Passivity-based Visual Motion Observer with Target Object Motion Model

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Background

Visual Perceptions

"The out there of conscious experience isn't really out there at all; it's inside the head. Our visual perceptions are a simulation of the real world." [1]

There are some ways to know the position relation in 3 dimension from information of two dimension. The famous way is stereo vision which uses two cameras. Though, not all animals use this way.

Visual processing of the brain includes high-level functions of building and manipulating an internal model of the outside world.

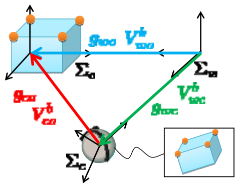
→A way is Visual Motion Observer

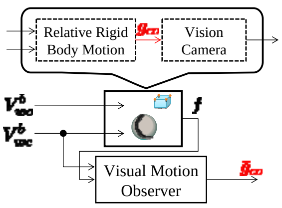
[1] J.Gray, *Consciousness: Creeping Up on the Hand Problem*, Oxford University Press, 2004

Introduction

Visual Motion Observer (VMO)

It estimates object's relative positions and orientations in 3 dimension from the image information.





Previous Work

It assumes the object has zero velocity

→There are the tracking errors when the object has a velocity

Object of this Work

Assuming the case of the object having a velocity by assuming the object's motion pattern

Outline

- Basis on Visual Motion Observer
- VMO with Target Object Motion Model
  - Target Object Motion Model
  - Constant Velocity
  - Generalized Motion
- Simulation
- Conclusion and Feature Works

Relative Pose and Rigid Body Motion

Pose and Body Velocity of Vision Camera

$$g_{wc} = (p_{wc}, e^{\hat{\xi}\theta_{wc}}) \quad V^b_{wc} = \begin{bmatrix} v^b_{wc} \\ \omega^b_{wc} \end{bmatrix} \quad \hat{V}^b_{wc} = g^{-1}_{wc}\dot{g}_{wc}$$

Pose and Body Velocity of Object

$$g_{wo} = (p_{wo}, e^{\hat{\xi}\theta_{wo}}) \quad V^b_{wo} = \begin{bmatrix} v^b_{wo} \\ \omega^b_{wo} \end{bmatrix} \quad \hat{V}^b_{wo} = g^{-1}_{wo}\dot{g}_{wo}$$

Pose and Body Velocity of Object relative to Vision Camera

$$g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}}) \quad V^b_{co} = \begin{bmatrix} v^b_{co} \\ \omega^b_{co} \end{bmatrix} \quad \hat{V}^b_{co} = g^{-1}_{co}\dot{g}_{co}$$

$$g = \begin{bmatrix} e^{\hat{\xi}\theta} & p \\ 0 & 1 \end{bmatrix}$$

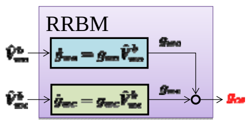
Position: $p \in R^3$

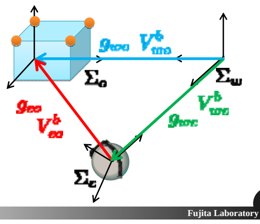
Orientation: $R = e^{\hat{\xi}\theta} \in SO(3)$

$$g_{co} = g^{-1}_{wc} g_{wo}$$

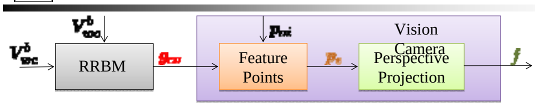
Relative Rigid Body Motion (RRBM)

$$\dot{g}_{co} = -\hat{V}^b_{wc} g_{co} + g_{co}\hat{V}^b_{wo}$$





Vision Camera



Object's Feature Points

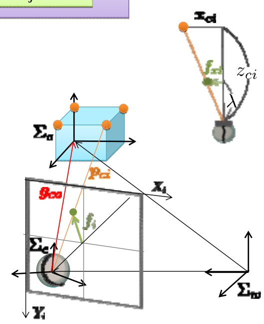
$$p_{oi} \ (i = 1, \cdots, m), m \geq 4$$

