Visual Feedback Attitude Control with Target Object Motion Model

Namba Yuto
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Introduction

Visual Feedback Control

A control strategy that uses computer vision data to control the motion of robots

Previous Works

Visual Feedback Tracking

Visual Motion Observer and Visual Feedback Control [1]

- The object is fixed
- Make the camera desired pose

Visual Motion Observer with Target Object Motion Models [2]

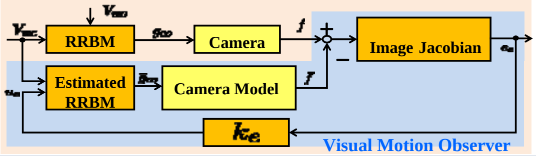
- The object is moving
- not consider the control

Consider the following case

Both the camera and object are moving

Make the camera desired pose

Visual Motion Observer [1,2]



Theorem

If $V_{wo}^b = 0$, then the equilibrium point $e_e = 0$ for the closed-loop system (A1) and (1) is asymptotically stable.

Visual Motion Observer (VMO)

negative feedback

$$u_e = k_e e_e \quad (1) \qquad k_e > 0$$

$$\begin{cases} u_e = k_e e_e \\ \bar{V}_{co}^b = -Ad_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e \\ e_e = J^{\dagger}(f - \bar{f}) \end{cases}$$

Passive $V_{wc}^b \to -e_r \quad e_r = \begin{bmatrix} p_{co} \\ e_R(R_{co}) \end{bmatrix}$

Not Passive $V_{wc}^b \to f \quad u_e = \begin{bmatrix} u_{ep} \\ u_{eR} \end{bmatrix}$

Passive $u_e \to -e_e \quad e_e = \begin{bmatrix} p_e \\ e_R(R_{ee}) \end{bmatrix}$

Outline

- Visual Motion Observer
- **Visual Feedback Control [1]**
- Visual Motion Observer with Target Object Motion Model [2]
- Visual Feedback Attitude Control with Target Object Motion Model

Control Error System [1]

Goal

$$g_{co} \to g_d \quad (\bar{g}_{co} \to g_{co} \quad + \quad \bar{g}_{co} \to g_d)$$

g_d : Constant Desired Pose

Control Error

$$g_{ec} = g_d^{-1} \bar{g}_{co}$$

available

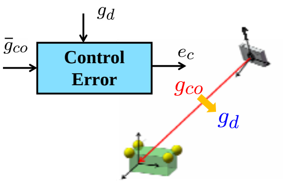
$$g_{ec} = I_4 \Longleftrightarrow \bar{g}_{co} = g_d$$

Control Error Vector

$$e_c = \begin{bmatrix} p_{ec} \\ e_R(R_{ec}) \end{bmatrix}$$

Control Error System

$$V_{ec}^b = -Ad_{(\bar{g}_{ec}^{-1})} Ad_{(g_d^{-1})} V_{wc}^b + u_e$$



$$\dot{g}_{ec} = \dot{g}_d^{-1} \bar{g}_{co} + g_d^{-1} \dot{\bar{g}}_{co}$$
$$= g_d^{-1} \dot{\bar{g}}_{co} \qquad (\because \dot{g}_d = 0)$$
$$= g_d^{-1}(-\hat{V}_{wc}^b \bar{g}_{co} + \bar{g}_{co} \hat{u}_e)$$

$$\hat{V}_{ec}^b = g_{ec}^{-1} \dot{g}_{ec} = \bar{g}_{co}^{-1} g_d \left(g_d^{-1}(-\hat{V}_{wc}^b \bar{g}_{co} + \bar{g}_{co} \hat{u}_e) \right)$$
$$= -\bar{g}_{co}^{-1} \hat{V}_{wc}^b \bar{g}_{co} + \hat{u}_e = \hat{\bar{V}}_{co}^b$$

: the same as EsRRBM

$$V_{ec}^b = -Ad_{(\bar{g}_{co}^{-1})} V_{wc}^b + u_e = -Ad_{(g_{ec}^{-1})} Ad_{(g_d^{-1})} V_{wc}^b + u_e$$

Total Error System [1]

Visual Feedback System

$$\begin{bmatrix} V_{ec}^b \\ V_{ee}^b \end{bmatrix} = \begin{bmatrix} -Ad_{(g_{ec}^{-1})} & I_6 \\ 0 & -Ad_{(g_{ee}^{-1})} \end{bmatrix} u_{ce} + \begin{bmatrix} 0 \\ I_6 \end{bmatrix} V_{wo}^b \quad (2)$$

