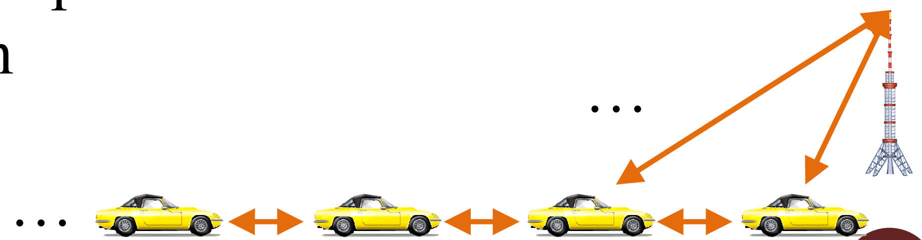
Vehicle-to-Vehicle-to-Infrastructure(V2V2I) communication

- The hybrid system of the V2V and V2I
- Vehicles are controlled by "Local" and "Global" information



Objectives

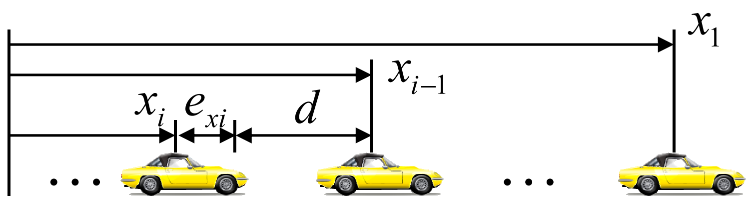
- Problem description of middle perspective
- Safety analysis of vehicle platoon
- Simulation



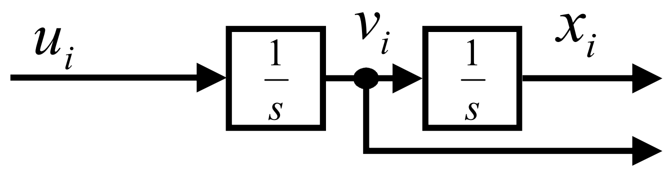
Problem Description

Vehicle platoon model[3,4]

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} I & TI \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u(k)$$



$x(k) = [x_1(k), \cdots, x_n(k)]^T \quad u(k) = [u_1(k), \cdots, u_n(k)]^T$



$v(k) = [v_1(k), \cdots, v_n(k)]^T \quad T$: Sampling time

Error[9,10]

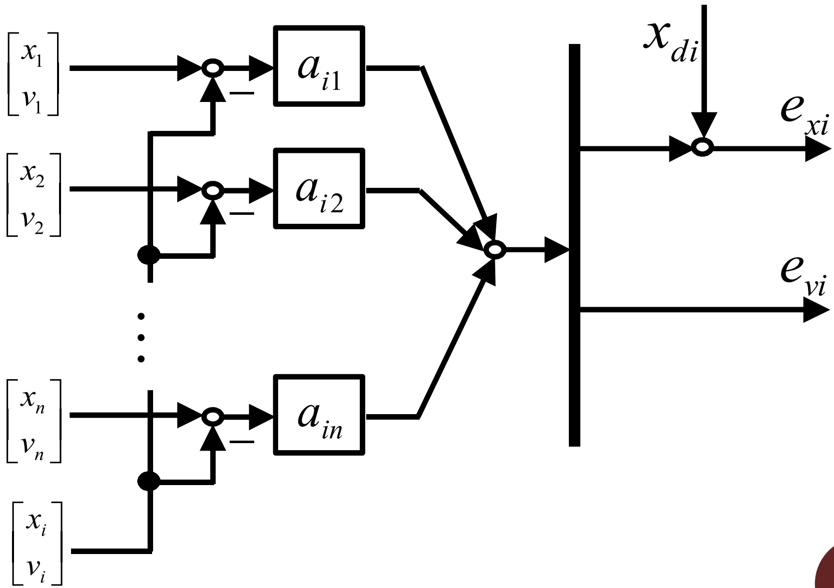
ith vehicle's position and velocity error

$$\begin{cases} e_{xi}(k) = \sum_{j=1}^{n} \alpha_{ij}(x_j(k) - x_i(k)) + x_{di} \\ e_{vi}(k) = \sum_{j=1}^{n} \alpha_{ij}(v_j(k) - v_i(k)) \end{cases}$$

x_{di} :Constant desired spacing

Weighted communication state

$$\begin{cases} 0 \le \alpha_{ij} \le 1 \\ \alpha_{ii} = -1 \end{cases} \quad \sum_{j \ne i} \alpha_{ij} = 1 \quad \sum_{j=1}^{n} \alpha_{ij} = 0$$



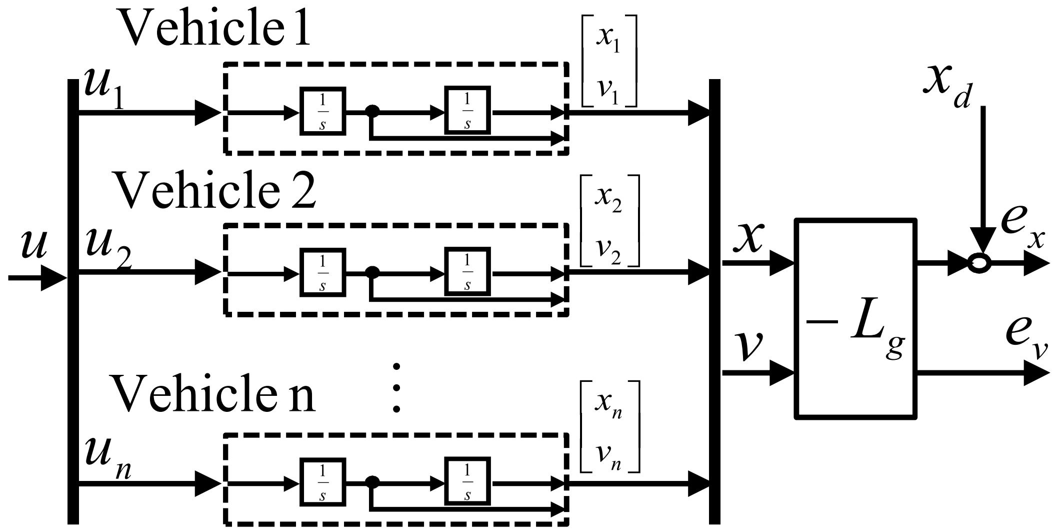
Problem Description

Platoon's position and velocity error

$$\begin{cases} e_x(k) = -L_g x(k) + x_d \\ e_v(k) = -L_g v(k) \end{cases}$$

$$x_d(k) = [x_{d1}(k), \cdots, x_{dn}(k)]^T$$

$$-L_g = \begin{bmatrix} -1 & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & -1 & \cdots & \alpha_{2n} \\ & & \ddots & \\ \alpha_{n1} & \alpha_{n2} & \cdots & -1 \end{bmatrix}$$



