Image Processing for Passivity Based Visual Motion Observer

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Outline

- Motivation
- Background
  - Visual Motion Observer
- Progress : Feature Information
- Feature Velocities
- Conclusion and Feature Works

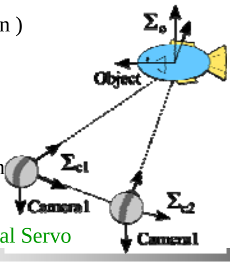
Motivation

How Animals Realize Real(External) World?

There are many methods to know real world.
- Vision - Sound - Sonic wave etc.

Then, they must be aware of real world.

- One approach (Vision)
  Stereo vision
  Two eyes are used to see object at 3D
  Reconstruction of an object pose from geometry relation



Position based Visual Servo

Motivation

- One approach (Vision)
  Stereo vision
  Two eyes are used to see object at 3D
  Shape is important

A question

All animals have stereo vision?
Some animals not always can see using two eyes

- Another approach (Vision)
  An eye is used to realize external world information
  Motion is important

One visual Sensor is used to estimate target motion
Visual Motion Observer (VMO)[4]

Background

Visual Sensor

Now: Camera sensor (visual information)

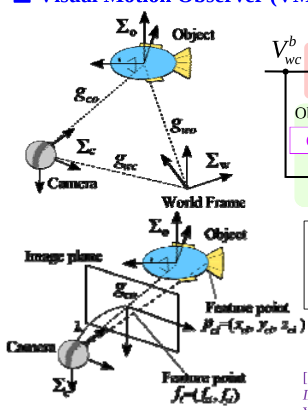
Advantages
- Rich information
- Easy understanding

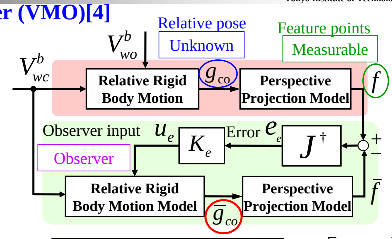
Application
- Target tracking
- Monitoring

http://www.security-d.com/cam_watch.html

Background

Visual Motion Observer (VMO)[4]





An observer to estimate the relative pose from visual measurements

$$g = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

J : Image jacobian

[1] M. Fujita, H. Kawai and M. W. Spong, *IEEE Transactions on Control Systems Technology*, Vol. 15, No.1, pp. 40-52, 2007.

Passivity-based Visual Motion Observer [4]

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Lemma : Passivity

If $V_{wo}^{b} = 0$, then the visual feedback system satisfies

$$\int_0^T -u_e^T \nu_e d\tau \geq -\beta_c \ , \quad \forall T > 0$$

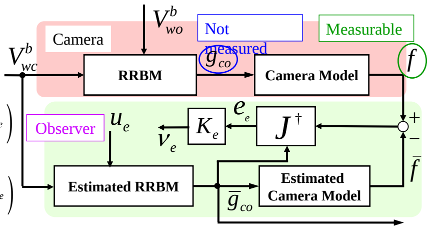
where ν_e is defined as $\nu_e = K_e e_e$ and β_c is positive scalar.

Proof :

Use energy function

$$V = \frac{1}{2}\|p_{ee}\|^2 + \frac{1}{2}\mathrm{tr}\left(I - e^{\hat{\xi}\theta_{ee}}\right)$$

$$\left(R_{ee} = e^{\hat{\xi}\theta_{ee}}\right)$$



[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE Transactions on Control Systems Technology*, Vol. 15, No1, pp. 40-52, 2007.

Topics : Feature Points

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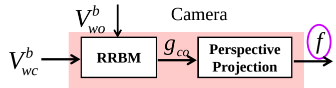
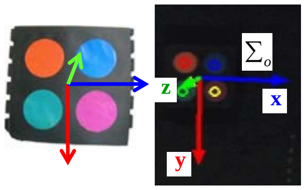
Previous Feature Points

Multiple point features on a known object are given.[1]



- Feature Points
  - Colored circles (four color)
  - Feature points on a plane
- Extraction algorithm
  - Extraction of each colored circle



Weak Points

- Difficulty of color variation
- Weakness against change of brightness
- Problem about Occlusion

Reconsideration

Topics : Feature Points

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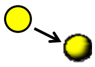
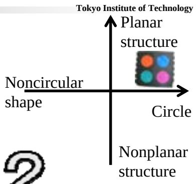
Reconsideration about Feature Points

Addition of feature points

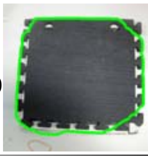
- 3D feature points
  - Example : the control of quad copter
- Letters (ex. Alphabets, Numbers)

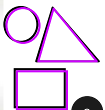




Extraction from target object

- Point based Methods
  - Speeded-Up Robust Features(SURF) etc.
- Edge based Methods
- Shape based Methods
  - Snake (Detection of object's shape)
  - Extraction of contours
- etc.