Feature Points

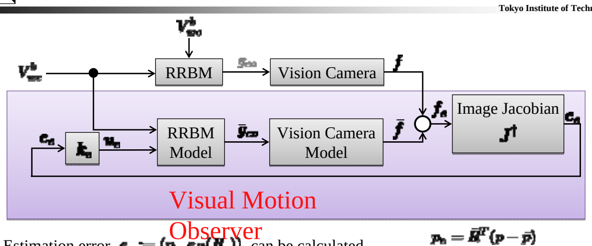
$$\begin{bmatrix} p_{ci} \\ 1 \end{bmatrix} = g_{co}\begin{bmatrix} p_{oi} \\ 1 \end{bmatrix}, \ p_c = \begin{bmatrix} p_{c1} \\ \vdots \\ p_{cm} \end{bmatrix}$$

Perspective Projection

$$f_i = \frac{\lambda}{z_{ci}}\begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}, \ f = \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix}$$

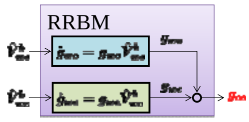


Visual Motion Observer



Estimation error $e_e := (p_{ec}, e_R(R_{ec}))$ can be calculated by product of Image Jacobian $J^{\dagger}$ and image information f_e

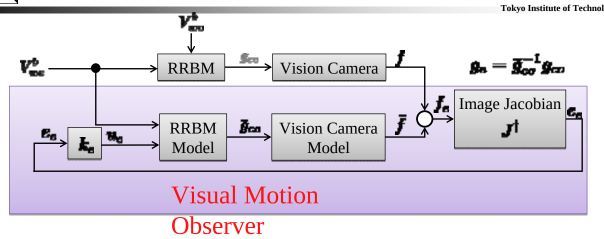
$$p_e = \bar{R}^T(p - \bar{p})$$
$$e_R(R_e) = \frac{1}{2}(R_e - R_e^T)^{\vee}$$
$$(R_e = \bar{R}^T R)$$



Formulated estimation mechanism

$$\dot{\bar{g}}_{co} = -\hat{V}^b_{wc}\bar{g}_{co} - \bar{g}_{co}\hat{u}_e$$
$$e_e = J^{\dagger}(\bar{g}_{co})f_e$$
$$u_e = -k_e e_e$$

Visual Motion Observer



$$g_e = \bar{g}_{co}^{-1} g_{co}$$

Theorem 1

If $V^b_{wo} = 0$,the equilibrium point $e_e = 0$ for the closed-loop system (1) and (2) is asymptotically stable.

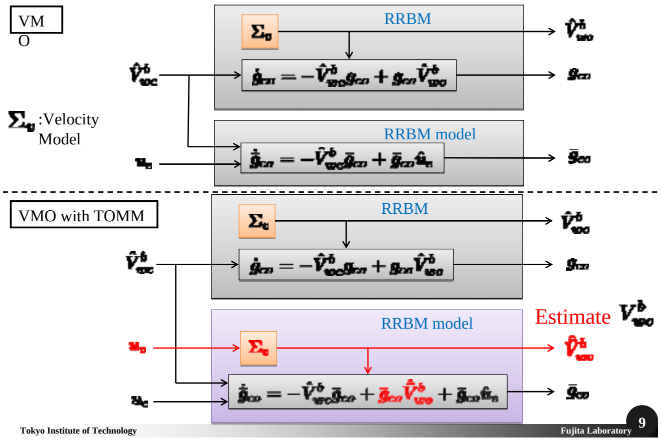
$$\dot{g}_e = \hat{u}_e + g_e\hat{V}^b_{wo} \quad (1)$$
$$u_e = -k_e e_e \quad (2)$$

→The visual motion observer leads the estimate $\bar{g}_{co}$ to the actual g_{co} .

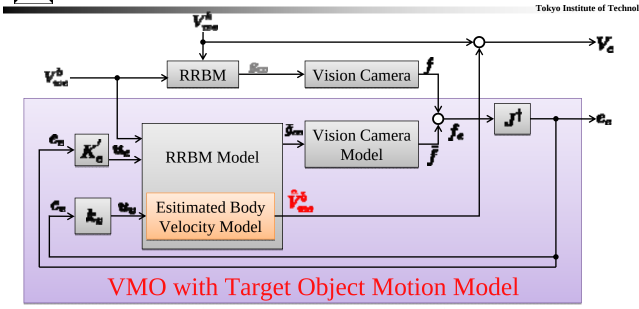
Though this framework assumes V^b_{wo} is a completely unknown disturbance so there is estimated error when the object move.

→Consideration of the object's motion pattern to cancel estimated error.