Control Error System

$$V_{ec}^b = -Ad_{(\bar{g}_{ec}^{-1})} Ad_{(g_d^{-1})} V_{wc}^b + u_e$$

\+

Estimation Error System

$$V_{ee}^b = -Ad_{(g_{ee}^{-1})} u_e + V_{wo}^b$$

(Goal) $e = 0 \Longrightarrow g_{co} = \bar{g}_{co} = g_d$ $(e_c = 0)$ $(e_e = 0)$ therefore, $e \to 0 \Longrightarrow g_{co} \to g_d$

Passivity

If $V_{wo}^b = 0$, then the Visual Feedback System (2) satisfies.

$$\int_0^T u_{ce}^T (\nu_{ce}) dt \geq -\beta_{ce}$$

where β_{ce} is a positive scalar.

Input

$$u_{ce} = \begin{bmatrix} Ad_{(g_d^{-1})} V_{wc}^b \\ u_e \end{bmatrix}$$

Error Vector

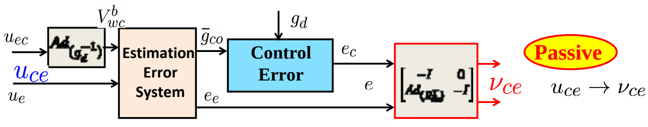
$$e = \begin{bmatrix} e_c \\ e_e \end{bmatrix}$$

Output

$$\nu_{ce} = \underbrace{\begin{bmatrix} -I & 0 \\ Ad_{(R_{ec}^T)} & -I \end{bmatrix}}_{N_{ce}} e$$

Storage Function

$$U_{ce} = \frac{1}{2}\|p_{ec}\|^2 + \phi(e^{\hat{\xi}\theta_{ec}}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee})$$



Visual Feedback Pose Control Law

$$u_{ce} = -\begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix} \begin{bmatrix} -I & 0 \\ Ad_{(R_{ec})} & -I \end{bmatrix} \begin{bmatrix} e_c \\ e_e \end{bmatrix} \quad (3)$$

$K_{ce} > 0$ $\quad \nu_{ce}$

based on the passivity property

$u_{ce} = -K_{ce}\nu_{ce}$
$= -K_{ce}(N_{ce}e)$

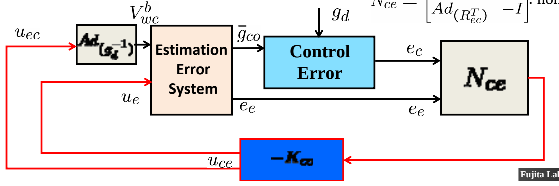
Theorem

If $V_{wo}^b = 0$, then the equilibrium point $e = 0$ for the closed-loop system (2) and (3) is asymptotically stable.

$U_{ce} = \frac{1}{2}\|p_{ec}\|^2 + \phi(R_{ec}) + \frac{1}{2}\|p_{ee}\|^2 + \phi(R_{ee})$ $\quad K_{ce} = \begin{bmatrix} K_c & 0 \\ 0 & K_e \end{bmatrix}$: positive definite

$\dot{U}_{ce} = u_{ce}^T \nu_{ce} = -e^T N_{ce}^T K_{ce} N_{ce} e < 0$

$N_{ce} = \begin{bmatrix} I & 0 \\ Ad_{(R_{ec}^T)} & -I \end{bmatrix}$: nonsingular



Outline

- Visual Motion Observer
- Visual Feedback Control [1]
- **Visual Motion Observer with Target Object Motion Model [2]**
- Visual Feedback Attitude Control with Target Object Motion Model

VMO with Target Object Motion Model[2]

TCST [1] $V_{wo}^b = 0$ ⇒ $\bar{V}_{wo}^b \to V_{wo}^b$

Target Object Motion

Actual Model

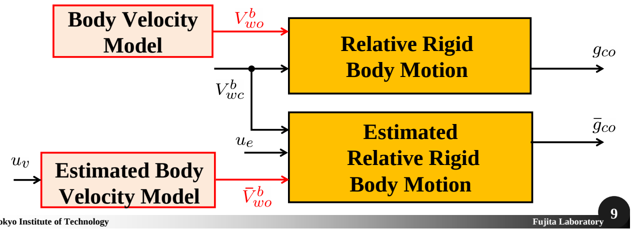
$V_{co}^b = -Ad_{(g_{co}^{-1})} V_{wc}^b + V_{wo}^b$