Result : Movie

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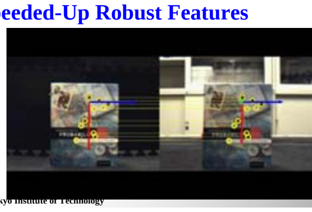
Face Detection

Ball Features

Speeded-Up Robust Features



Snake and Optical Flow

Sorry!
There are no movies to watch

Good Points and Weak Points

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Method	Extraction Difficulty	Extraction Time[ms]	Implementation to VMO	Accuracy of VMO
Ball Features	△	60	※Finished	△
Snake	△～×	Over 200	In review	(△～×)
SURF	○～△	124～40	Finished	△
Face Detection	○～△	100～80	To be prepared	(△)
Optical Flow	○～×	100～80	To be prepared	(△)
Colored Circle	○～△	60	Finished	○～△

⇧

Note : Self review

Topics : Optical Flow

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Optical Flow

Optical flow is a reliable approximation to two-dimensional image motion

Estimation of the displacement field between two sequence images

It is similar to a human eye

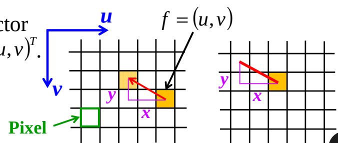
Detail

Extraction of a pixel displacement in a image sequence

Then, the difference is calculated

The value of a difference vector is movement of a pixel $f = (u,v)^T$.

$$\dot{f} = \frac{d}{dt} f = (\dot{u}, \dot{v})^T = \begin{pmatrix} x \\ y \end{pmatrix}$$

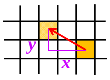


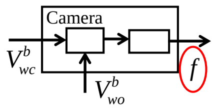
Application to Visual Motion Observer

Phase Lead Compensation

Optical flow : Extraction of a pixel displacement in a image sequence

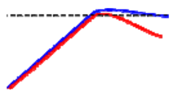
$$\boxed{\dot{f} = \frac{d}{dt} f = (\dot{u}, \dot{v})^T = (x, y)^T}$$





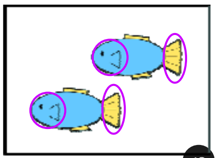
We can use feature points position f and their velocity $\dot{f}$

This leads to phase lead compensation



Problem in these days

- Image processing time is long : over 100ms
  Same problem occur[8]
  Consideration about particular area
- Miss calculation because of extraction miss
  Feature works



3D Target Motion Model

VMO Integrating Internal Representation of 3D Target Motion Model [5]

RRBM Model with Target Model

$\dot{\bar{g}}_{co} = -\hat{V}_{wc}^b \bar{g}_{co} + \bar{g}_{co}\bar{V}_{wo}^b - \bar{g}_{co}\hat{u}_e$

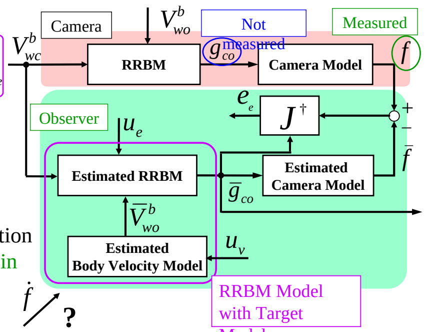
Object velocity model

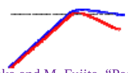
c.f. RRBM

$\dot{g}_{co} = -\hat{V}_{wc}^b g_{co} + g_{co}\hat{V}_{wo}^b$

Expansion

Phase lead compensation to increase gain margin





[5] T. Hatanaka and M. Fujita, "Passivity-based Visual Motion Observer Integrating Internal Representation of 3D Target Object Motion," *Proc. of the 2012 American Control Conference*, Montreal, Canada, 2012.