Target Object Motion Model (TOMM)



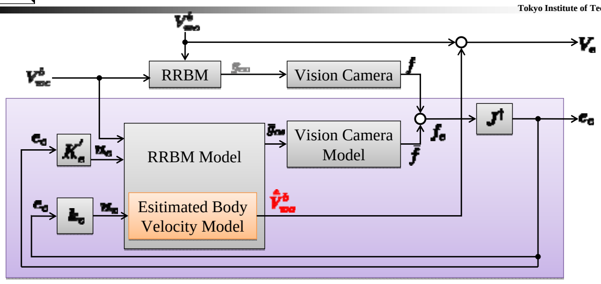
VMO with Target Object Motion Model



Formulated estimation mechanism

$$\dot{\hat{V}}^b_{wo} = -u_v$$
$$\dot{\bar{g}}_{co} = -\hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co}(\hat{V}_{wo} - \hat{u}_e)$$
$$e_e = J^{\dagger}(\bar{g})f_e$$
$$u_v = -k_v e_v, \quad u_e = -K'_e e_e$$

Assume Constant Velocity



Theorem 2

Consider the estimation error system (3) for a constant velocity model (4) with the input

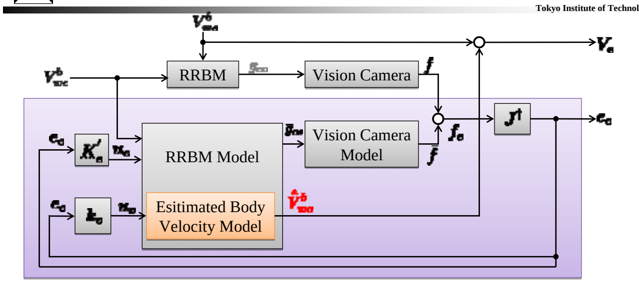
$$u_v = -k_v e_v, \quad u_e = -K'_e e_e, \quad K'_e := \begin{bmatrix} \hat{\theta}_{wo} - k_e I_3 & 0 \\ 0 & k_e I_3 \end{bmatrix}.$$

Then, we have $\lim_{t \to \infty}(V_e, e_e) = 0$.

$$\dot{V}_e = u_v \quad (3a)$$
$$\dot{g}_e = \hat{u}_e g_e - \hat{V}^b_{wo} g_e + g_e \hat{V}^b_{wo} \quad (3b)$$

$$\dot{V}^b_{wo} = 0$$
$$\dot{g}_{co} = -\hat{V}^b_{wc} g_{co} + g_{co}\hat{V}^b_{wo} \quad (4)$$

Assume Constant Velocity

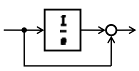


Formulated estimation mechanism

$$\dot{\hat{V}}^b_{wo} = -u_v$$
$$\dot{\bar{g}}_{co} = -\hat{V}^b_{wc}\bar{g}_{co} + \bar{g}_{co}(\hat{V}_{wo} - \hat{u}_e)$$
$$e_e = J^{\dagger}(\bar{g})f_e$$
$$u_v = -k_v e_v, \quad u_e = -K'_e e_e \quad (5)$$

View $u'_e = \bar{V}^b_{wo} - u_e$ as new input of the observer

$$u'_e = -\int u_v - u_e$$
$$= k_v \int e_e + K'_e e_e$$



The same structure as the proportional-integral contoroller

Generalized Motion Model

Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx)$$

Periodic functions can be represented by the sum of a set of oscillating functions

Consider the case the target object body velocity (approximately) is given

$$V_{wo} = \begin{bmatrix} v_{wo} \\ w_{wo} \end{bmatrix} = \begin{bmatrix} c_v + \sum_{i=1}^{n} a_{v,i} \sin w_i t + b_{v,i} \cos w_i t \quad (=: z_i) \\ c_w + \sum_{i=1}^{n} a_{w,i} \sin w_i t + b_{w,i} \cos w_i t \end{bmatrix} \quad (6)$$

$$(a_{w,i}, a_{v,i}, b_{w,i}, b_{v,i}, c_w, c_v \in \mathbb{R}^3)$$

$$v_{wo}(t) = \sin(at)$$

$$\begin{cases} \frac{d}{dt}\begin{bmatrix} v_{wo} \\ \dot{v}_{wo} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a^2 & 0 \end{bmatrix}\begin{bmatrix} v_{wo} \\ \dot{v}_{wo} \end{bmatrix} & (\ddot{v}_{wo}(t) = -a^2 \sin(at)) \\ \frac{d}{dt}\begin{bmatrix} \bar{v}_{wo} \\ \dot{\bar{v}}_{wo} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a^2 & 0 \end{bmatrix}\begin{bmatrix} \bar{v}_{wo} \\ \dot{\bar{v}}_{wo} \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_{vv} \end{cases}$$

