Distributed Camera Localization and Pose Estimation : Averaging Performance Analysis

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Problem Settings

Pose representation

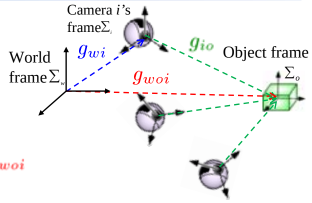
Pose of camera i : g_{wi}

Pose of target (object) : g_{woi}

Pose of target relative to camera i:

$$g_{io} = g_{wi}^{-1} g_{woi}$$

Camera i measure relative pose $\bar{g}_{io}$



Camera Set N cameras $i \in \mathcal{V} := \{1, \cdots, N\}$ $|\mathcal{V}_k|$: number of $\mathcal{V}_k$

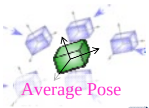
1. Pose of camera g_{wi} is known $i \in \mathcal{V}_k \subset \mathcal{V}$ $|\mathcal{V}_k| \geq 2$

   We can measure target pose (noisy) $\bar{g}_{woi} = g_{wi}\bar{g}_{io}$

Objective (Target Estimation)

Estimating g_{woi} which is close to the average $\bar{g}^*$

$\bar{g}^*$: Average of target pose



Problem Settings

Camera Set

2. Pose of camera g_{wi} is unknown $i \in \mathcal{V}_u = \mathcal{V} \backslash \mathcal{V}_k$

   We cannot measure target pose $\bar{g}_{woi}$, but get $\bar{g}_{io}$

   ⇨ No measurements about g_{wi}

   Use average of object pose $\bar{g}^*$

Objective (Localization)

Estimating g_{wi} which $g_{woi} = g_{wi}\bar{g}_{io}$ is close to the average $\bar{g}^*$

⇨ Estimating g_{woi} which is close to the average $\bar{g}^*$

Simultaneously get camera pose by setting $g_{wi} = g_{woi}\bar{g}_{io}^{-1}$

Consider only estimating the target pose g_{woi}

Objective

Objective (Target Estimation and Localization)

Estimating g_{woi} which is close to the average $\bar{g}^*$

1. Pose of camera g_{wi} is known $i \in \mathcal{V}_k \subset \mathcal{V}$ $|\mathcal{V}_k| \geq 2$

   Defining the average $\bar{g}^*$

   Modifying the measurements $\bar{g}_{woi}$

2. Pose of camera g_{wi} is unknown

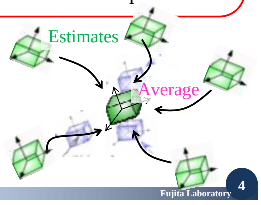
   Setting $g_{wi} = g_{woi}\bar{g}_{io}^{-1}$ ⇨ Estimate camera pose

Average of target pose

$$\bar{g}^* = (\bar{p}^*, \bar{R}^*) := \arg \min_{g \in SE(3)} \sum_{i \in \mathcal{V}_k} \psi(g^{-1}\bar{g}_{woi})$$

$$\psi(g) := \frac{1}{2}||I_4 - g||_F^2 = \frac{1}{2}||p||^2 + \phi(R)$$

$$\phi(R) := \frac{1}{2}||I_3 - R||_F^2 = \mathrm{tr}(I_3 - R)$$



Distributed Pose Estimation

Proposed Update Equations of Target Pose g_{woi}

$$\dot{g}_{woi} = g_{woi}\hat{u}_i$$ Gradient descent of $\psi(g_{woi}^{-1}\bar{g}_{woi})$

$$u_i = \begin{cases} k_e E_R(g_{woi}^{-1}\bar{g}_{woi}) + k_s \sum_{j \in \mathcal{N}_i} E_R(g_{woi}^{-1}g_{woj}) & i \in \mathcal{V}_k \\ k_s \sum_{j \in \mathcal{N}_i} E_R(g_{woi}^{-1}g_{woj}) & i \in \mathcal{V}_u \end{cases} \quad (1)$$

Pose synchronization [3]

$$E_R(g) := \begin{bmatrix} p^T & \mathrm{sk}(R)^\vee \end{bmatrix}^T \qquad \mathrm{sk}(M) = \frac{1}{2}(M - M^T)$$

Also estimate camera pose g_{wi} by updating g_{woi}

$g_{wi} = g_{woi}\bar{g}_{io}^{-1}$ and (1) $i \in \mathcal{V}_u$

Averaging analysis about target pose g_{woi}

[3] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in *SE(3)*," *IEEE Transactions on Control Systems Technology*, Vol. 17, No. 5, pp.1119–1134, 2009.

Assumptions (Averaging Performance)

Assumption 1 (Communication Graph)

The communication graph $G = (\mathcal{V}, \mathcal{E})$ is fixed, balanced and strongly connected.

Assumption 2 (Target Object Pose)

- The object is static. $V_{wo}^b = 0$
- There exists a pair $(i, j) \in \mathcal{V} \times \mathcal{V}$ such that $\bar{g}_{woi} \neq \bar{g}_{woj}$
- $\bar{R}^{*T}\bar{R}_{woi} > 0 \quad \forall i \in \mathcal{V}$ holds true

  The relative angle between is smaller than $\pi/2$

Main Theorem (Averaging Performance)

Theorem 1

Suppose the estimates g_{woi} are updated according to the update equation (1) and that the initial estimates satisfy $R_{woi}^T \bar{R}^* > 0$.
Given any $\alpha_R > |\mathcal{V}_u|/|\mathcal{V}_k|$, under Assumptions 1 and 2, if the gain $k = k_e/k_s$ is sufficiently small, then for all $\epsilon \in (0,1)$ and sufficiently large times T,

$$\frac{1}{|\mathcal{V}|}\sum_{i\in\mathcal{V}}\phi(\bar{R}^{*T}R_{woi}) < (\alpha'_R + \alpha_R)\frac{1}{|\mathcal{V}_k|}\sum_{i\in\mathcal{V}_k}\phi(\bar{R}^{*T}\bar{R}_{woi})$$

Error between average and estimates — Error between average and measurements

$$\alpha'_R = \begin{cases} 1-(1-\epsilon)(\sqrt{\beta}-\sqrt{kW})^2 & \text{if } k \le \frac{\beta}{W} \\ 1 & \text{otherwise} \end{cases}$$

$$\beta := 1 - \sqrt{2(\phi(\bar{R}^{*T}\bar{R}_{woh}) + c)}$$

holds true.