$\dot{V}_{wo}^b = 0$ the target object has
$(V_{wo}^b = c)$ a constant velocity

Estimated Model

$\bar{V}_{co}^b = -Ad_{(\bar{g}_{co}^{-1})} V_{wc}^b + \bar{V}_{wo}^b - u_e$

$\dot{\bar{V}}_{wo}^b = -u_v \quad u_e = \begin{bmatrix} u_{ep} \\ u_{eR} \end{bmatrix} \quad u_v = \begin{bmatrix} u_{vp} \\ u_{vR} \end{bmatrix}$



Estimation Error System[2]

Goal $\bar{g}_{co} \to g_{co}, \quad \bar{V}_{wo}^b \to V_{wo}^b$

Estimation Error

$g_e = \bar{g}_{co}^{-1} g_{co}$

$V_e^b = V_{wo}^b - \bar{V}_{wo}^b$

Estimation Error System

$V_{ee}^b = Ad_{(g_{ee}^{-1})} u_e - Ad_{(g_{ee}^{-1})} \bar{V}_{wo}^b + V_{wo}^b$

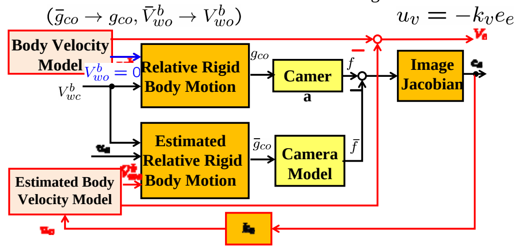
$\dot{V}_e^b = u_v = -k_v e_e$

design u_e so that $\lim_{t\to\infty}(e_e, V_e) = 0$.

$(\bar{g}_{co} \to g_{co}, \bar{V}_{wo}^b \to V_{wo}^b)$

negative feedback

$u_v = -k_v e_e$



Division into the Orientation Part [2]

We consider only the orientation part.

Orientation Part

$\dot{\omega}_e = -k_v e_R(R_{ee})$

$\dot{R}_{ee} = \hat{u}_{eR} R_{ee} - \hat{\bar{\omega}}_{wo} R_{ee} + R_{ee}\hat{\omega}_{wo}$

be independent of the position evolution

$\dot{\phi}(R_{ee}) = e_R^T(R_{ee})\omega_e + e_R^T(R_{ee})u_{eR}$

Storage Function

$U_R = \phi(R_{ee}) + S_\omega \quad S_\omega = \frac{1}{2k_v}\|\omega_e\|^2$ ⇒ $\dot{S}_\omega = \frac{1}{k_v}\omega_e^T \dot{\omega}_e = \frac{1}{k_v}\omega_e^T u_{vR}$

Inner loop

$u_{vR} = -k_v e_R(R_{ee}) \quad \dot{S}_\omega = -\omega_e^T e_R(R_{ee}) \quad \dot{U}_R = u_{eR}^T e_R(R_{ee})$

$= -e_R^T(R_{ee})\omega_e$ **Passive** $u_{eR} \to e_R(R_{ee})$

Negative feedback

$u_{eR} = -k_e e_R(R_{ee})$ ⇒ $(e_R(R_{ee}), \omega_e) \to 0$

LaSalle

Outline

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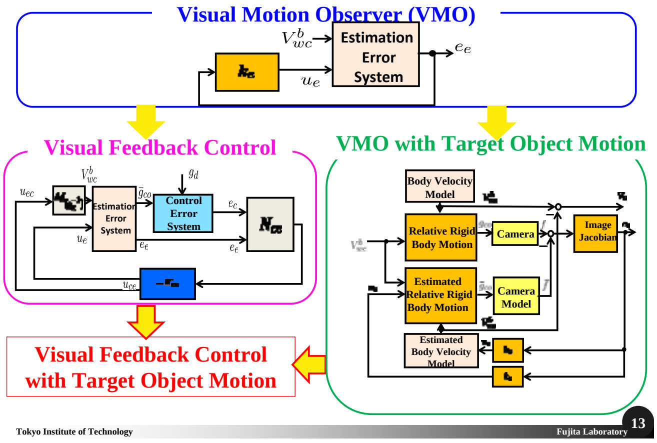
Visual Feedback Control with Target Object Motion



Total Error System with Velocity Error System

Assumption

$\omega_{wo} = \text{const}(\dot\omega_{wo} = 0)$

$v_{wo} = 0, \bar v_{wo} = 0$ (Future work $v_{wo} = \text{const}$ almost achieved)