Addition of feature velocities

Perspective Projection (Pinhole Camera Model)

$$\begin{bmatrix} f_{xi} \\ f_{yi} \end{bmatrix} = \frac{\lambda}{z_{ci}} \begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}$$

Previous work[4]

$$\frac{d}{dt} \Rightarrow \begin{bmatrix} \dot{f}_{xi} \\ \dot{f}_{yi} \end{bmatrix} = \frac{\lambda}{z_{ci}} \begin{bmatrix} \dot{x}_{ci} - \dot{z}_{ci}\dfrac{x_{ci}}{z_{ci}} \\ \dot{y}_{ci} - \dot{z}_{ci}\dfrac{y_{ci}}{z_{ci}} \end{bmatrix}$$

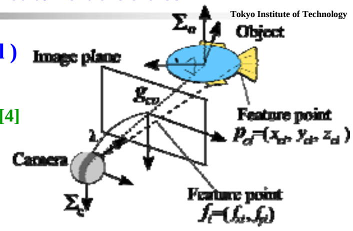
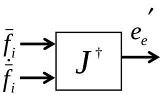


Image Jacobian with Time Differential Form

$f_{ei} = f_i - \bar{f}_i$

$\dot{f}_{ei} = \dot{f}_i - \dot{\bar{f}}_i$



$$\begin{bmatrix} f_{xi} \\ f_{yi} \\ \dot{f}_{xi} \\ \dot{f}_{yi} \end{bmatrix} = f(p_{ci}, \dot{p}_{ci}) = \begin{bmatrix} \dfrac{\partial f}{\partial x_{ci}} & \dfrac{\partial f}{\partial y_{ci}} & \cdots & \dfrac{\partial f}{\partial \dot{z}_{ci}} \end{bmatrix}\Bigg|_{x_{ci}=\bar{x}_{ci}, \cdots, \dot{z}_{ci}=\dot{\bar{z}}_{ci}} \begin{bmatrix} p_{ci} - \bar{p}_{ci} \\ \dot{p}_{ci} - \dot{\bar{p}}_{ci} \end{bmatrix}$$

Previous work[7]

Next, Derivative $\dot{p}_{ci} - \dot{\bar{p}}_{ci}$

Addition of feature velocities

The Relation between the Actual Feature Point and the Estimated One

$$p_{ci} - \bar{p}_{ci} = \begin{bmatrix} \bar{R}_{co} & -\bar{R}_{co}(\bar{R}_{co} p_{ci})^\wedge \end{bmatrix} \begin{bmatrix} p_{ee} \\ e_R(R_{ee}) \end{bmatrix}$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}^\wedge = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$f_i - \bar{f}_i = \begin{bmatrix} \dfrac{\lambda}{\bar{z}_{ci}} & 0 & -\dfrac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \dfrac{\lambda}{\bar{z}_{ci}} & -\dfrac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} (p_{ci} - \bar{p}_{ci})$$

Estimated Error Vector $e_e = \begin{bmatrix} p_{ee} \\ e_R(R_{ee}) \end{bmatrix}$

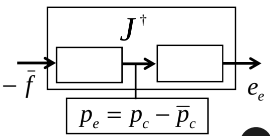
$p_{ee} = \bar{R}_{co}(p - \bar{p})$

$e_R(R_{ee}) = \frac{1}{2}(R_{ee} - R_{ee}^T)^\vee$

Previous work[4] in detail

I consider same numerical method to calculate $\dot{p}_{ci} - \dot{\bar{p}}_{ci}$

$f_e = f - \bar{f}$



Addition of Feature Velocities

Body velocity of the target object : $V_{wo} = \begin{bmatrix} \omega & v \\ 0 & 0 \end{bmatrix}$ Observer Input : $u_e = \begin{bmatrix} \bar{\omega} & \bar{v} \\ 0 & 0 \end{bmatrix}$

Camera (fixed)

$$p_{ci} = g_{co} p_{oi} \;\underset{\frac{d}{dt}}{\Rightarrow}\; \dot{p}_{ci} = \dot{g}_{co} p_{oi} = g_{co}\hat{V}_{wo} \begin{bmatrix} p_{oi} \\ 1 \end{bmatrix} = R_{co}\hat{\omega}\, p_{oi} + R_{co} v$$

$$(\because \dot{g}_{co} = -v_{wc} g_{co} + g_{co}\hat{V}_{wo} = g_{co}\hat{V}_{wo})$$

Camera model (fixed)

$$\bar{p}_{ci} = \bar{g}_{co} p_{oi} \;\underset{\frac{d}{dt}}{\Rightarrow}\; \dot{\bar{p}}_{ci} = \dot{\bar{g}}_{co} p_{oi} = \bar{g}_{co}\hat{u}_e \begin{bmatrix} p_{oi} \\ 1 \end{bmatrix} = \bar{R}_{co}\hat{\bar{\omega}}\, p_{oi} + \bar{R}_{co}\bar{v}$$

$$(\because \dot{\bar{g}}_{co} = -v_{wc}\bar{g}_{co} + \bar{g}_{co}\hat{u}_e = g_{co}\hat{V}_{wo})$$