Generalized Motion Model

The time evolution of w_{wo} is represented by the linear time invariant system

$$\begin{cases} \dot{x}_w = A x_w \\ w_{wo} = C x_w \end{cases} \quad A := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & I_{3n} \\ 0 & -\mathrm{diag}(w_1^2, \cdots, w_n^2) \otimes I_3 & 0 \end{bmatrix}, \; C := [\mathbf{1}_{n+1}^T \otimes I_3 \;\; 0]$$

$$x_w = \begin{bmatrix} z_0 \\ z \\ \dot{z} \end{bmatrix} = \begin{bmatrix} c_w \\ (z_1, \cdots, z_n)^T \\ (\dot{z}_1, \cdots, \dot{z}_n)^T \end{bmatrix}, \; \mathbf{1}_n = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^3$$

Model is

$$\begin{cases} \dot{\bar{x}}_w = A \bar{x}_w - B u_{vw}, \; B := C^T \\ \bar{w}_{wo} = C \bar{x}_w \end{cases}$$

The estimation error system of w_{wo}

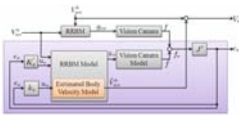
$$\begin{cases} \dot{x}_{ew} = A x_{ew} + B u_{vw} \\ w_e = C x_{ew} \end{cases} \quad (7) \qquad (x_{ew} = x_w - \bar{x}_w)$$

$$\begin{cases} \dot{x}_{ev} = A x_{ev} + B u_{vv} \\ v_e = C x_{ev} \end{cases} \quad (8) \qquad (x_{ev} = x_v - \bar{x}_v)$$

Generalized Motion Model

Theorem 3

Consider the estimation error system (3b) , (7) and (8) for a velocity model (6) with the input (5).
Then, we have $\lim_{t\to\infty}(V_e, e_e) = 0$.



$$V_{wo} = \begin{bmatrix} v_{wo} \\ w_{wo} \end{bmatrix} = \begin{bmatrix} c_v + \sum_{i=1}^{n} a_{v,i} \sin w_i t + b_{v,i} \cos w_i t \\ c_w + \sum_{i=1}^{n} a_{w,i} \sin w_i t + b_{w,i} \cos w_i t \end{bmatrix} \quad (6) \qquad u_v = -k_v e_v, \; u_e = -K_e' e_e \quad (5)$$

$$\dot{g}_{eo} = \hat{g}_{eo} g_{eo} - \hat{\bar{V}}^b_{wo} g_{eo} + g_{eo} \hat{V}^b_{wo} \quad (3b)$$

The estimation error system of w_{wo}

$$\begin{cases} \dot{x}_{ew} = A x_{ew} + B u_{vw} \\ w_e = C x_{ew} \end{cases} \quad (7)$$

The estimation error system of v_{wo}

$$\begin{cases} \dot{x}_{ev} = A x_{ev} + B u_{vv} \\ v_e = C x_{ev} \end{cases} \quad (8)$$

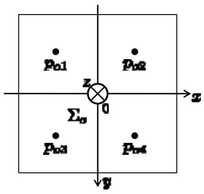
Simulation

Input :

1. A sine wave
2. Sum of sine waves
3. White noise + Low pass filter
4. White noise + Low pass filter (other situation)

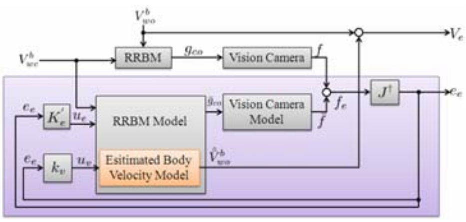
Simulation: Setting

Feature Points



$$p_{o1} = \begin{bmatrix} -0.066 \\ -0.066 \\ 0 \end{bmatrix} [\mathrm{m}] \qquad p_{o2} = \begin{bmatrix} 0.066 \\ -0.066 \\ 0 \end{bmatrix} [\mathrm{m}]$$

$$p_{o3} = \begin{bmatrix} -0.066 \\ 0.066 \\ 0 \end{bmatrix} [\mathrm{m}] \qquad p_{o4} = \begin{bmatrix} 0.066 \\ 0.066 \\ 0 \end{bmatrix} [\mathrm{m}]$$