Goal

$R_{ec} = I_3(\bar R_{co} = R_d = I_3),\quad R_{ee} = I_3(\bar R_{co} = R_{co}),\quad \bar\omega_{wo} = \omega_{wo}$

TCST [1] ACC [2]

Estimation Error System

$$\begin{bmatrix} v_{ee} \\ \omega_{ee}\end{bmatrix} = \begin{bmatrix} -R_{ee}^T & R_{ee}^T\hat{\bar p}_{ee} \\ 0 & -R_{ee}^T\end{bmatrix}\begin{bmatrix} u_{ep} \\ u_{eR} + \bar\omega_{wo}\end{bmatrix} + \begin{bmatrix} 0 \\ \omega_{wo}\end{bmatrix}$$

Control Error System (Orientation Part)

$\omega_{ec} = -R_{ec}^T\omega_{wc} + u_{eR} + \bar\omega_{wo}$

Target Object Motion Model

Target Velocity Estimation Error System (Orientation Part)

$\dot\omega_e = -u_{vR}$

Visual Feedback Control with Object Motion

Total Error System

$$\begin{bmatrix} \dot\omega_e \\ \omega_{ec} \\ v_{ee} \\ \omega_{ee}\end{bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & -R_{ec}^T & 0 & I \\ 0 & 0 & -R_{ee}^T & R_{ee}^T\hat{\bar p}_e \\ 0 & 0 & 0 & -R_e^T\end{bmatrix}\begin{bmatrix} u_{vR} \\ \omega_{wc} \\ u_{ep} \\ u_{eR} + \bar\omega_{wo}\end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_{wo}\end{bmatrix}$$ ignore

Input

$\omega_{wc},\quad u_{eR},\quad u_{vR}$

TCST [1]

$\omega_{wc} = k_c e_R(R_{ec})$

$u_{eR} = k_e\,(e_R(R_{ee}) - e_R(R_{ec}))$

ACC [2]

$u_{vR} = k_v e_R(R_{ee})$

$u_{eR} = k_e e_R(R_{ee})$

$u_{vR} =\quad \omega_{wc} =\quad u_{eR} =$

?

Decision of u_{vR}, ω_{wc}

$u_{vR} = ?$ → $u_{vR} = k_v e_R(R_{ee})$ the same as [2]

$\dot\phi(R_{ee}) = e_R^T(R_{ee})\omega_e - e_R^T(R_{ee})u_{eR}$

$S_\omega = \frac{1}{2k_v}\|\omega_e\|^2 \quad \dot S_\omega = \frac{1}{k_v}\omega_e^T\dot\omega_e = -\frac{1}{k_v}\omega_e^T u_{vR}$

$u_{vR} = k_v e_R(R_{ee})$ → $\dot\phi(R_{ee}) + \dot S_\omega = -e_R^T(R_{ee})u_{eR}$

$\omega_{wc} = ?$ → $\omega_{wc} = \omega'_{wc} + R_{ec}\bar\omega_{wo}$

new input

Control Error System

$\omega_{ec} = -R_{ec}^T\omega_{wc} + u_{eR} + \bar\omega_{wo}$

$\omega_{wc} = \omega'_{wc} + R_{ec}\bar\omega_{wo}$

→ $\omega_{ec} = -R_{ec}^T(\omega'_{wc} + R_{ec}\bar\omega_{wo}) + u_{eR} + \bar\omega_{wo}$

$= -R_{ec}^T\omega'_{wc} + u_{eR}$ the same as [1]

Validity of this approach is verified by passivity (explain later)

Interpretation of ω_{wc}

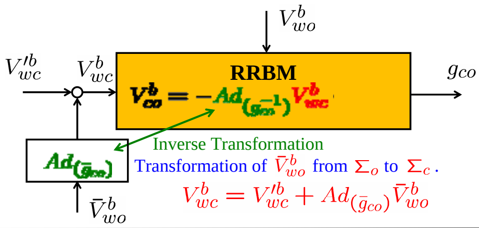
$\omega_{wc} = \omega'_{wc} + R_{ec}\bar\omega_{wo}$

$R_{ec} = \bar R_{co}(\because R_d = I)$

$R_{ec}\bar\omega_{wo} = \bar R_{co}\bar\omega_{wo}$

Transformation of ω_{wo} from Σ_o to Σ_c

Namely, $\bar R_{co}\bar\omega_{wo}$ cancels the target motion ,and ω'_{wc} is a new input for the desired attitude.



Transformation of $\bar V_{wo}^b$ from Σ_o to Σ_c.