Input from Infrastructure

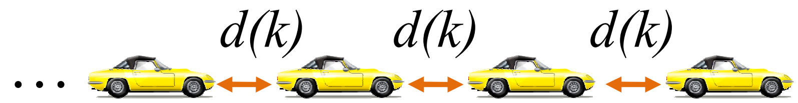
*i*th vehicle's input

$$u_i(k) = k'_p\, e_{xi}(k) + k'_v\, e_{vi}(k)$$

$$k'_v = k_v T \quad k'_p = k_p T$$

Platoon's input

$$u(k) = k'_p\, e_x(k) + k'_v\, e_v(k)$$



Reference relative position

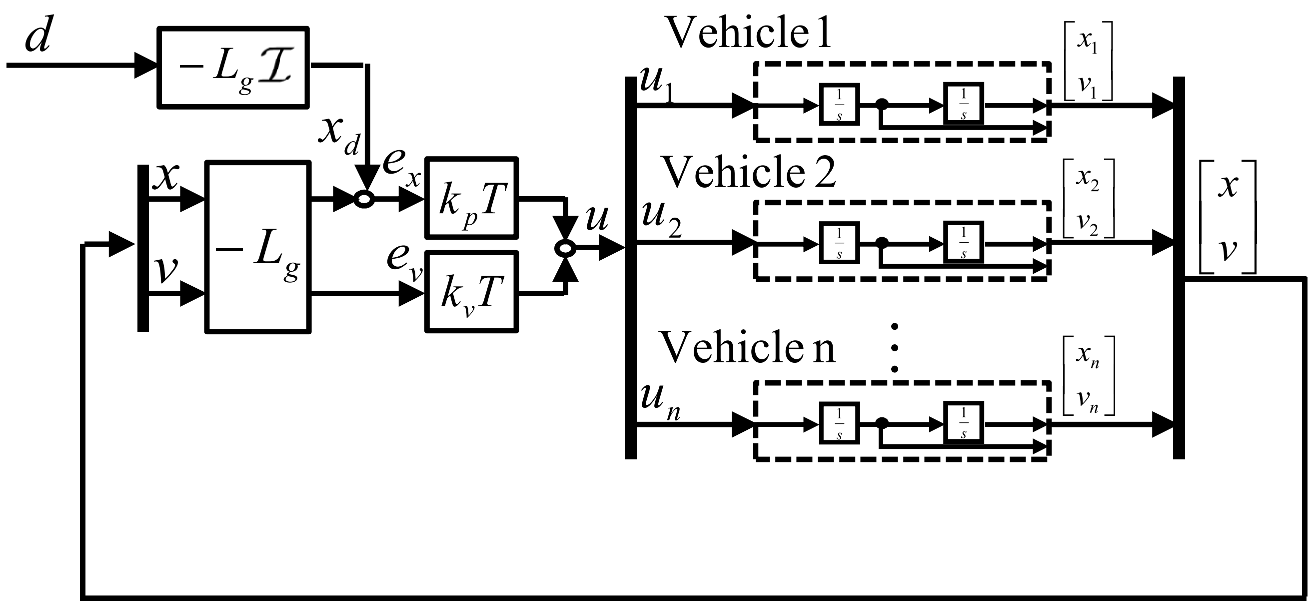
*i*th vehicle's input

$$x_{di} = \sum_{j=1}^{n} j a_{ij} d(k)$$

Platoon's input

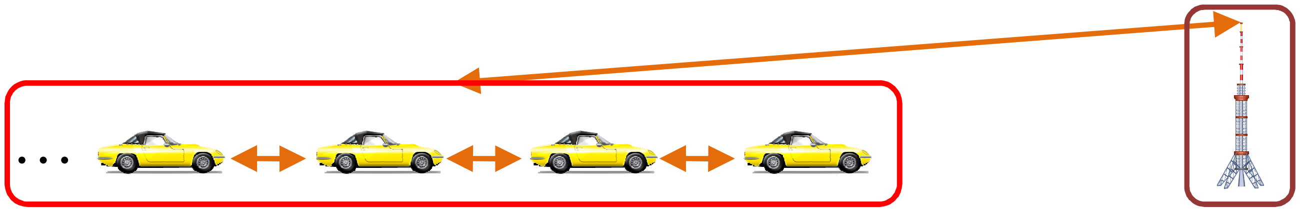
$$x_d = -L_g \mathcal{I}\, d(k) \quad \mathcal{I} = [1,2,3,\cdots]^T$$

Problem Description



State equation of vehicle platoon

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} I & TI \\ -k'_p L_g & I-k'_v L_g \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} - k'_p \begin{bmatrix} 0 \\ L_g\mathcal{I} \end{bmatrix} d(k) \quad \cdots(1)$$



Vehicle platoon

Input from Infrastructure

Safety Analysis

Definition of safety

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} x(k)+Tv(k) \\ v(k)-k'_p L_g x(k)-k'_v L_g v(k)-k'_p L_g \mathcal{I}\, d(k) \end{bmatrix}$$