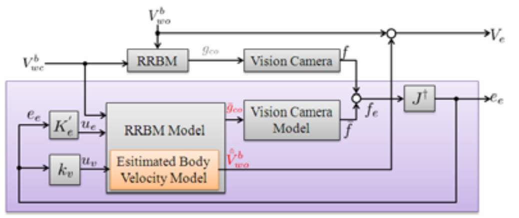
Simulation: Setting

Initial value

$$p_{co} = \begin{bmatrix} 0 \\ 0 \\ 2.25 \end{bmatrix} [\mathrm{m}] \qquad R_{co} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Initial value of estimation

$$\bar{p}_{co} = \begin{bmatrix} 0.3 \\ 0.3 \\ 2 \end{bmatrix} [\mathrm{m}] \qquad \bar{R}_{co} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

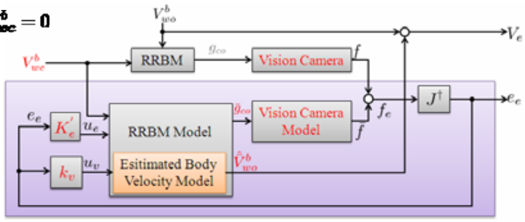


Simulation: Setting

Rest of setting

Gain $k_e = 1,\ k_v = 1$ — Focal Length $\lambda = 0.0033$ [m] — Sampling Time $T = 0.01$ [s]

Body Velocity of Camera $V_{wc}^b = 0$



Examine some conbinations of V_{wo} and Estimated Body Velocity Model

Afterward, consider that the object has only x-axis's linear velocity

Estimated Body Velocity Model

$v_{wo}(t) = \sin(at)$

$$\frac{d}{dt}\begin{bmatrix} v_{wo} \\ \dot{v}_{wo} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a^2 & 0 \end{bmatrix}\begin{bmatrix} v_{wo} \\ \dot{v}_{wo} \end{bmatrix} \quad (\ddot{v}_{wo}(t) = -a^2 \sin(at))$$

$$\frac{d}{dt}\begin{bmatrix} \bar{v}_{wo} \\ \dot{\bar{v}}_{wo} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a^2 & 0 \end{bmatrix}\begin{bmatrix} \bar{v}_{wo} \\ \dot{\bar{v}}_{wo} \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_{wo}$$

$$\frac{d}{dt}\begin{bmatrix} v_e \\ \dot{v}_e \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a^2 & 0 \end{bmatrix}\begin{bmatrix} v_e \\ \dot{v}_e \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_{wo} \quad (v_e = v_{wo} - \bar{v}_{wo})$$

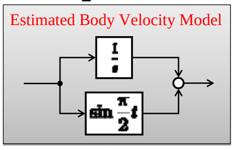
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} v_e \\ \dot{v}_e \end{bmatrix}$$

$$e_e \rightarrow \boxed{\begin{matrix} \dot{x} = Ax + Bu \\ y = Cx \end{matrix}} \rightarrow \bar{V}_{wo} \quad := \quad e_e \rightarrow \boxed{\sin at} \rightarrow \bar{V}_{wo}$$

Simulation (1): A sine wave

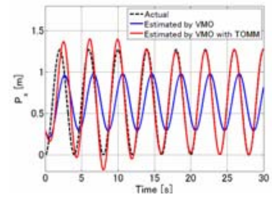
$v_{wo_x} = \sin \frac{\pi}{2} t$

Estimated Body Velocity Model

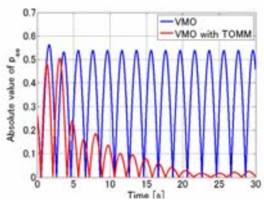


$p_{ee} = \bar{R}^T(p - \bar{p}),\ R_{ee} = \bar{R}^T R$

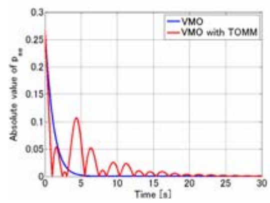
Position (x)



Estimation error of position (x)



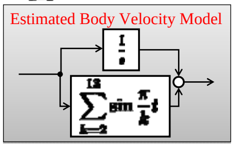
Estimation error of position (y)