$V_{wc}^b = V_{wc}'^b + Ad_{(\bar g_{co})}\bar V_{wo}^b$

Decision of u_{eR}, ω'_{wc}

$u_{eR} = ?$

$\omega'_{wc} = ?$

TCST [1]

$\omega_{wc} = k_c e_R(R_{ec})$

$u_{eR} = k_e\,(e_R(R_{ee}) - e_R(R_{ec}))$

ACC [2]

$u_{eR} = k_e e_R(R_{ee})$

Total Error System (Using $\omega_{wc} = \omega'_{wc} + R_{ec}\bar\omega_{wo}$.)

$$\begin{bmatrix} \dot\omega_e \\ \omega_{ec} \\ v_{ee} \\ \omega_{ee}\end{bmatrix} = \begin{bmatrix} -I_3 & 0 & 0 & 0 \\ 0 & -R_{ec}^T & 0 & I_3 \\ 0 & 0 & -R_{ee}^T & R_{ee}^T\hat{\bar p}_e \\ 0 & 0 & 0 & -R_{ee}^T\end{bmatrix}\begin{bmatrix} u_{vR} \\ \omega'_{wc} \\ u_{ep} \\ u_{eR}\end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ R_{ee}^T\hat{\bar p}_e\bar\omega_{wo} \\ \omega_{wo} - R_{ee}^T\bar\omega_{wo}\end{bmatrix} \quad (4)$$

can be eliminated by ACC approach

Orientation Part

$$\begin{bmatrix} \omega_{ec} \\ \omega_{ee}\end{bmatrix} = \begin{bmatrix} -R_{ec}^T & I \\ 0 & -R_{ee}^T\end{bmatrix}\begin{bmatrix} \omega'_{wc} \\ u_{eR}\end{bmatrix}$$

the same total estimation system as TCST

Passivity Approach

Input

$u_{ceR} = \begin{bmatrix} \omega'_{wc} \\ u_{eR}\end{bmatrix}$

Output

$\nu_{ce} = N_{ce}e \quad N_{ce} = \begin{bmatrix} -I & 0 \\ R_{ec}^T & -I\end{bmatrix} \quad e = \begin{bmatrix} e_R(R_{ec}) \\ e_R(R_{ee})\end{bmatrix}$

Passivity

Passivity

If $u_{vR} = k_v e_R(R_{ee})$
then, the total error system (4) with input u_{ceR} and output ν_{ce} is passive.

Proof

$$U_{ecR} = \phi(R_{ee}) + S_\omega + \phi(R_{ec})$$

$$\dot{U}_{ecR} = \dot{\phi}(R_{ee}) + \dot{S}_\omega + \dot{\phi}(R_{ec})$$

$$= -e_R^T(R_{ee})u_{eR} + e_R^T(R_{ec})(-\omega_{wc} + R_{ec}u_{eR} + R_{ec}\bar{\omega}_{wo})$$

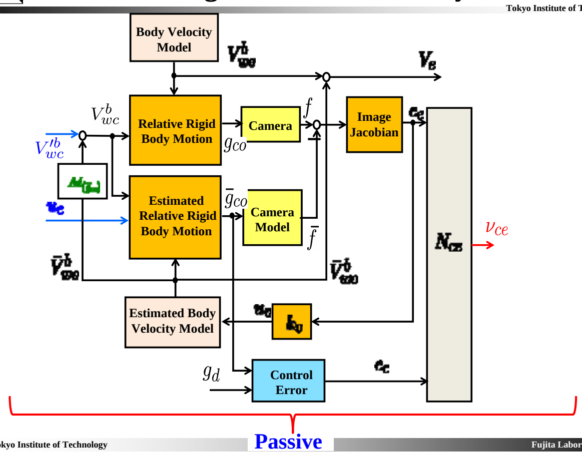
$$= \begin{bmatrix} e_R^T(R_{ec}) & e_R^T(R_{ee}) \end{bmatrix} \begin{bmatrix} -I & R_{ec} \\ 0 & -I \end{bmatrix} \begin{bmatrix} \omega_{wc} - R_{ec}\bar{\omega}_{wo} \\ u_{eR} \end{bmatrix}$$

$$= \begin{bmatrix} e_R^T(R_{ec}) & e_R^T(R_{ee}) \end{bmatrix} \begin{bmatrix} -I & R_{ec} \\ 0 & -I \end{bmatrix} \begin{bmatrix} \omega'_{wc} \\ u_{eR} \end{bmatrix} \quad \omega_{wc} = \omega'_{wc} + R_{ec}\bar{\omega}_{wo}$$

