Controller Design for an Omni-directional Mobile Robot

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Introduction

Experiment System at Fujita Lab.

Previous Robots

- Two-wheel Pendulum Robots
- have non-holonomic constraints

⇒ exact verification ?

New Robots

- Omni-directional Robots
- Improvements
  - (a) No non-holonomic constraints
  - (b) Always Stable Equilibrium

⇒ need to design a Local Controller satisfying the design specification

Parameters

States (Pose) from overhead camera

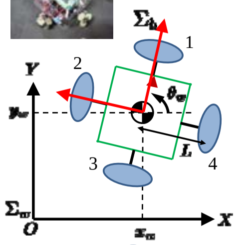
$x_w, y_w \in \mathbb{R}$: position

$\theta_w \in \mathbb{R}$: attitude

Body velocity (control targets)

$v_x^b, v_y^b \in \mathbb{R}$: linear velocity

$\omega^b \in \mathbb{R}$: angular velocity

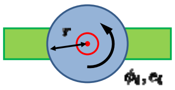


Wheel angles

$\phi_1, \phi_2, \phi_3, \phi_4 \in \mathbb{R}$

Input voltages

$e_1, e_2, e_3, e_4 \in \mathbb{R}$

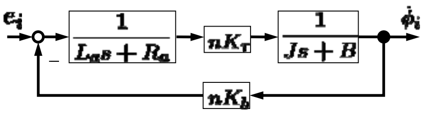


Length

L ($= 0.085$ [m]) : distance between center of robot and one of the wheel

r ($= 0.024$ [m]) : radius of the wheel

Actuator Model



Suspose

$L_a, B \to 0$ (from Product Information)

⇒ $\omega_i(t) = \frac{nK_\tau}{R_a J s + nK_\tau nK_b} e_i(t)$

$R_a J s \to 0$

(unconsidered dynamics delay)

L_a : inner inductance

R_a : inner resistor [Ω]

K_τ : torque constant

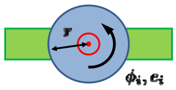
J : inertial of wheel

B : friction

K_b : back EMF constant [Vs]

n : gear rate

Actuator Model $\omega_i(t) = \frac{1}{nK_b} e_i(t)$



Overview of Controller

Data Gotten from Camera

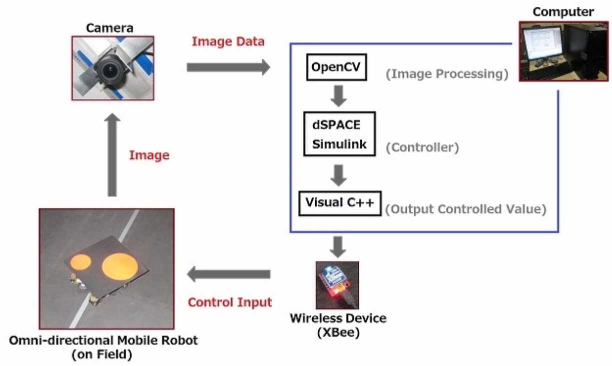
$\begin{bmatrix} x_w \\ y_w \\ \theta_w \end{bmatrix}$ → D/A → $\begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{\theta}_w \end{bmatrix}$ → $T = \begin{bmatrix} \cos\theta_w & \sin\theta_w & 0 \\ -\sin\theta_w & \cos\theta_w & 0 \\ 0 & 0 & 1 \end{bmatrix}$ → measurement value $v_b = \begin{bmatrix} v_x^b \\ v_y^b \\ \omega^b \end{bmatrix}$

Velocity and Position Controller

Input value

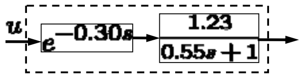
$u = \begin{bmatrix} v_x^{\prime b} \\ v_y^{\prime b} \\ \omega^{\prime b} \end{bmatrix}$ → $\frac{1}{r}\begin{bmatrix} 0 & -1 & -L \\ 1 & 0 & -L \\ 0 & 1 & -L \\ -1 & 0 & -L \end{bmatrix}$ → $\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\phi}_4 \end{bmatrix}$ → nK_b → $\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$

Experimental Environment

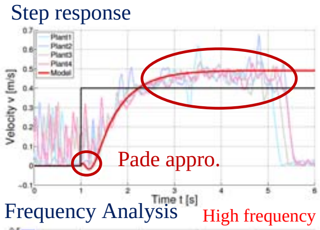


Plant Model

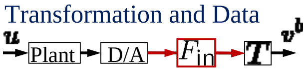
Time Delay + 1st order system



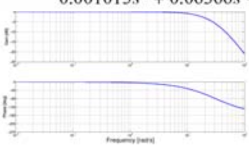
Step response



Transformation and Data

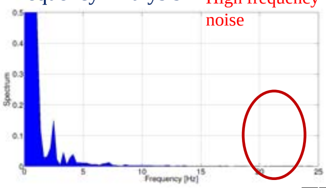


$$F_{in}(s) = \frac{1}{0.001013s^2 + 0.06366s + 1}$$



Frequency Analysis

High frequency noise



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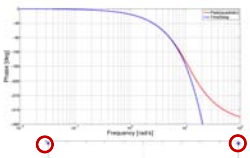
- Introduction
- System Expression
  - Motion Model
  - Experiment Environment
- Controller Design
  - Velocity Controller (FB / FF / 2DOF)
  - Position Controller (FB / FF / 2DOF)
  - Option (FF with StateFeedack / MPT)
  - Comparison
- Summary and Future Work