From these equations,

$$\dot{p}_{ci} - \dot{\bar{p}}_{ci} = R_{co}\hat{\omega}\, p_{oi} + R_{co} v - (\bar{R}_{co}\hat{\bar{\omega}}\, p_{oi} + \bar{R}_{co}\bar{v})$$

Eliminate actual rotation

$$= \bar{R}_{co}\{(R_{ee}\hat{\omega} - I)p_{oi} + R_{ee} v - \bar{v}\} \quad (\because R_{ee} = \bar{R}_{co}^{-1} R_{co})$$

Addition of Feature Velocities

$$\dot{p}_{ci} - \dot{\bar{p}}_{ci} = \bar{R}_{co}\{(R_{ee}\hat{\omega} - I)p_{oi} + R_{ee} v - \bar{v}\}$$

$$= \bar{R}_{co}\{(R_{ee}\hat{\omega} - \hat{\bar{\omega}} R_{ee})p_{oi} + \hat{\bar{\omega}} R_{ee} p_{oi} - p_{oi} + (R_{ee} v - \bar{v} - \hat{\bar{\omega}} p_{ee}) + \hat{\bar{\omega}} p_{ee}\}$$

Kinematic Model of Estimation Error

$$\hat{V}_{ee} = -\hat{u}_e g_{ee} + g_{ee}\hat{V}_{wo} = -\begin{bmatrix} \hat{\bar{\omega}} & \bar{v} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} R_{ee} & p_{ee} \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} R_{ee} & p_{ee} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} v_{ee} = -\hat{\bar{\omega}} p_{ee} - \bar{v} + R_{ee} v \\ \hat{\omega}_{ee} = -\hat{\bar{\omega}} R_{ee} + R_{ee}\hat{\omega} \end{cases} \qquad \left(V_{ee} = \begin{bmatrix} \omega_{ee} & v_{ee} \\ 0 & 0 \end{bmatrix} \right)$$

v_{ee} and ω_{ee} seem to be variables $\Rightarrow$ Add to estimation error

c.f. $e_e = \begin{bmatrix} p_{ee} \\ e_R(R_{ee}) \end{bmatrix}$

Addition of Feature Velocities

Kinematic Model of Estimation Error

$$\begin{cases} v_{ee} = -\hat{\bar{\omega}} p_{ee} - \bar{v} + R_{ee} v \\ \omega_{ee} = -\hat{\bar{\omega}} R_{ee} + R_{ee} \hat{\omega} \end{cases} \quad \left(V_{ee} = \begin{bmatrix} \omega_{ee} & v_{ee} \\ 0 & 0 \end{bmatrix} \right)$$

$$\dot{p}_{ci} - \dot{\bar{p}}_{ci} = \bar{R}_{co} \left\{ \hat{\omega}_{ee} p_{oi} + \hat{\bar{\omega}} R_{ee} p_{oi} - \hat{\bar{\omega}} p_{oi} + v_{ee} + \hat{\bar{\omega}} p_{ee} \right\}$$
$$\vdots$$
$$= \bar{R}_{co} \left\{ \hat{\bar{\omega}} p_{ee} - (\bar{R}_{co} p_{oi})^{\wedge} \hat{\bar{\omega}} e_R(R_{ee}) + v_{ee} - \hat{p}_{oi} \omega_{ee} \right\}$$

$$\therefore \dot{p}_{ci} - \dot{\bar{p}}_{ci} = \bar{R}_{co} \begin{bmatrix} \hat{\bar{\omega}} & -(\bar{R}_{co} p_{oi})^{\wedge} \hat{\bar{\omega}} & I & -\hat{p}_{oi} \end{bmatrix} \begin{bmatrix} p_{ee} \\ e_R(R_{ee}) \\ v_{ee} \\ \omega_{ee} \end{bmatrix}$$