$$= e^T N_{ce}^T u_{ce} = u_{ce}^T \nu_{ce} \quad (\nu_{ce} = N_{ce} e)$$

Passive $\quad u_{ceR} \to \nu_{ce}$

Block Diagram of The Total System



Control Law

Control Law
based on the passivity property

$$u_{ceR} = -K_{ce}N_{ce}e, \quad K_{ce} = \begin{bmatrix} k_c I_3 & 0 \\ 0 & k_e I_3 \end{bmatrix} > 0$$
$$u_{vR} = k_v e_R(R_{ee}) \qquad (5)$$

$$\begin{pmatrix} \omega_{wc} = k_c e_R(R_{ec}) + \bar{R}_{co}\bar{\omega}_{wo} \\ (\omega'_{wc} = k_c e_R(R_{ec})) \\ u_{eR} = k_e\,(c_R(R_{ee}) - c_R(R_{ec})) \end{pmatrix}$$

i.e. TCST + u_{vR}

Theorem

Consider the total error system (4) with control law (5).
Then , $\lim_{t\to\infty}(\omega_e, e_R(R_{ec}), e_R(R_{ee})) = 0$ holds and hence both estimates converge to their actual values and the actual orientation to the same one as the target object.

Sketch of Proof

K_{ce} : positive definite
N_{ce} : nonsingular

Using the above control input, then

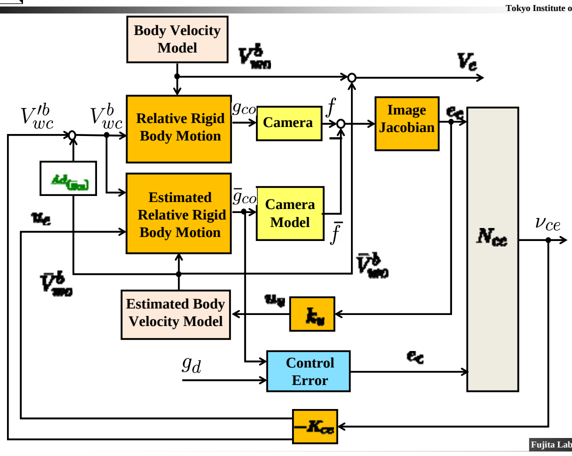
$$\dot{U}_{ceR} = -e^T N_{ce}^T K_{ce} N_{ce} e < 0 \ (e \neq 0) \Rightarrow e_R(R_{ec}), e_R(R_{ee}) \to 0$$

Using the LaSalle's invariance principle (refer to [2]).
$\omega_e \to 0$

$$\bar{\omega}_{wo} = \omega_{wo}, \bar{R}_{co} = R_{co}, \bar{R}_{co} = R_d = I$$

If we consider the desired attitude R_d, the same approach can be taken.

Block Diagram of The Visual Feedback Control



Summary and Future Works

Summary

- Study of previous works [1,2]
- Visual feedback attitude control with target motion model
  - Estimation error system
  - Control error system
  - Target velocity estimation error system
  - Analysis of the passivity
  - Visual feedback attitude control law
  - Analysis of the stability

Future Works

- $v_{wo} = 0 \Rightarrow v_{wo} = \text{const}$
- Another target motion model $V_{wo}^b \neq \text{const}$
- Simulation (middle term presentation)
- Experiment (final presentation)
- Multi-agent (Attitude Synchronization)

References

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE Transactions on Control Systems Technology, Vol.15, No.1, pp.40-52, 2007.*

[2]T. Hatanaka and M. Fujita, "Passivity-based Visual Motion Observer Integrating Internal Representation of 3D Target Object Motion," *Proc. of the 2012 American Control Conference, 2012*(submitted).

Appendix

Relative Pose and Body Velocity

Notation and Definition

Set the frames
- Σ_w : the world frame
- Σ_c : the camera frame
- Σ_o : the object frame

$p_{co} \in \mathcal{R}^3$: the position vector from Σ_c to Σ_o.

$e^{\hat{\xi}\theta_{co}} \in SO(3)$: the rotation matrix from Σ_c to Σ_o.

$v^b_{co} \in \mathcal{R}^3$: the body linear velocity from Σ_c to Σ_o.

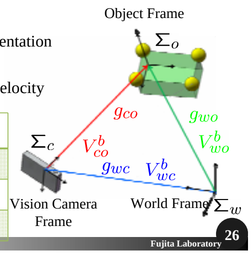
$\omega^b_{co} \in \mathcal{R}^3$: the body angular velocity from Σ_c to Σ_o.