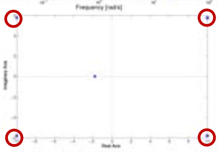
Design Specification (velocity)

Pade Approximation

$$e^{-0.30s} \simeq \frac{s^2 - 20s + 133.3}{s^2 + 20s + 133.3}$$

(considering phase)





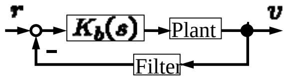
Velocity

- PM=67[deg]
- GM=10-15[dB]
- Steady Position Error $e_s < 10^{-3}$
- Gain Crossover $\omega_{gc} \leq$ 0.75-1.3[rad/s] (considering phase lag, sampling rate)
- Peak gain
  - Sensitivity $\leq$ 1.5(=3.5[dB])
  - Comp. Sens. $\simeq$ 1.3(=2.3[dB])
- Band width (Comp. Sens.) $\leq$ 4[rad/s]
- (Overshoot $\leq$ 5%)

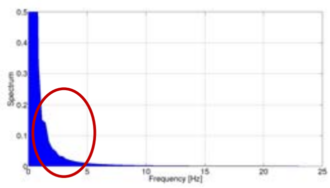
Analysis

- Bode plot for open loop
- Time Step Response
- Gang of Four
- Pole/Zero Map

Velocity Feedback control

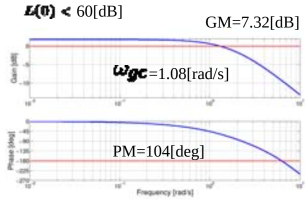


$$\text{Filter} = \frac{1}{0.04938s^2 + 0.4444s + 1}$$



Bode Plot for open loop

$L(0) <$ 60[dB]

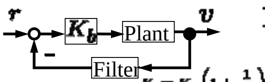


Not satisfy Stable Position Error
- PI control or Phase lag control

$$\lim_{s \to 0} L(s) \geq 60[\text{dB}]$$

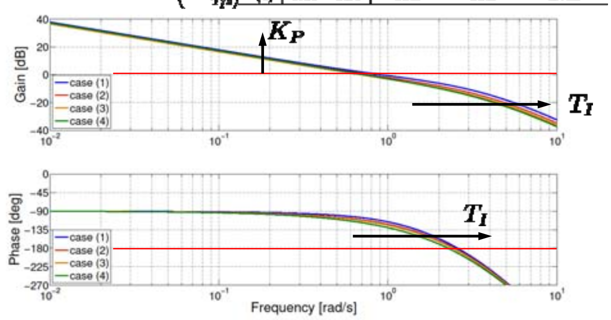
In this seminar,
deal with PI control only

PI velocity FB Control (Bode Plot)

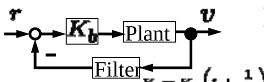


$K_b = K_P\left(1 + \frac{1}{T_I s}\right)$

case	K_P	T_I	GM [dB]	PM [deg]	ω_{gc} [rad/s]
(1)	0.64	1.00	8.73	67.0	0.915
(2)	0.49	0.82	10.3	66.3	0.778
(3)	0.41	0.75	11.4	67.2	0.692
(4)	0.36	0.60	11.2	60.9	0.729



PI velocity FB Control (Step Response)



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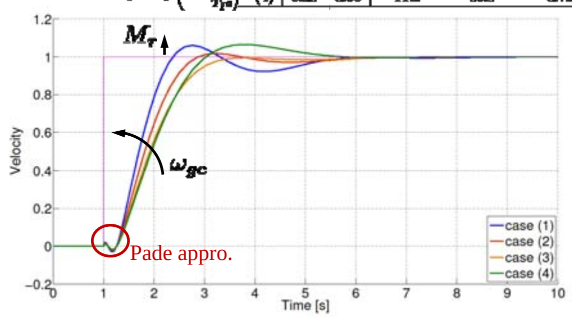
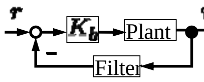
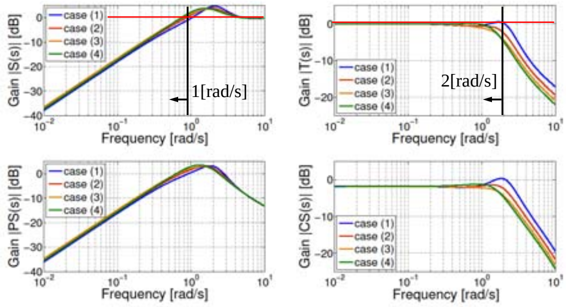