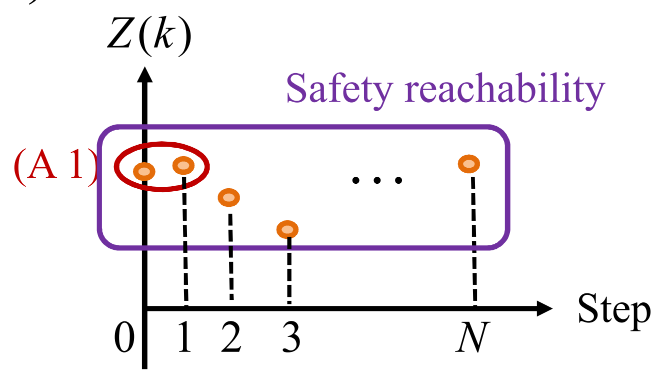
$$\begin{bmatrix} x(k+2) \\ v(k+2) \end{bmatrix} = \begin{bmatrix} x(k)+2Tv(k)-T(k'_p L_g x(k)+k'_v L_g v(k))-k'_p TL_g \mathcal{I}\, d(k) \\ -k'_p L_g(x(k)+Tv(k))+k'_v L_g(k'_p L_g x(k)+k'_v L_g v(k)+k'_p L_g \mathcal{I}\, d(k))-k'_p TL_g \mathcal{I}\, d(k) \end{bmatrix}$$

Assumption (A1):

The system (1) satisfies $Z(0)>0, Z(1)>0$,

where $Z(k)=Dx(k)$, $D=\begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \\ & & & 1 & -1 \\ 0 & & \cdots & & 0 \end{bmatrix}$.

ex) $n=2$



Definition 1: Safety reachability

The system (1) is safety reachable if given any $Z(0)$ satisfying (A1) there exists a $d(k)$, $\forall 0,1,\cdots,N-2$ such that

$$Z(k)>0,\ \forall 2,3,\cdots,N.$$

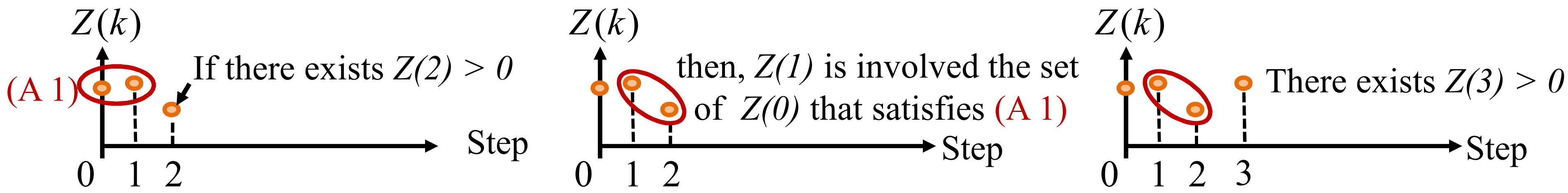
Safety Analysis

Main result

Theorem 1:

The system (1) is safety reachable iff given any $Z(0)$ satisfies (A1) there exists a $d(0) > 0$ that satisfies $Z(2) > 0$

Sketch of proof: Sufficient condition



Theorem 2:

In order for given any $Z(0)$ satisfies (A1) there exists a $d(0) > 0$ that satisfies $Z(2) > 0$, it is necessary that any $i = 1, 2, \cdots, n-1$ satisfies

$$\left\{ -DL_g \mathcal{I} \right\}_i \neq 0.$$

Sketch of proof:

$$Z(2) = Dx(2) = Dx(0) + 2TDv(0) - T(k'_p\, DL_g x(0) + k'_v\, DL_g v(0)) \underline{- k'_p\, TDL_g \mathcal{I}}\, d(0)$$

If there exists i satisfying $\left\{ -DL_g \mathcal{I} \right\}_i = 0$, $x_i(2) - x_{i+1}(2)$ doesn't depend on $d(0)$

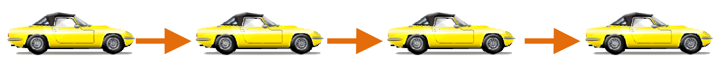
Safety Analysis

Safety unreachability communication structure

Considering the case $\{-DL_g\mathcal{I}\}_i = 0$

ex) Predecessor following

$$-DL_g\mathcal{I} = D\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & & \\ & 1 & -1 & \\ & & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = D\begin{bmatrix} 0 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Corollary 1:

The system (1) is safety unreachable iff given $i = 1, 2, \cdots, n-1$

$-L_g$ satisfies the following condition

$$\begin{bmatrix} a_{i(i-1)} - a_{i(i+1)} & a_{i(i-2)} - a_{i(i+2)} & \cdots \end{bmatrix} = \begin{bmatrix} a_{(i+1)i} - a_{(i+1)(i+2)} & a_{(i+1)(i-1)} - a_{(i+1)(i+3)} & \cdots \end{bmatrix}$$

with $a_{ij} = 0, j < 0$.

Sketch of proof:

$$\{-L_g\mathcal{I}\}_i = \sum_{j=1}^{n} ja_{ij} = \sum_{j=1}^{n} ja_{ij} - i\sum_{j=1}^{n} a_{ij} = -(a_{i(i-1)} - a_{i(i+1)}) - 2(a_{i(i-2)} - a_{i(i+2)})\cdots$$

$$\{-DL_g\mathcal{I}\}_i = \sum_{j=1}^{n} j(a_{ij} - a_{(i+1)j}) - \sum_{j=1}^{n} ia_{ij} - (i+1)a_{(i+1)j} = 0$$

Relatively-equal communication structure makes the system (1) safety unreachable

Safety Analysis

Safety reachability communication structure

There exists communication structures satisfying $\{-DL_g\mathcal{I}\}_i \neq 0$

ex) Leader and Predecessor following

$$-DL_g\mathcal{I} = D\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & & & \\ 1/2 & 1/2 & -1 & & \\ 1/2 & & 1/2 & -1 & \\ 1/2 & & & 1/2 & -1 \end{bmatrix}\begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix} = D\begin{bmatrix}0\\-1\\-3/2\\-2\\-5/2\end{bmatrix} = \begin{bmatrix}1\\1/2\\1/2\\1/2\\0\end{bmatrix}$$

Theorem 3:

If $-L_g$ satisfies the following condition:

$$\sum_{j=1}^{n} ja_{1j} > \cdots > \sum_{j=1}^{n} ja_{nj}$$

Then there exists a $d(0) > 0$ that satisfies $Z(2) > 0$.