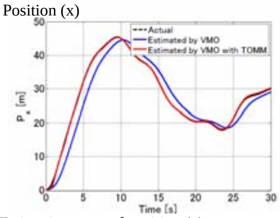
Simulation (2): Sum of sine waves

$v_{wo_x} = \sum_{k=2}^{13} \sin \frac{\pi}{k} t$

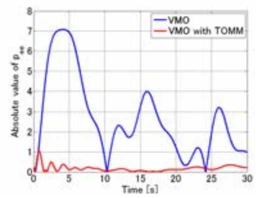
Estimated Body Velocity Model



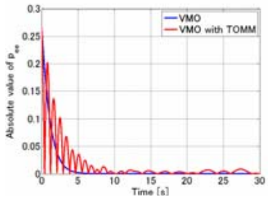
Position (x)



Estimation error of position (x)



Estimation error of position (y)

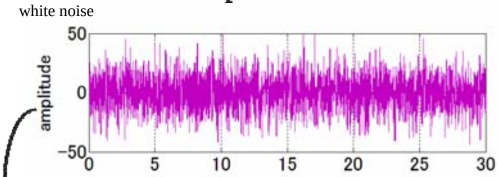


Simulation (3): White noise + Low Pass Filter

Make common signal by taking out a certain frequency region from white noise

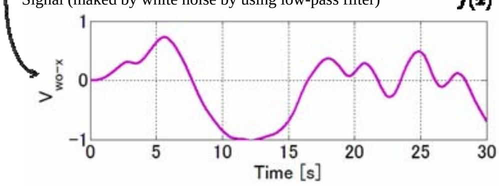
under $\frac{\pi}{2}$[rad/s]

white noise



low-pass filter $w_c = \frac{\pi}{2}$[rad/s]

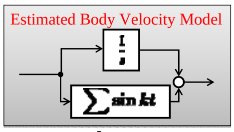
Signal (maked by white noise by using low-pass filter) $f(x)$



Simulation (3): White noise + Low Pass Filter

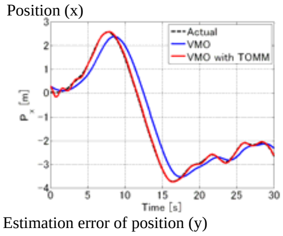
Dividing equally the frequency domain into twelve

Estimated Body Velocity Model

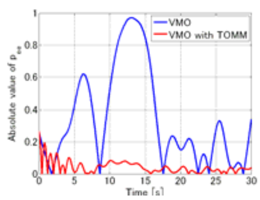


$k = \frac{\pi}{2} \cdot \frac{i}{12}\ (i = 1, \cdots, 12)$

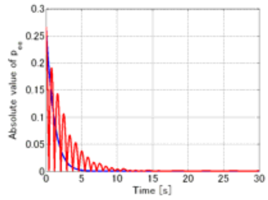
Position (x)



Estimation error of position (x)

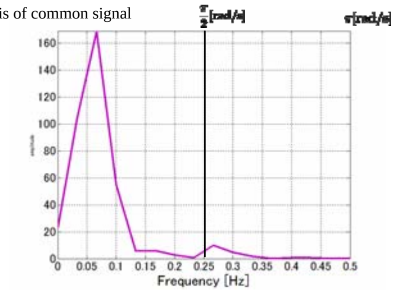


Estimation error of position (y)



Simulation(3): Frequency Analysis

Frequency Analysis of common signal



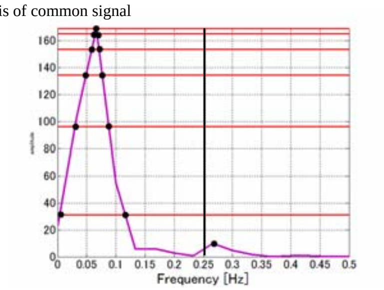
This signal has most frequency components from 0 [Hz] to 0.12 [Hz] and also a small peak in 0.265 [Hz]

(0.12[Hz]=0.754[rad/s],0.265[Hz]=1.67[rad/s])

→There is a distribution so we should make Estimated Body Velocity Model by considering the distribution

Simulation(3): Frequency Analysis

Frequency Analysis of common signal



This signal has most frequency components from 0 [Hz] to 0.12 [Hz] and also a small peak in 0.265 [Hz]

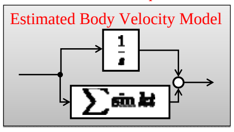
(0.12[Hz]=0.754[rad/s],0.265[Hz]=1.67[rad/s])

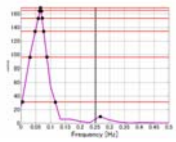
→There is a distribution so we should make Estimated Body Velocity Model by considering the distribution