PI velocity FB Control (Gang of Four)

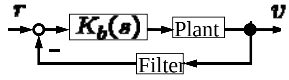


$K_b = K_P\left(1+\frac{1}{T_I s}\right)$

case	K_P	T_I	M_{rs} [dB]	M_r [dB]
(1)	0.64	1.00	4.73	0.52
(2)	0.49	0.82	3.97	0
(3)	0.41	0.75	3.50	0
(4)	0.36	0.60	3.83	0.13



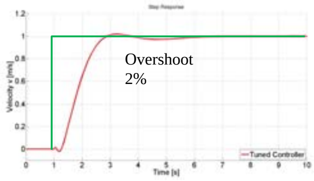
PI velocity FB control



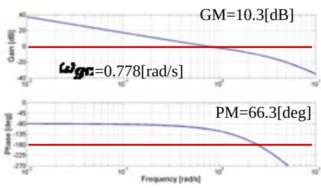
$K_b = 0.49\left(1+\frac{1}{0.82s}\right)$

Case(2)

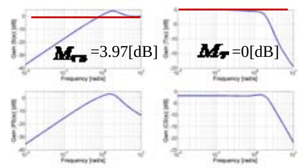
Time response



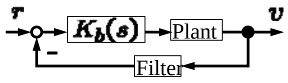
Bode Plot



Gang of Four



PI velocity FB control

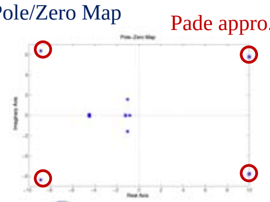


$K_b = 0.49\left(1+\frac{1}{0.82s}\right)$

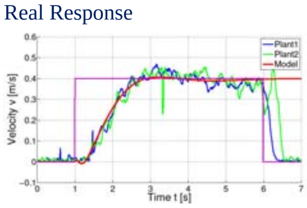
Case(2)

Pole/Zero Map

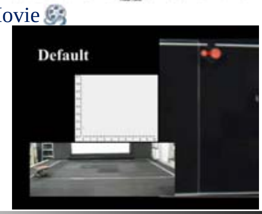
Pade appro.



Real Response



Movie



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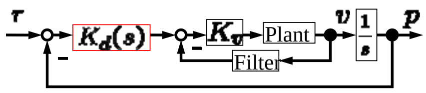
Position Feedback control



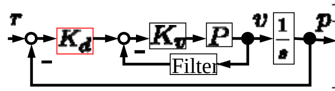
Design Specification

- PM $\geq$ 20[deg]
- GM = 3-10[dB]
- Steady Position Error $e_s < 10^{-3}$
- Gain Crossover $\omega_{gc} \leq$ 0.778[rad/s]
- Peak gain
  - Sensitivity $\leq$ 1.5(=3.5[dB])
  - Comp. Sens. $\simeq$ 1.3(=2.3[dB])
- Band width (Comp. Sens.) $\leq$ 4[rad/s]
- (Overshoot $\leq$ 5%)

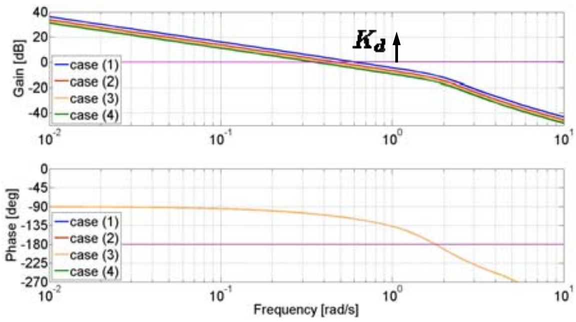
Analysis

- Bode plot for open loop
- Time Step Response
- Gang of Four
- Pole/Zero Map

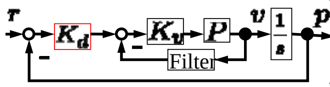
P position FB Control (Bode Plot)



case	K_d	GM [dB]	PM [deg]	ω_{gc} [rad/s]
(1)	0.47	13.2	67.3	0.454
(2)	0.60	11.1	61.9	0.573
(3)	0.68	10	58.7	0.645
(4)	0.73	9.41	56.7	0.69



P position FB Control (Step Response)