Sketch of proof:

$$Z(2) = Dx(0) + 2TDv(0) - T(k'_p DL_g x(0) + k'_v DL_g v(0)) - k'_p TDL_g\mathcal{I}\, d(0) > 0$$

$$\Leftrightarrow -DL_g\mathcal{I}\, d(0) \geq -\frac{D}{k'_p T}\left(x(0) + 2Tv(0) - k'_p TL_g x(0) - k'_v TL_g v(0)\right)$$

If given any $i = 1, 2, \cdots, n-1$ satisfies $\{-DL_g\mathcal{I}\}_i > 0$, $d(0)$ has an only lower bound

Latter vehicle should lay more weight on the leader information

Simulation

Comparative Verification

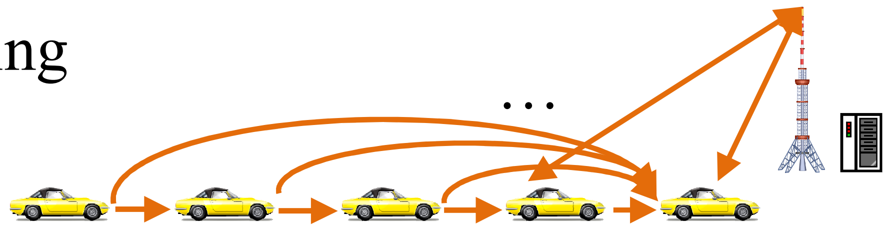
Safety unreachability ex:

Predecessor following

Safety reachability ex:

Leader and Predecessor following

- Constant control
- Infrastructure control



Simulation Settings

Constant control

$$d(k) = d_{ref}$$

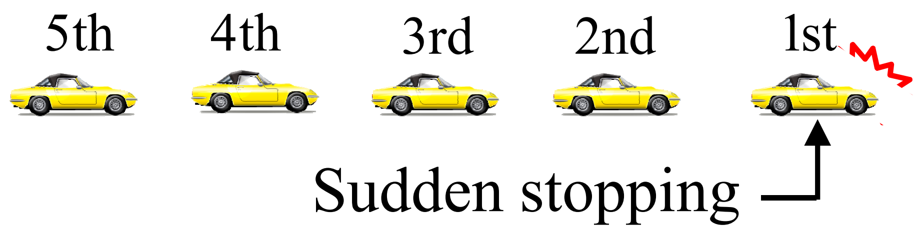
Infrastructure control

$$d(k) = \begin{cases} d_{ref} & (d_{ref} \geq d_{min}(k)) \\ d_{min}(k) & (d_{min}(k) > d_{ref}) \end{cases}$$

$$d_{min}(k) = \min_{d(k)} d(k) + \delta$$

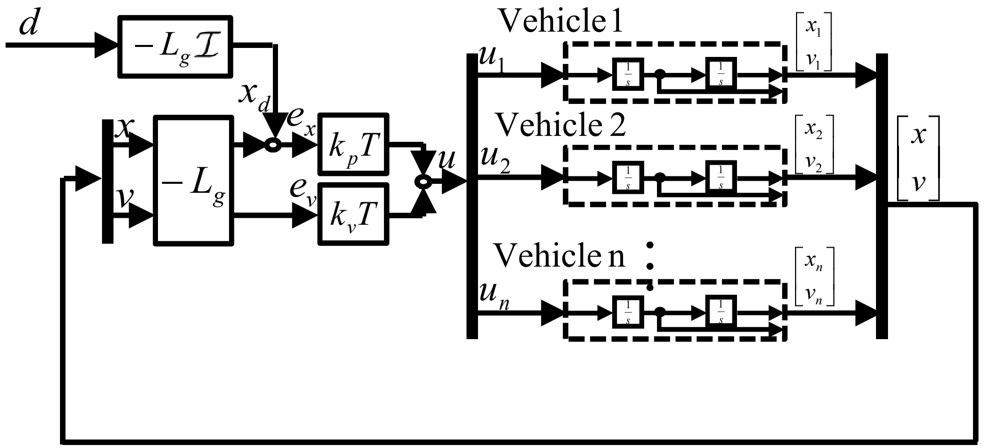
$$s.t. \; -(d(k)+\delta) DL_g \mathcal{I} \geq -\frac{D}{k'_p T}\left(x(k) + 2Tv(k) - k'_p TL_g x(k) - k'_v TL_g v(k)\right)$$

Speed profile



Summary

- Problem description of middle perspective



$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} I & TI \\ -k'_p L_g & I - k'_v L_g \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} - k'_p \begin{bmatrix} 0 \\ L_g \mathcal{I} \end{bmatrix} d(k)$$

- Safety analysis of vehicle platoon

Theorem 1
→ Theorem 2
  → Corollary 1
→ Theorem 3

Reference

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Appendix

Simulation

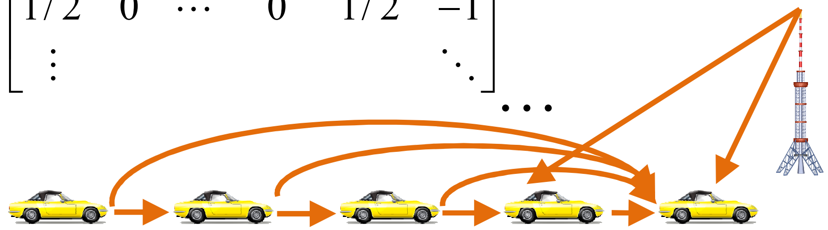
Comparative Verification

Precede following

$$L_g = \begin{bmatrix} \ddots & & & \\ & 1 & -1 & \\ & & & \ddots \end{bmatrix}$$