Conduct simulation by using ● frequencies once again

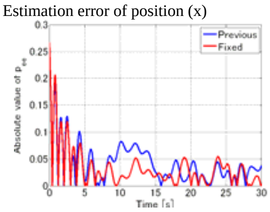
Simulation (3): White noise + Low Pass Filter

Consider the distribution of frequencies

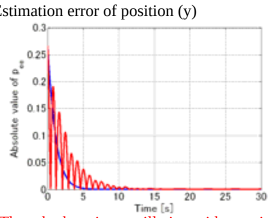




Estimation error of position (x)

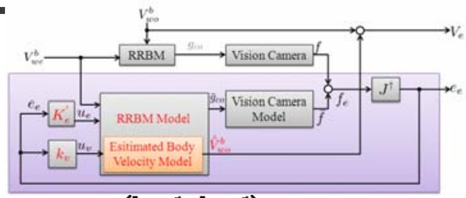


Estimation error of position (y)

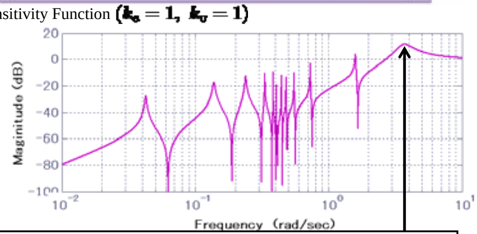


Though, there is a oscillation with a period about 1.5[s] ≒ 4[rad/s]

Analysis of Sensitivity Function



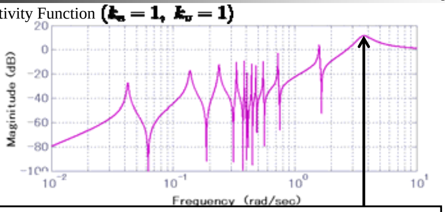
Sensitivity Function $(k_e = 1,\ k_v = 1)$



There is a peak in 4[rad/s] and this is the same as the oscillation period

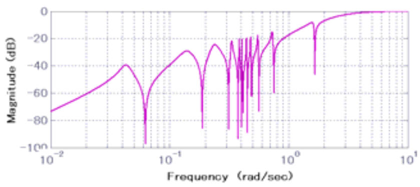
Analysis of Sensitivity Function

Sensitivity Function $(k_e = 1,\ k_v = 1)$



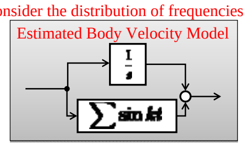
There is a peak in 4[rad/s] and this is the same as the oscillation period

Sensitivity Function $(k_e = 4, k_v = 0.5)$

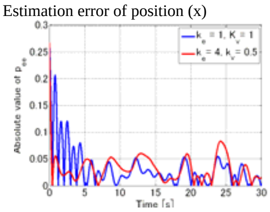


Simulation (3): White noise + Low Pass Filter

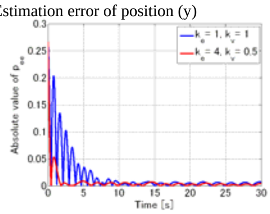
Consider the distribution of frequencies and adjust the gains



Estimation error of position (x)



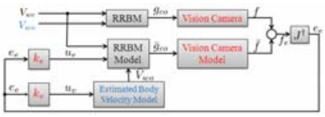
Estimation error of position (y)



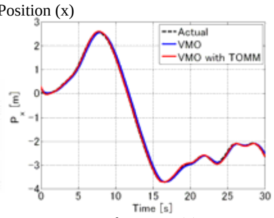
→Attenuate the oscillation by considering the sensitivity function

Comparison

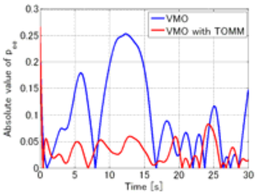
VMO $(k_e = 4)$
VMO with TOMM $(k_e = 4,\ k_v = 0.5)$



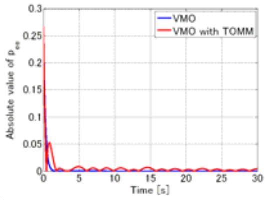
Position (x)



Estimation error of position (x)



Estimation error of position (y)



Other Situation

Initial value

$$p_{co} = \begin{bmatrix} 0 \\ 0 \\ 2.25 \end{bmatrix} \text{[m]}$$

$$R_{co} = e^{\hat{\xi}\theta} \ : \xi = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ \theta = \frac{\pi}{7}$$