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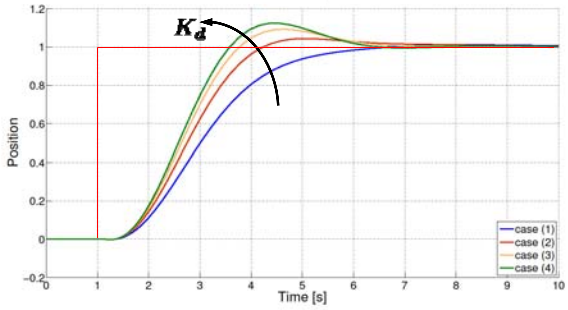
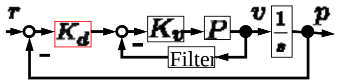
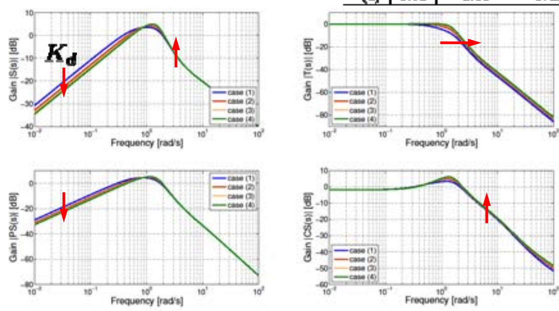


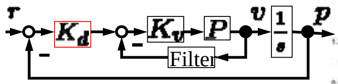
P position FB Control (Gang of Four)



case	K_d	M_{rs} [dB]	M_r [dB]
(1)	0.47	3.82	0
(2)	0.60	4.28	0.0046
(3)	0.68	4.74	0.345
(4)	0.73	5.05	0.452



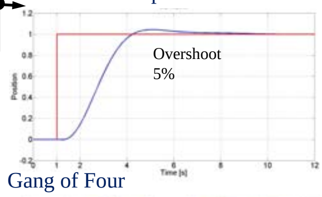
P position FB control



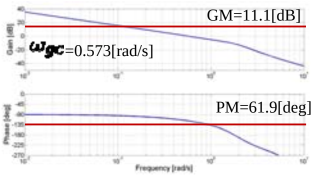
$K_d = 0.60$

Case(2)

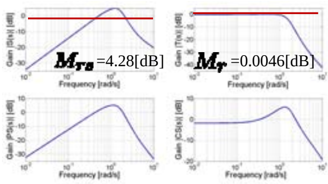
Time response



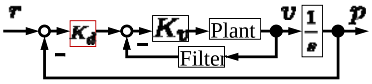
Bode Plot



Gang of Four

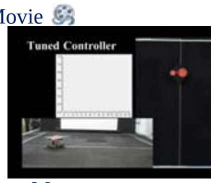


P position FB control



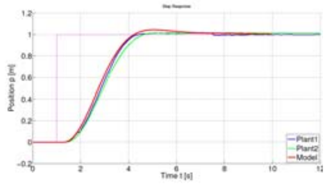
$K_d = 0.60$

Movie



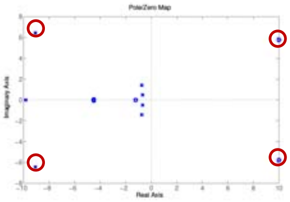
without friction support,
Include 5% Error

Real Response
(with friction support)



Pole/Zero Map

Pade appro.



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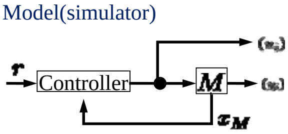
Feedforward controller of 2DOF

Model(simulator)



u_i : ideal input
y_i : ideal output
x_M: inner states of model

Design

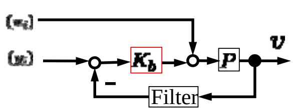
- Close loop system : Stabilization
- Design state feedback controller (following ref. signal r ideally)

Join

input and output of model u_i ,y_i to Real Feedback system

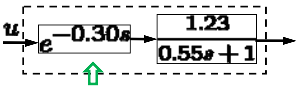
Model Following Control

Real Feedback System



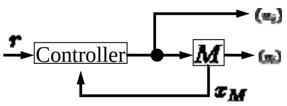
State feedback controller

Plant model



Pade appro.
(quadratic)

Model(simulator)



u_i : ideal input
y_i : ideal output
x_M : inner states of model

Approach

- State equation of plant model

$$\frac{dx}{dt} = Ax + Bu \quad x \in \mathbb{R}^3$$
$$y = Cx \quad u \in \mathbb{R}, \; y \in \mathbb{R}$$

(Controller Canonical Form)

- (A, B) controllable
- (A, C) observable
- rank condition :

$$\text{rank}\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = 3 + 1$$

⇩

State feedback design is usable

LQ regulator

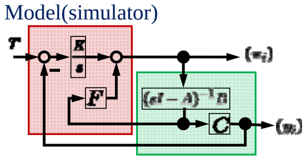
Servo system $X = [x^T \;\; \eta]^T$

$$\frac{dX}{dt} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} X + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} X \quad x \in \mathbb{R}^3$$
$$u = Fx + K\eta \quad u \in \mathbb{R} \; y \in \mathbb{R}$$