Leader and Precede following

$$L_g = \begin{bmatrix} \vdots & & & \ddots & \ddots & \\ 1/2 & 0 & \cdots & 1/2 & -1 & 0 \\ 1/2 & 0 & \cdots & 0 & 1/2 & -1 \\ \vdots & & & & & \ddots \end{bmatrix}$$



Simulation Settings

Parameter

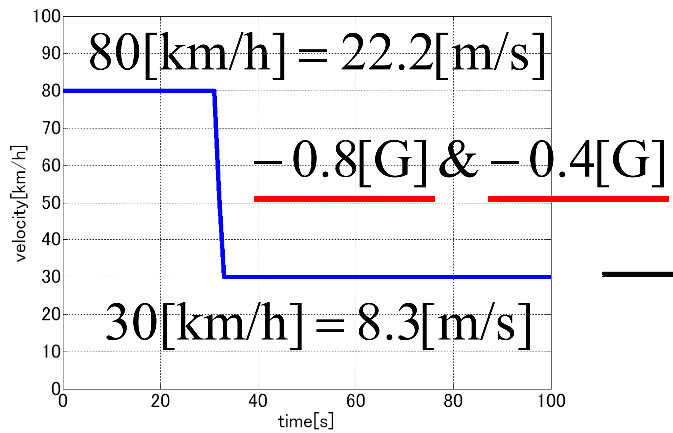
$k_p = 2, k_v = 2$

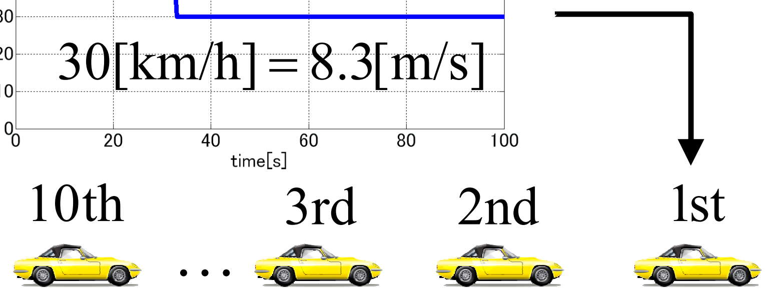
$d = 25[\mathrm{m}]$ →Time head way

80[km/h]: 1.13[s]

30[km/h]: 3.00[s]

Speed profile

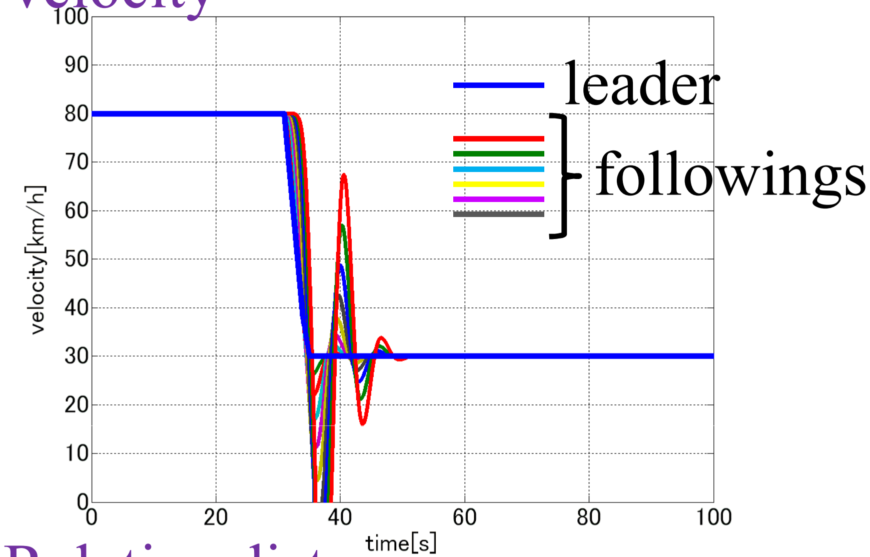




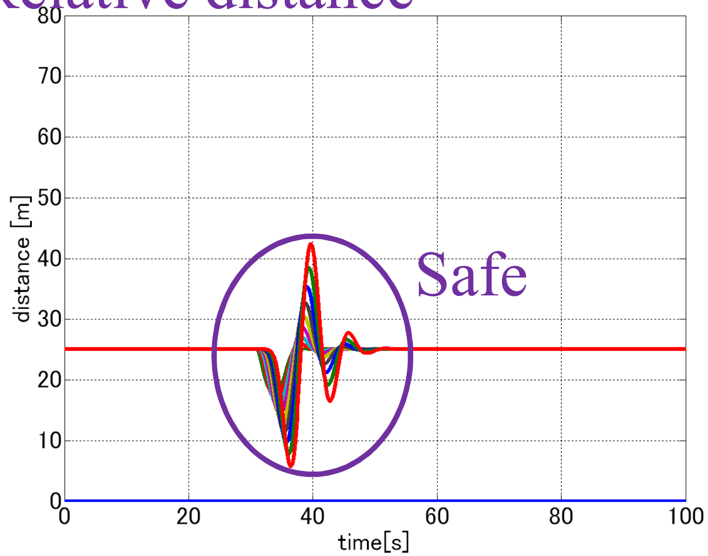
Simulation Result (-0.4[G])