Addition of Feature Velocities

The Relation between Feature Points on Image Plane and Them in Space

$$\begin{bmatrix} f_{xi} \\ f_{yi} \\ \dot{f}_{xi} \\ \dot{f}_{yi} \end{bmatrix} = \left. \begin{bmatrix} \dfrac{\partial f}{\partial x_{ci}} & \dfrac{\partial f}{\partial y_{ci}} & \cdots & \dfrac{\partial f}{\partial \dot{z}_{ci}} \end{bmatrix} \right|_{x_{ci} = \bar{x}_{ci}, \cdots, \dot{z}_{ci} = \dot{\bar{z}}_{ci}} \begin{bmatrix} p_{ci} - \bar{p}_{ci} \\ \dot{p}_{ci} - \dot{\bar{p}}_{ci} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\lambda}{\bar{z}_{ci}} & 0 & -\dfrac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} & 0 & 0 & 0 \\ 0 & \dfrac{\lambda}{\bar{z}_{ci}} & -\dfrac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} & 0 & 0 & 0 \\ -\dfrac{\lambda \dot{\bar{z}}_{ci}}{\bar{z}_{ci}^2} & 0 & -\dfrac{\lambda \dot{\bar{x}}_{ci}}{\bar{z}_{ci}^2} + \dfrac{2\lambda \dot{\bar{z}}_{ci} \bar{x}_{ci}}{\bar{z}_{ci}^3} & \dfrac{\lambda}{\bar{z}_{ci}} & 0 & -\dfrac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & -\dfrac{\lambda \dot{\bar{z}}_{ci}}{\bar{z}_{ci}^2} & -\dfrac{\lambda \dot{\bar{y}}_{ci}}{\bar{z}_{ci}^2} + \dfrac{2\lambda \dot{\bar{z}}_{ci} \bar{y}_{ci}}{\bar{z}_{ci}^3} & 0 & \dfrac{\lambda}{\bar{z}_{ci}} & -\dfrac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} p_{ci} - \bar{p}_{ci} \\ \dot{p}_{ci} - \dot{\bar{p}}_{ci} \end{bmatrix}$$

Addition of Feature Velocities

Image Jacobian of the *i* th Feature Point

$$\begin{bmatrix} f_{xi} \\ f_{yi} \\ \dot{f}_{xi} \\ \dot{f}_{yi} \end{bmatrix} = \begin{bmatrix} \dfrac{\lambda}{\bar{z}_{ci}} & 0 & -\dfrac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} & 0 & 0 & 0 \\ 0 & \dfrac{\lambda}{\bar{z}_{ci}} & -\dfrac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} & 0 & 0 & 0 \\ -\dfrac{\lambda \dot{\bar{z}}_{ci}}{\bar{z}_{ci}^2} & 0 & -\dfrac{\lambda \dot{\bar{x}}_{ci}}{\bar{z}_{ci}^2} + \dfrac{2\lambda \dot{\bar{z}}_{ci} \bar{x}_{ci}}{\bar{z}_{ci}^3} & \dfrac{\lambda}{\bar{z}_{ci}} & 0 & -\dfrac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & -\dfrac{\lambda \dot{\bar{z}}_{ci}}{\bar{z}_{ci}^2} & -\dfrac{\lambda \dot{\bar{y}}_{ci}}{\bar{z}_{ci}^2} + \dfrac{2\lambda \dot{\bar{z}}_{ci} \bar{y}_{ci}}{\bar{z}_{ci}^3} & 0 & \dfrac{\lambda}{\bar{z}_{ci}} & -\dfrac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} \begin{bmatrix} I & -(\bar{R}_{co} p_{oi})^{\wedge} & 0 & 0 \\ \hat{\bar{\omega}} & -(\bar{R}_{co} p_{oi})^{\wedge} \hat{\bar{\omega}} & I & -\hat{p}_{oi} \end{bmatrix} \begin{bmatrix} \bar{R}_{co} & 0 & 0 & 0 \\ 0 & \bar{R}_{co} & 0 & 0 \\ 0 & 0 & \bar{R}_{co} & 0 \\ 0 & 0 & 0 & \bar{R}_{co} \end{bmatrix} \begin{bmatrix} p_{ee} \\ e_R(R_{ee}) \\ v_{ee} \\ \omega_{ee} \end{bmatrix}$$

$$\boxed{J_i}$$

Image Jacobian with Feature Velocities

Previous Work[4]

$$f_e = f - \bar{f} \longrightarrow \boxed{J^{\dagger}} \longrightarrow e_e$$

$$e_e = \begin{bmatrix} p_{ee} \\ e_R(R_{ee}) \end{bmatrix}$$

Passivity : ○

Proof : Use energy function

$$V = \frac{1}{2} \| p_{ee} \|^2 + \frac{1}{2} \mathrm{tr}(I - R_{ee})$$

Observer Input

$$u_e = -K(-e_e) = Ke_e$$

This Work with Feature Velocities

$$\begin{matrix} f_e = f - \bar{f} \\ \dot{f}_e = \dot{f} - \dot{\bar{f}} \end{matrix} \longrightarrow \boxed{J'^{\dagger}} \longrightarrow e_e'$$

$$e_e' = \begin{bmatrix} p_{ee} \\ e_R(R_{ee}) \\ v_{ee} \\ \omega_{ee} \end{bmatrix}$$