- $\eta \in \mathbb{R}$: state for following
- $r \in \mathbb{R}$: external input

Model(simulator)



u_i : ideal input y_i : ideal output

LQ optimal regulator

$$J = \int_0^\infty (X^T Q X + u^T R u) dt$$

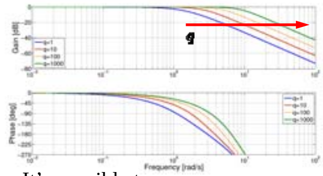
⇩ min J

$$u = Fx + K\eta$$

Compose the simulator

$$Q = \text{diag}(0, 0, 0, q) \quad R = 1$$

Bode Plot of TF from r to y



It's possible to
design band width arbitrarily

Velocity 2DOF with state feedback

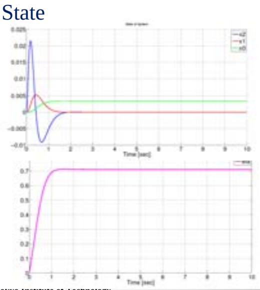
Design

- weight matrix $R = 1$

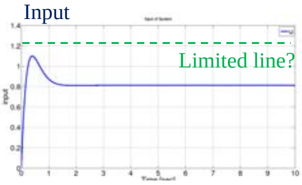
$$J = \int_0^\infty (X^T Q X + u^T R u) dt$$

$$Q = \text{diag}(1, 100, 250000, 60)$$

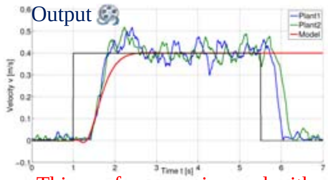
State



Input



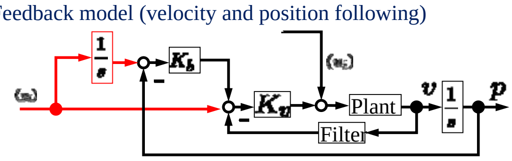
Output



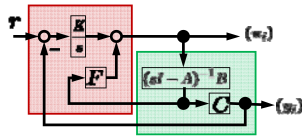
This performance is equal with
one of using transfer function

State feedback controller

Feedback model (velocity and position following)



Model(simulator)



u_i : ideal input
y_i : ideal output

Servo system $X = [x^T \;\; \eta]^T$

$$\frac{dX}{dt} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} X + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} X \quad u = Fx + K\eta$$

LQ optimal regulator

$$J = \int_0^\infty (X^T Q X + u^T R u) dt$$

$$R = 1$$

Position 2DOF with state feedback

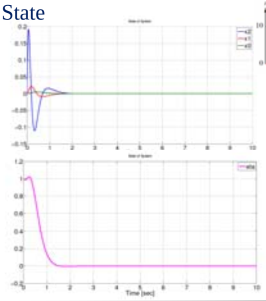
Design

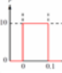
- weight matrix $R = 1$

$$J = \int_0^\infty (X^T Q X + u^T R u) dt$$

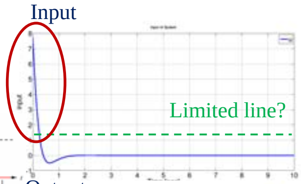
$$Q = \text{diag}(1, 100, 250000, 60)$$

State





Input



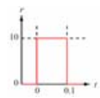
Output

Overshoot : 20% (too large)
Rising response : bad and delay

Position 2DOF with state feedback

Analysis

Input

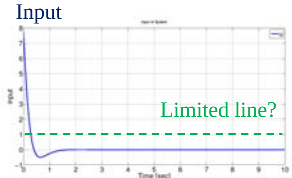


- signal shaping
  (rectangle, triangle, trapezoid)
  ⇨ same result
- input value
  over-input (limited input)
  ⇨ bad rising
- input method
  1 velocity output system
  ⇨ 2 output system
  (velocity / position)

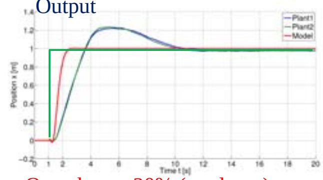
Change weight-matrix

⇨ Hybrid predictive control

Input



Output



Overshoot : 20% (too large)
Rising response : bad and delay

Predictive Control

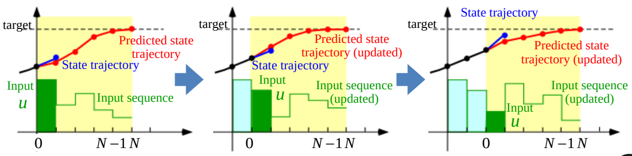
Predictive Control

1) Solve optimal control of fine horizon by the state x_0 at the time k
2) Input the first step of the obtained input now
3) In the next time step, return 1) and repeat