Precede following

Velocity

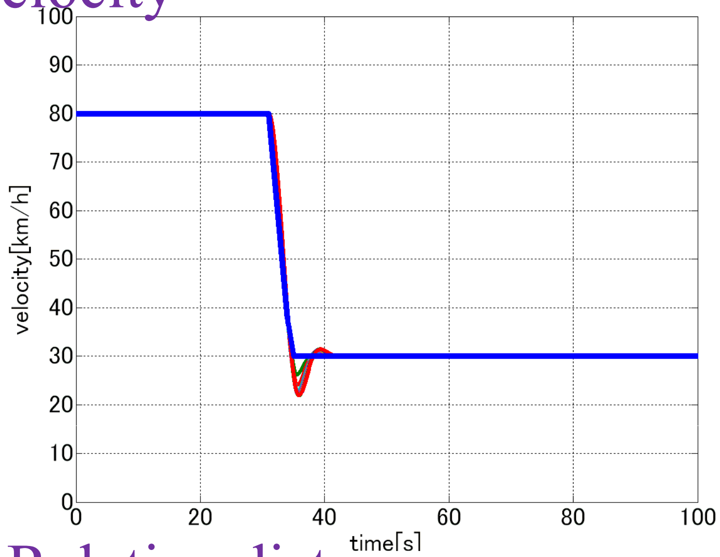


Relative distance

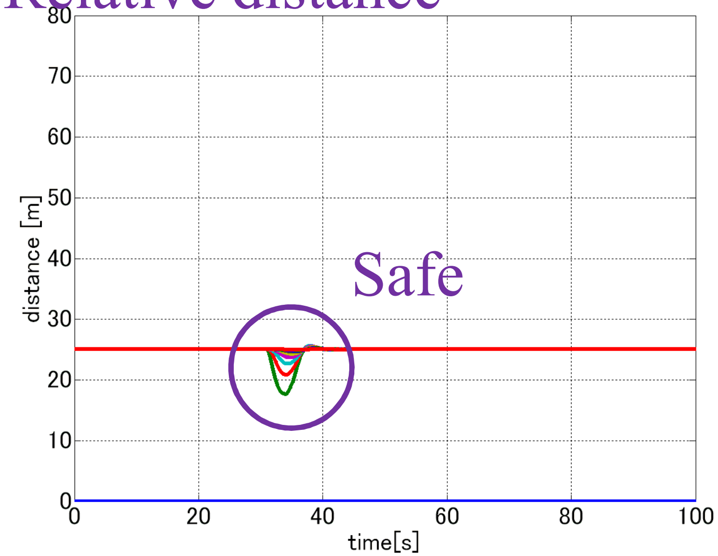


Leader and Precede following

Velocity



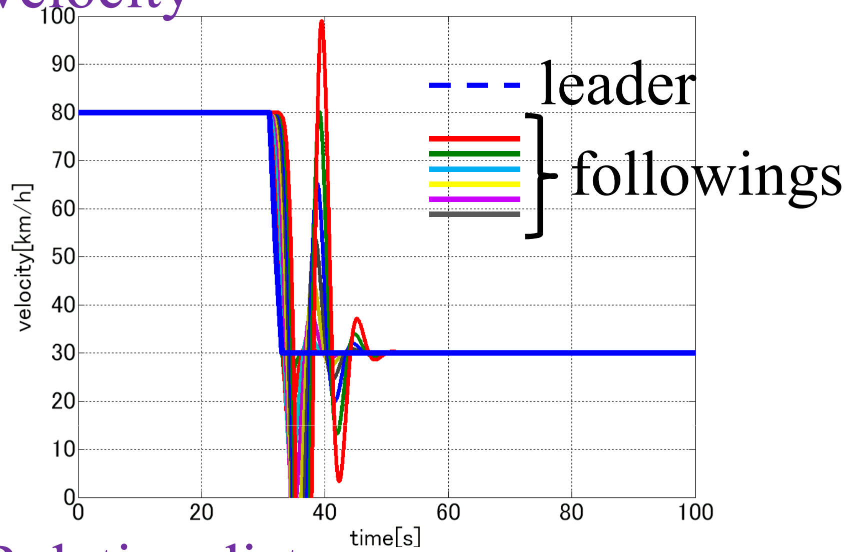
Relative distance



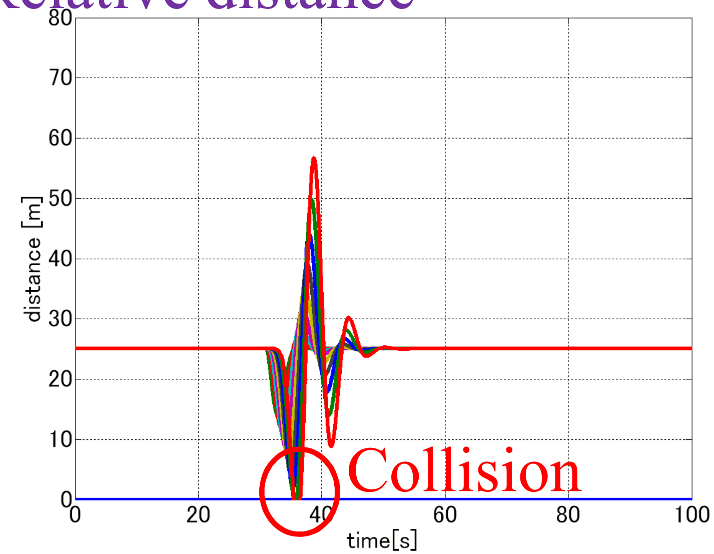
Simulation Result (-0.8[G])