Passivity : not proven yet

Observer Input

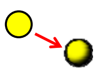
Continued : Feature work

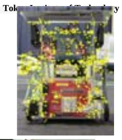
Progress and Feature Works

Progress

Reconsideration about feature points

- 3D feature points
- Speeded-Up Robust Features(SURF)





Consideration about optical flow

It continues to Yuta's bachelor thesis

Feature Works

- Exploration of image processing methods for VMO
  - Shape based - Points based - Time series based etc.
- Efficient analysis of image processing methods on VMO
- Robust feature points for VMO

Thank you !

References

Vision and Internal Representation Model

[1] D. Marr, "Vision: A Computational Investigation into Human Representation and Processing of Visual Information," New York, 1982.

[2] J. Gray, "Consciousness: Creeping Up on the Hard Problem," Oxford University Press, 2004.

[3] R. Sheldrake, "The Sense of Starting at: And Other Aspects of the Extended Mind," New York, Crown Publishers, 2003.

Visual Motion Observer

[4] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis", *IEEE Transactions on Control Systems Technology*, Vol. 15, No.1, p.p. 40-52, 2007.

[5] T. Hatanaka and M. Fujita, "Passivity-based Visual Motion Observer Integrating Internal Representation of 3D Target Object Motion," *Proc. of the 2012 American Control Conference*, Montreal, Canada, 2012.

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[7] H. Kawai, T. Murao and M. Fujita, "Passivity-based Visual Motion Observer with Panoramic Camera for Pose Control," *Journal of Intelligent and Robotic Systems*, Vol. 64, No. 3-4, p.p. 561-583, 2011.

References

Optical Flow

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Visual Servo

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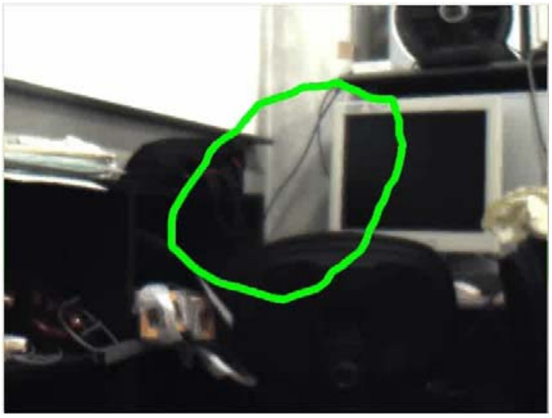
Appendix



Extension of Speeded-Up Robust Features

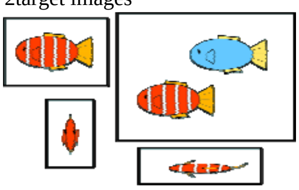
Increment of Target Information

Previous Work

1 target image + 1 camera image

Feature Work

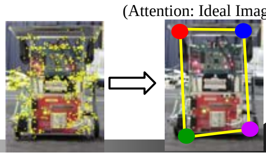
More than 2target images + 1 camera image



More Robust Image Processing

- Extraction of more robust features
- Increase of matching feature points
- Transformation from feature points to obvious form (ex. rectangle …)

(Attention: Ideal Image)



Estimation Error

$$f_e = f - \bar{f} = \begin{pmatrix} f_1 - \bar{f}_1 \\ f_2 - \bar{f}_2 \\ \vdots \\ f_n - \bar{f}_n \end{pmatrix} \in R^{2\times n}$$

$$f_e \rightarrow \boxed{J^{\dagger}} \rightarrow e_e$$

n : The number of feature points, $n \geq 3$

$$e_e = \left(p_e^{\ T}, e_R^T\left(e^{\hat{\xi}\theta}\right)\right)^T \in R^6$$

Pseudo Inverse of image jacobian $J^{\dagger}$

A kind of least squares method from $f_e \in R^{2\times n}$ to $e_e \in R^6$

When the number of feature points n change, the answer of least squares method is different in spite of accurate data.

Main problem of oscillatory estimation error ?