Utility Function $\min_{U^N} (U^N)^T H U^N + h(x_0) U^N$

Constraints $M U^N \le m(x_0)$

(Input, state, termination, …)



Hybrid Predictive Control (MPT Setting)

Plant model

$$\frac{dx}{dt} = Ax + Bu \qquad x \in \mathbb{R}^3$$
$$y = Cx \qquad u \in \mathbb{R}, \quad y \in \mathbb{R}$$: velocity

Transformed System (*)

$$\frac{d}{dt}\begin{bmatrix} x' \\ x_p \end{bmatrix} = \begin{bmatrix} A' & 0 \\ 1\ 0\ 0 & 0 \end{bmatrix}\begin{bmatrix} x' \\ x_p \end{bmatrix} + \begin{bmatrix} B' \\ 0 \end{bmatrix} u$$

$$y' = \begin{bmatrix} y_v \\ y_p \end{bmatrix} = \begin{bmatrix} C' & 0 \\ 0^T & 1 \end{bmatrix}\begin{bmatrix} x' \\ x_p \end{bmatrix}$$

$$x' = \begin{bmatrix} x_v & x_1 & x_2 \end{bmatrix}^T$$

$$A' = T_m A T_m^{-1}$$
$$B' = T_m B$$
$$C' = C T_m^{-1}$$
$$T_m = \begin{bmatrix} & C & \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

MPT system

model

Discrete system (*) (T=0.1[s])

constraints

Input $u \in [-0.74, 0.74]$

State $x_p \in [-0.64, 0.64]$

Evaluation parameters

Steps $N = 10$

Weight $R = 1$

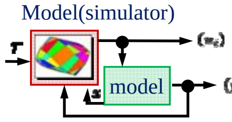
matrix $Q = \mathrm{diag}(1, 100, 250000, 3)$

Output weight

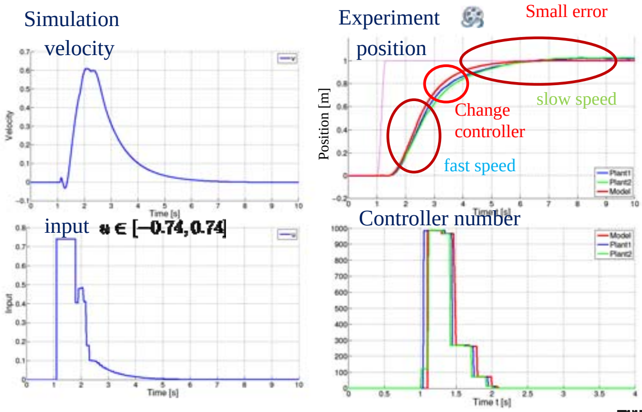
$Q_y = \mathrm{diag}(100, 300)$

tracking mode

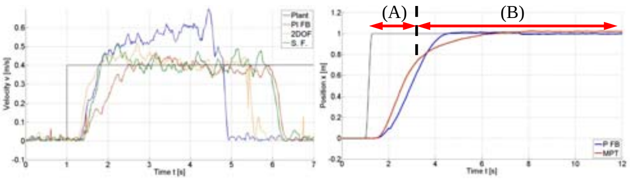
Model(simulator)



Hybrid Predictive Control (Solution)



Conclusion (comparison)



PI Feedback

Improve follow-ability / Stability

Time constant

(compare with PI feedback)

2DOF	Improve 37.5%
State Feedback	Improve 62.5%

(A)

Time constant improves 15%

(compare with P feedback)

(B)

Lower Overshoot

(compare with P feedback)

Summary and Future Work

Summary

Show 2D System Dynamics

Compose Experiment System (cooperation)

Design Local Controller about Linear motion

(velocity and position : nominal model)

Compose Adaptive Cruise Control

Omni-robot has high reproducibility.

Future Work

Design Local Controller about Rotation motion

Robust performance of Linear motion

Verification of Linear and Rotation motion

Installation of Encoder (Identification)

Reference / Appendix

Reference

[1] 杉江俊治, 藤田政之, “フィードバック制御入門,” コロナ社, 1999.
[2] 大明準治, “ロボット運動制御のための2 リンク2 慣性系の非干渉化同定法,” 慶応義塾大学, 博士論文, 2010.