Relative Pose and Body Velocity

$$g = \begin{bmatrix} e^{\hat{\xi}\theta} & p \\ 0 & 1 \end{bmatrix} = g(p, e^{\hat{\xi}\theta})$$: the homogeneous representation

$$V^b = \begin{bmatrix} v \\ \omega \end{bmatrix} \underset{\text{vee}}{\overset{\text{wedge}}{\rightleftarrows}} \hat{V}^b = g^{-1}\dot{g} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}$$: the body velocity

	Pose	Body Velocity
Σ_w to Σ_c	$g_{wc} = (p_{wc}, e^{\hat{\xi}\theta_{wc}})$	$\hat{V}^b_{wc} = g_{wc}^{-1}\dot{g}_{wc}$
Σ_w to Σ_o	$g_{wo} = (p_{wo}, e^{\hat{\xi}\theta_{wo}})$	$\hat{V}^b_{wo} = g_{wo}^{-1}\dot{g}_{wo}$
Σ_c to Σ_o	$g_{co} = (p_{co}, e^{\hat{\xi}\theta_{co}})$	$\hat{V}^b_{co} = g_{co}^{-1}\dot{g}_{co}$

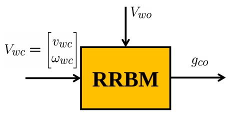


Relative Rigid Body Motion

Relative Rigid Body Motion (RRBM)

$$\dot{g}_{co} = -\hat{V}^b_{wc} g_{co} + g_{co}\hat{V}^b_{wo}$$

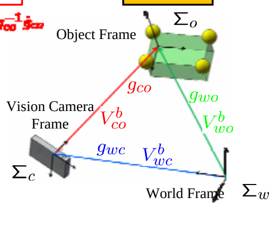
$$V^b_{co} = -Ad_{(g_{co}^{-1})} V^b_{wc} + V^b_{wo}$$



$\hat{V}^b_{co} = g_{co}^{-1}\dot{g}_{co}$

$$Ad_{(g_{co})} = \begin{bmatrix} R_{co} & \hat{p}_{co}R_{co} \\ 0 & R_{co} \end{bmatrix}$$

$$(g_{co}^{-1}\hat{V}^b_{co} g_{co})^\vee = Ad_{(g_{co}^{-1})} V^b_{co}$$



Passivity

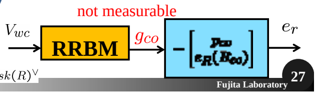
If the target object is the static ($V^b_{wo} = 0$), then the RRBM satisfies.

$$\int_0^T (V^b_{wc})^T(-e_r)dt \geq -\beta_\gamma$$

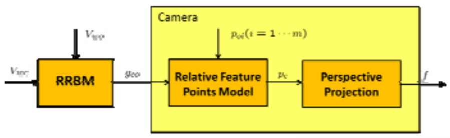
where β_r is a positive scalar.

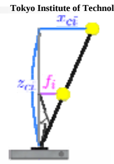
Passive $V^b_{wc} \rightarrow -e_r$

$e_r = \begin{bmatrix} p_{co} \\ e_R(R_{co}) \end{bmatrix}$ $e_R(R) = sk(R)^\vee$



Camera Model





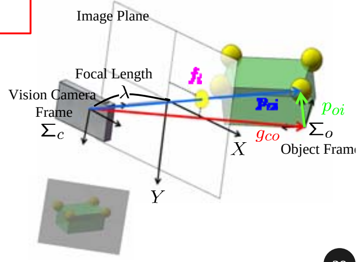
Relative Feature Points Model

$$p_{ci} = g_{co}p_{oi}$$

$$p_{oi} = \begin{bmatrix} x_{oi} \\ y_{oi} \\ z_{oi} \end{bmatrix} \quad p_{ci} = \begin{bmatrix} x_{ci} \\ y_{ci} \\ z_{ci} \end{bmatrix}$$

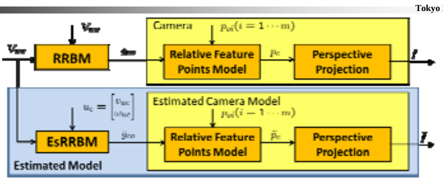
Perspective Projection

$$f_i = \frac{\lambda}{z_{ci}}\begin{bmatrix} x_{ci} \\ y_{ci} \end{bmatrix}$$



Not Passive $V^b_{wc} \rightarrow f$

Estimated Model



Estimated Relative Rigid Body Motion

$$\bar{V}^b_{co} = -Ad_{(\bar{g}_{co}^{-1})} V^b_{wc} + u_e$$

Observer Input

Relative Rigid Body Motion

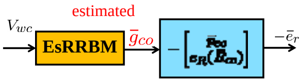
$$V^b_{co} = -Ad_{(g_{co}^{-1})} V^b_{wc} + V^b_{wo}$$