Precede following

Velocity

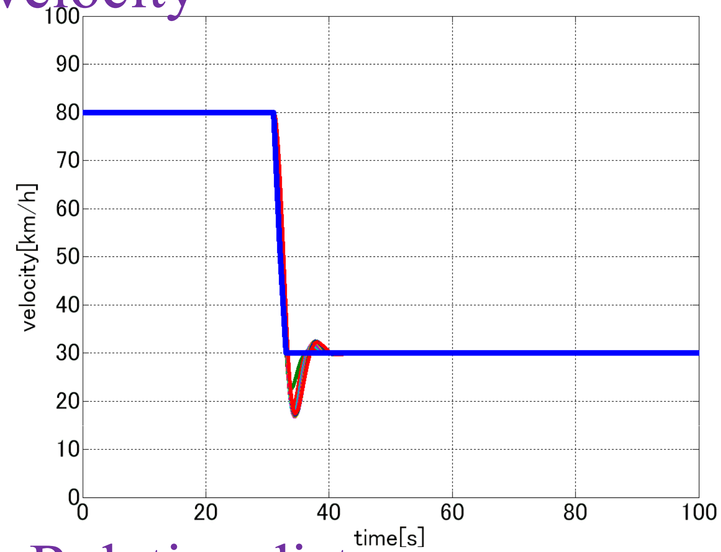


Relative distance

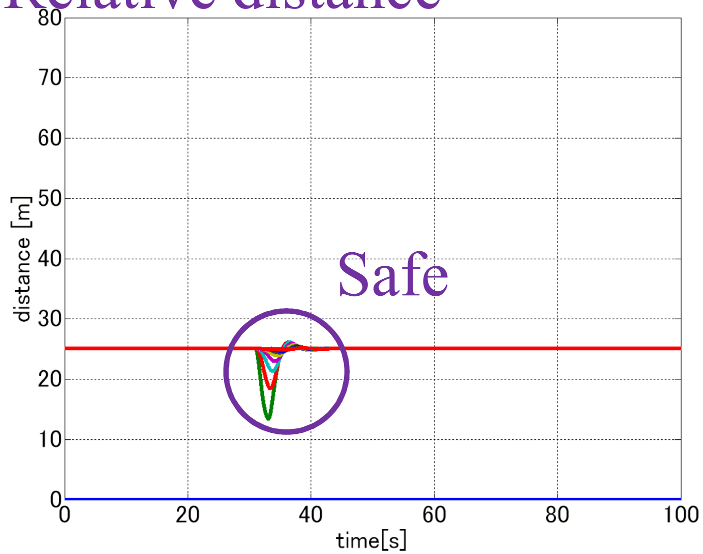


Leader and Precede following

Velocity



Relative distance



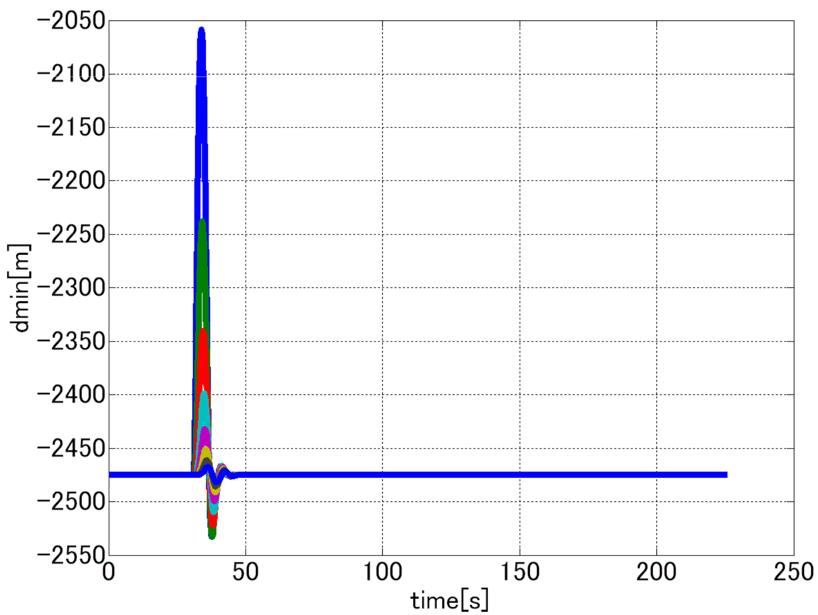
Simulation Result

Leader and Precede following

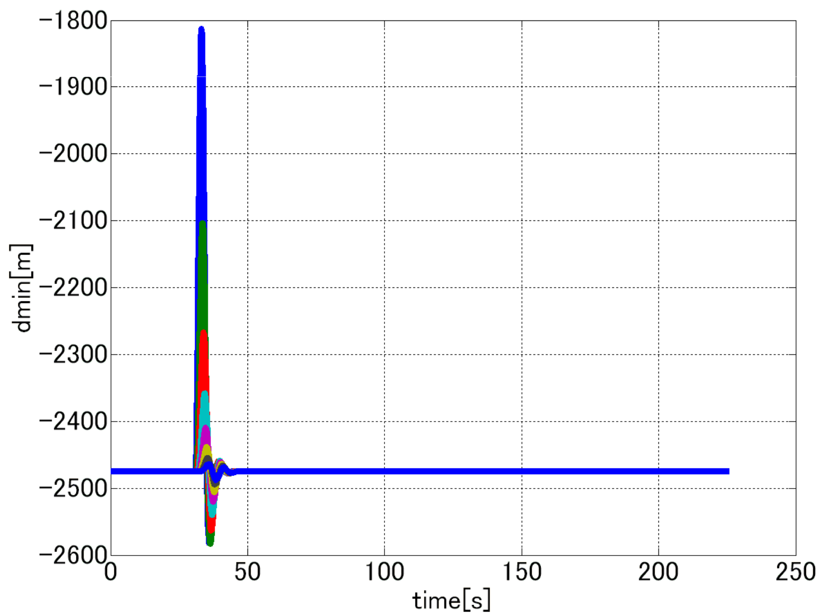
$$dDL_gJ > -\frac{D}{k'_p t}\left(x(k) + 2tv(k) + k'_p tL_g x(k) + k'_v tL_g v(k)\right)$$

$$\Rightarrow d_{\min} DL_gJ = -\frac{D}{k'_p t}\left(x(k) + 2tv(k) + k'_p tL_g x(k) + k'_v tL_g v(k)\right)$$

dmin -0.4[G]



dmin -0.8[G]