Appendix

System Expression

- Kinematics Model
- System Dynamics (Motion Equations)

Controller Design

- Velocity Controller (2DOF model)

Application

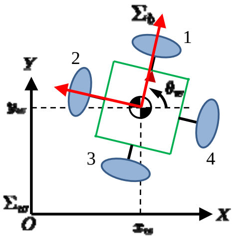
- Cooperative Control (Adaptive Cruise Control)

Kinematics Model

Rigid Body Motion

$$\begin{bmatrix}\dot{x}_w\\ \dot{y}_w\\ \dot{\theta}_w\end{bmatrix} = \overbrace{\begin{bmatrix}\cos\theta_w & -\sin\theta_w & 0\\ \sin\theta_w & \cos\theta_w & 0\\ 0 & 0 & 1\end{bmatrix}}^{R(\theta_w)} \begin{bmatrix}v_x^b\\ v_y^b\\ \omega^b\end{bmatrix}$$

$R(\theta_w)$: Transformation from Σ_b to Σ_w



Transformation
from body velocity
to wheel angular velocity

$$\begin{bmatrix}\dot{\phi}_1\\ \dot{\phi}_2\\ \dot{\phi}_3\\ \dot{\phi}_4\end{bmatrix} = \frac{1}{r}\begin{bmatrix}0 & -1 & -L\\ 1 & 0 & -L\\ 0 & 1 & -L\\ -1 & 0 & -L\end{bmatrix}\begin{bmatrix}v_x^b\\ v_y^b\\ \omega^b\end{bmatrix}$$

System Dynamics

System Model : Two-dimensional Plane Motion

Motion Equations (with Euler-Lagrange M.E.)

$$\begin{aligned} M_0\ddot{x} &= F_x\\ M_0\ddot{y} &= F_y\\ J_c\ddot{\theta}+D_a\dot{\theta} &= F_\theta \end{aligned}$$

M_0 : The whole mass
J_c : Moment of inertia around the center
D_a : Viscosity friction coefficient

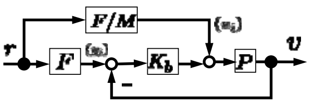
Identification (regression model)

$$\begin{bmatrix}\ddot{x} & 0 & 0\\ 0 & \ddot{\theta} & \dot{\theta}\end{bmatrix}\begin{bmatrix}M_0\\ J_c\\ D_a\end{bmatrix} = \begin{bmatrix}F_x\\ F_\theta\end{bmatrix}$$

Problem
Measure with overhead camera is inexact

Solution
install Encoders

Velocity 2DOF control



$P(s) =$ (Plant) × (Filter)

Analysis and Design
Change to the left block diagram
Transfer function G from r to v

$$G = F + FS(P/M - 1)$$

If $P \approx M$ or $s(\omega) \to 0$,
then Feedforward is effective

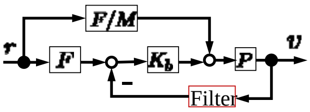
Design of Feedforward F

- Feedforward controller F : Stable
- F/M : Stable
  - u_i : ideal input
  - y_i : ideal output
- Real Transfer function G'

$$G' = \frac{G}{\text{Filter}}$$

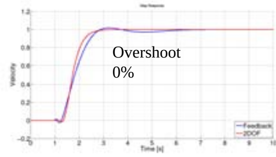
[Theorem]
Suppose P stable.
$F = PR$ and $R(s)$ is stable
$\Leftrightarrow$ system is stable.

Velocity 2DOF control (1)



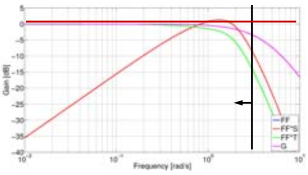
$$F = P(s)\frac{0.447s+0.813}{0.055s+1}$$

Time response

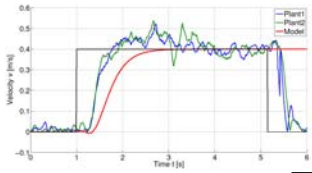


Gain Plot

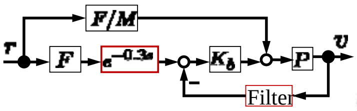
Band width
3[rad/s]



Real Response(Movie)

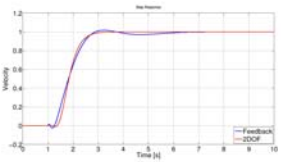


Velocity 2DOF control (2)



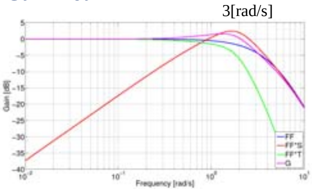
$$F = \frac{1}{0.008s^3+0.12s^2+0.6s+1}$$

Time response

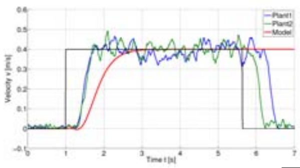


Gain Plot

Band width
3[rad/s]



Real Response



Adaptive Cruise Control

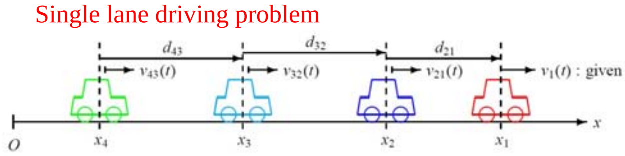
Adaptive Cruise Control

a system which controls a relative distance and velocity from a forward vehicle and is fascinated with the congestion aviodance

purpose
to realize a autonomous-distributed system which achieves driving such as minimizing a crash risk. (crash avoidance)