Passivity

If the target object is the static ($V^b_{wo} = 0$), then the EsRRBM satisfies.

$$\int_0^T (V^b_{wc})^T(-\bar{e}_r)dt \leq -\beta_\gamma$$

where β is a positive scalar.



Passive $V^b_{wc} \rightarrow -\bar{e}_r$

Estimation Error

Estimation Error

$$g_{ee} = \bar{g}_{co}^{-1}g_{co} = \begin{bmatrix} R_{ee} & p_{ee} \\ 0 & 1 \end{bmatrix}$$

not measurable

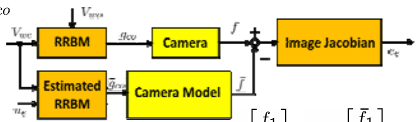
$$g_e = I_4 \Longleftrightarrow \bar{g}_{co} = g_{co}$$

Estimation Error Vector

$$e_e = \begin{bmatrix} p_{ee} \\ e_R(R_e) \end{bmatrix} = \begin{bmatrix} \bar{R}^T_{co}(p_{co} - \bar{p}_{co}) \\ e_R(\bar{R}^T_{co}R_{co}) \end{bmatrix}$$

Image Information Error

$$f_e = f - \bar{f}$$



$$f = \begin{bmatrix} f_1 \\ \vdots \\ f_m \end{bmatrix} \quad \bar{f} = \begin{bmatrix} \bar{f}_1 \\ \vdots \\ \bar{f}_m \end{bmatrix}$$

$$f_{ei} = \begin{bmatrix} \frac{\lambda}{\bar{z}_{ci}} & 0 & -\frac{\lambda \bar{x}_{ci}}{\bar{z}_{ci}^2} \\ 0 & \frac{\lambda}{\bar{z}_{ci}} & -\frac{\lambda \bar{y}_{ci}}{\bar{z}_{ci}^2} \end{bmatrix} R_{co}\begin{bmatrix} I & -\hat{\bar{p}}_{oi} \end{bmatrix} e_e$$

J_i : Image Jacobian

the relation between the actual image information and the estimated one can be expressed

$$f_e = Je_e \qquad \therefore e_e = J^\dagger f_e$$

estimation error e_e can be calculated

Estimation Error System

Estimation Error System

$$\dot{g}_{ee} = -\hat{u}_e g_{ee} + g_{ee}\hat{V}_{wo}^b$$
$$V_{ee}^b = -Ad_{(g_{ee}^{-1})}u_e + V_{wo}^b \quad \text{(A1)}$$

Passivity

If the target object is the static ($V_{wo}^b = 0$), then the Estimation Error System satisfies.

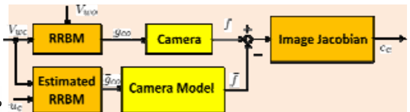
$$\int_0^T (u_e)^T(-e_e)dt \geq -\beta_e$$

where β_e is a positive scalar.

$$g_{ee} = \bar{g}_{co}^{-1}g_{co}$$
$$\dot{g}_e = \dot{\bar{g}}_{co}^{-1}g_{co} + \bar{g}_{co}^{-1}\dot{g}_{co}$$
$$= -\bar{g}_{co}^{-1}\dot{\bar{g}}_{co}g_{ee} + g_{ee}g_{co}^{-1}\dot{g}_{co}$$
$$= -\hat{u}_e g_{ee} + g_{ee}\hat{V}_{wo}^b$$
$$\hat{V}_{ee}^b = g_{ee}^{-1}\dot{g}_{ee}$$
$$= -g_{ee}^{-1}\hat{u}_e g_{ee} + \hat{V}_{wo}^b$$
$$V_{ee}^b = -Ad_{(g_{ee}^{-1})}u_e + V_{wo}^b$$

Passive $u_e \rightarrow -e_e$

$$u_e = \begin{bmatrix} v_{ue} \\ \omega_{ue} \end{bmatrix} \qquad e_e = \begin{bmatrix} p_{ee} \\ e_R(R_e) \end{bmatrix}$$



Estimation Error System