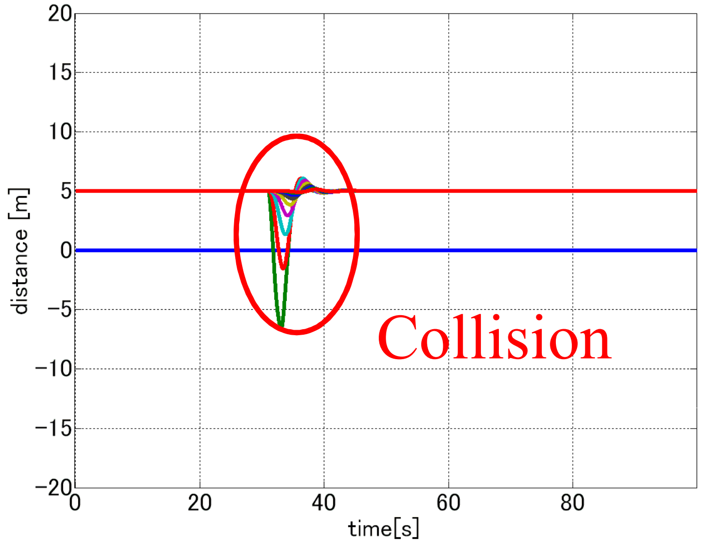


dmin has much luxury

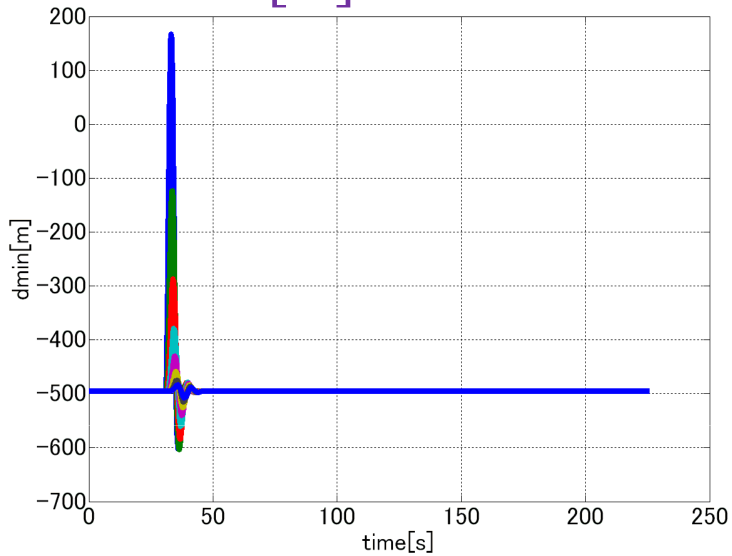
Simulation Result(-0.8[G])

$d = 25[\mathrm{m}] \rightarrow 5[\mathrm{m}]$

Relative distance



dmin -0.8[G]



dmin feedback vehicle control

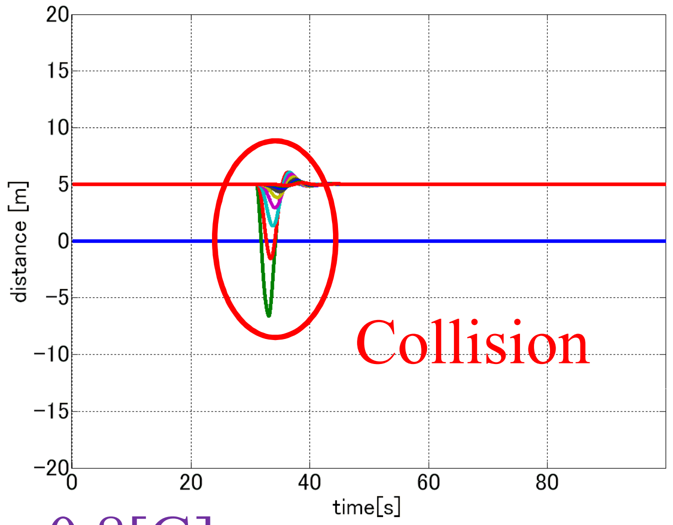
$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} I & tI \\ k'_p L_g & I + k'_v L_g \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + k'_p \begin{bmatrix} 0 \\ L_g J \end{bmatrix} d'_{\min}$$

$$d'_{\min} = \begin{cases} d_{\min} & d_{\min} > d \\ d & d_{\min} < d \end{cases}$$

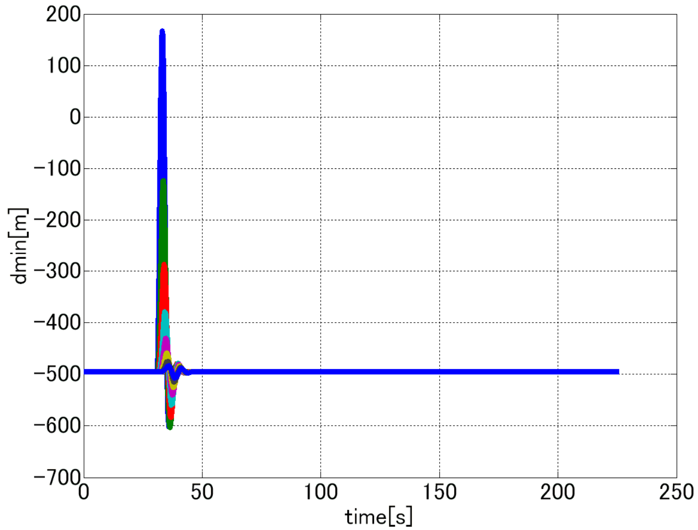
Simulation Result(-0.8[G]) $d = 5$[m]

Constant d vehicle control

Relative distance

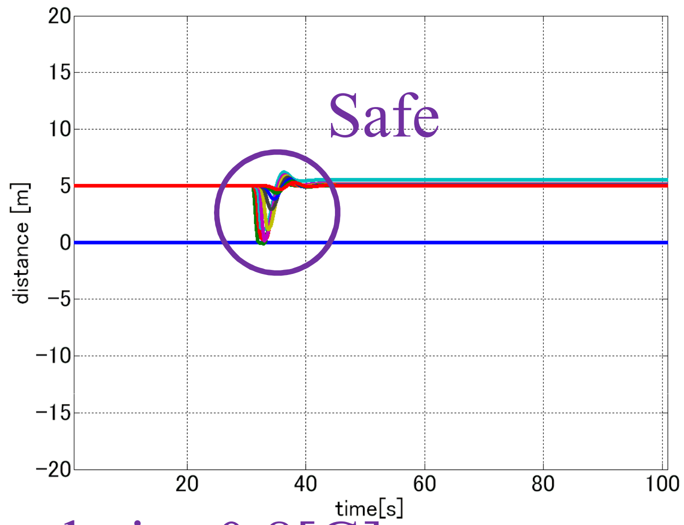


dmin -0.8[G]



dmin feedback vehicle control

Relative distance



dmin -0.8[G]