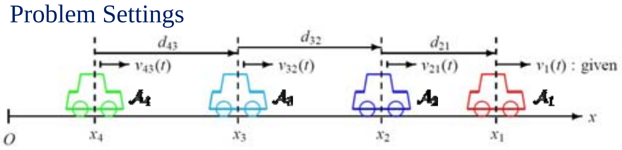
Problem Settings

Single lane driving problem



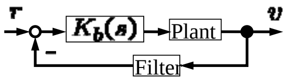
Adaptive Cruise Control

Problem Settings



Follower agent A_i gets
distance and velocity
of the agent A_{i-1}

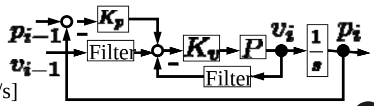
A_1 (leader) controller



Simulation parameters

Initial distance : 0.3 [m]
Initial velocity : 0 [m/s]
Finish distance : 0.4 [m]
Leader velocity : v_1 =0.2 [m/s]

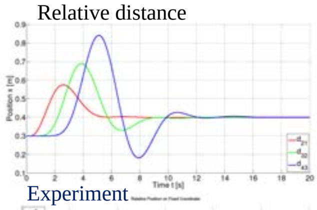
Other (follower i) controller



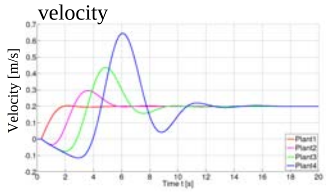
Simple Controller

Simulation

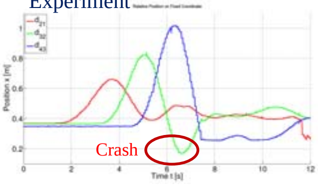
Relative distance

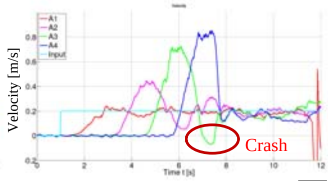


velocity

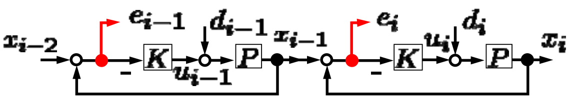


Experiment





String Stability



Assumption

- LTI SISO plant/controller
- Each loop has relative degree
- Homogeneous loop

System

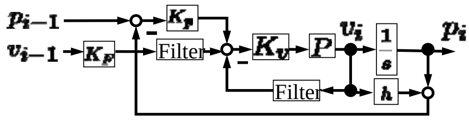
- LTI MIMO plant/controller
- Each loop has relative degree
- Homogeneous loop

Sufficient Condition

$$\left\| \frac{e_i(s)}{e_{i-1}(s)} \right\|_\infty < 1$$

The perturbation doesn't propagate to following vehicles

String Stability (including Time Headway)



Transfer function from 2input v_{i-1}, p_{i-1} to v_i

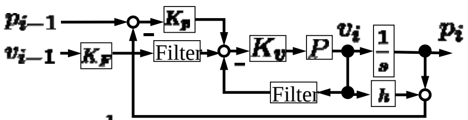
$$v_i = \frac{1}{(1+PK_vF)/PK_v + K_d/s}(FK_Fv_{i-1} + K_dx_{i-1})$$

simplify

$$x_i = (T_p + (hs+1)S_pT_vFK_F)\frac{x_{i-1}}{hs+1}$$

$$v_i = (T_p + (hs+1)S_pT_vFK_F)v_{i-1}$$

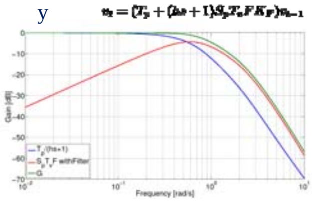
String Stability (including Time Headway)



Analysis $K_F = \frac{1}{1.6s+1}$

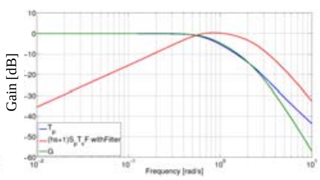
Velocit
y

$$v_i = (T_p + (hs+1)S_pT_vFK_F)v_{i-1}$$



Position

$$x_i = (T_p + (hs+1)S_pT_vFK_F)\frac{x_{i-1}}{hs+1}$$



Controller with String Stability

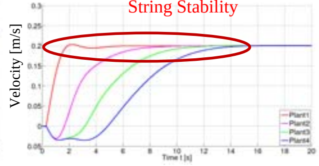
Simulation

Relative distance

Same distance

velocity

Achieve String Stability



Experiment