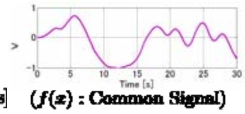
Initial value of estimation

$$\bar{p}_{co} = \begin{bmatrix} 0.3 \\ 0.3 \\ 2 \end{bmatrix} \text{[m]}$$

$$\bar{R}_{co} = e^{\hat{\xi}\theta} \ : \xi = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ \theta = \frac{\pi}{5}$$

Body Velocity

$$V_{wo}^b = \begin{bmatrix} v_{wox} \\ v_{woy} \\ v_{woz} \\ \omega_{wox} \\ \omega_{woy} \\ \omega_{woz} \end{bmatrix} = \begin{bmatrix} 0.5 \\ f(x)+0.3 \\ 0 \\ 0.3 \\ 0 \\ f(x) \end{bmatrix} \text{[m/s]}$$



$(f(x)$: Common Signal)

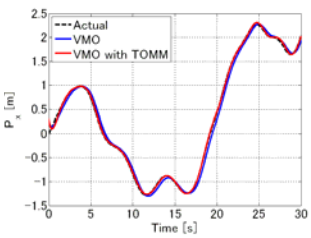
Gain

$k_e = 4, k_v = 0.5$

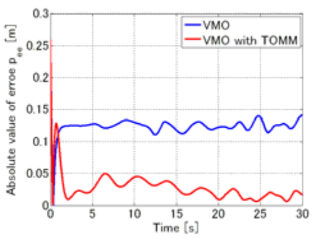
Simulation (4)

Position (x)

Position (x)



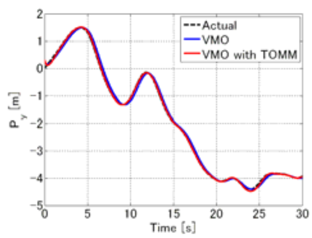
Estimation error (x)



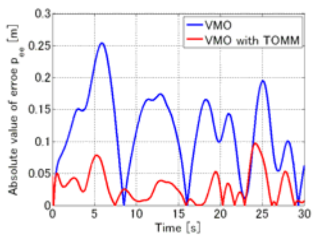
Simulation (4)

Position (y)

Position (y)



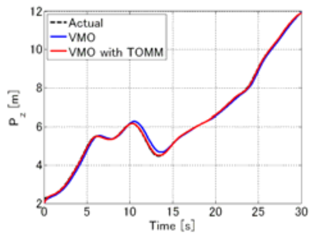
Estimation error (y)



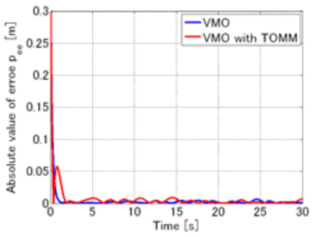
Simulation (4)

Position (z)

Position (z)



Estimation error (z)

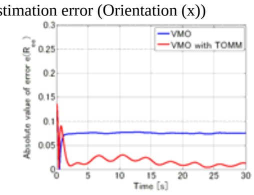


$$p_{ee} = \bar{R}^T (p - \bar{p})$$

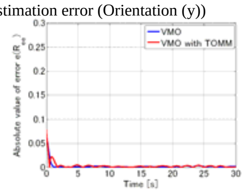
Simulation (4)

Orientation

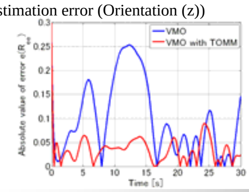
Estimation error (Orientation (x))



Estimation error (Orientation (y))



Estimation error (Orientation (z))



Conclusion and Future Works

Conclusion

Target Object Motion Model

・Assuming constant velocity, the Estimated Body Velocity Model is the same structure as the proportional-integral controller

・Assuming generalized motion, we can use the Fourier series as the Estimated Body Velocity by analyzing the signal in frequency domain

・Verifications of value of VMO with TOMM by simulation

Future Works

・Experiment
・Consideration of automatically analyzing the component of frequency domain
・Not only the estimation error of pose information but also velocity information

References

[1] J.Gray, *Consciouness: Creeping Up on the Hand Problem*, Oxford University Press, 2004

[2] T. Hatanaka and M. Fujita "Passivity-based Visual Motion Observer Integrating Internal Representation of 3D Target Object Motion," Proc. of the 2012 American Control Conference, Montreal, Canada, 2012.

[3] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE Transactions on Control Systems Technology*, Vol. 15, No.1, pp. 40-52, 2007.