$|\mathcal{V}_k|$: number of known pose camera

$|\mathcal{V}_u|$: number of unknown pose camera

Lemma1

Lemma 1 (Positively Invariance)

Under assumption 1and 2 and $\bar{R}^{*T}R_{woi} > 0 \ \ \forall t \ge 0$ holds,
Then for any positive scalar c, there exists a finite time $\tau(c)$ such that

$$\phi(\bar{R}^{*T}R_{woi}) \le \phi(\bar{R}^{*T}\bar{R}_{woh}) + c \qquad \forall t \ge \tau(c) \quad i \in \mathcal{V}_k$$

$$h := \arg\max_j \phi(\bar{R}^{*T}\bar{R}_{woj})$$

Proof

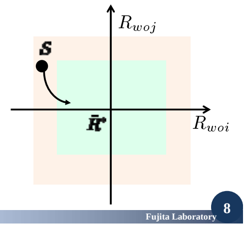
Energy function: $U := \max_{i\in\mathcal{V}_k}\phi(\bar{R}^{*T}R_{woi})$

Under assumption 2

$$\mathcal{S} = \{(R_{woi})_{i\in\mathcal{V}} | \bar{R}^{*T}R_{woi} > 0 \quad \forall i \in \mathcal{V}\}$$

is positively invariant

i.e. if $\bar{R}^{*T}R_{woi} > 0$ holds at the initial time then it also holds for all subsequent time



Lemma2 (Averaging Performance)

Lemma 2

Suppose the estimates g_{woi} are updated according to the update equation (1) and that the initial estimates satisfy $R_{woi}^T \bar{R}^* > 0$.
Given any $\alpha_R > |\mathcal{V}_u|/|\mathcal{V}_k|$, under Assumptions 1 and 2, if the gain $k = k_e/k_s$ is sufficiently small, then for all sufficiently large times T,

$$\frac{1}{|\mathcal{V}|}\sum_{i\in\mathcal{V}}\phi(\bar{R}^{*T}R_{woi}) \le (1 + \alpha_R)\frac{1}{|\mathcal{V}_k|}\sum_{i\in\mathcal{V}_k}\phi(\bar{R}^{*T}\bar{R}_{woi})$$

Error between average and estimates — Error between average and measurements

holds true.

$|\mathcal{V}_k|$: number of known pose camera

$|\mathcal{V}_u|$: number of unknown pose camera

Proof of Lemma2

Energy function

$$U_R := \sum_{i\in\mathcal{V}}\phi(\bar{R}^{*T}R_{woi})$$

Update equation of orientation

$$\dot{R}_{woi} = \begin{cases} R_{woi}(k_e\mathrm{sk}(R_{woi}^T\bar{R}_{woi}) + k_s\sum_{j\in\mathcal{N}_i}\mathrm{sk}(R_{woi}^T R_{woj})) & i \in \mathcal{V}_k \\ R_{woi}\mathrm{sk}(k_s\sum_{j\in\mathcal{N}_i}R_{woi}^T R_{woj}) & i \in \mathcal{V}_u \end{cases}$$

Derivative of the energy function

$$\begin{aligned}\dot{U}_R &= -\sum_{i\in\mathcal{V}}\mathrm{tr}(\bar{R}^{*T}\dot{R}_{woi}) \\ &= -\frac{1}{2}k_e\sum_{i\in\mathcal{V}_k}\underbrace{\mathrm{tr}(\bar{R}^{*T}\bar{R}_{woi} - \bar{R}^{*T}R_{woi}\bar{R}_{woi}^T R_{woi})}_{\Phi_1} \\ &\quad -\frac{1}{2}k_s\underbrace{\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\mathrm{tr}(\bar{R}^{*T}R_{woj} - \bar{R}^{*T}R_{woi}R_{woj}^T R_{woi})}_{\Phi_2}\end{aligned}$$

Proof of Lemma2

Lemma3 $\forall R_1, R_2, R_3 \in SO(3) \qquad \mathrm{sym}(M) = \frac{1}{2}(M + M^T)$

$$\frac{1}{2}\mathrm{tr}(R_1^T R_2 - R_1^T R_3 R_2^T R_3) \ge \phi(R_1^T R_3) - \phi(R_1^T R_2) + \lambda_{\min}(\mathrm{sym}(R_1^T R_3))\phi(R_2^T R_3)$$

$$\frac{1}{2}\mathrm{tr}(\Phi_1) \ \ge\ \phi(\bar{R}^{*T}R_{woi}) - \phi(\bar{R}^{*T}\bar{R}_{woi}) + \sigma_i\phi(R_{woi}^T\bar{R}_{woi})$$

$$\sigma_i = \lambda_{\min}(\mathrm{sym}(\bar{R}^{*T}R_{woi}))$$

$$\frac{1}{2}\sum_{i\in\mathcal{V}}\mathrm{tr}(\Phi_2) \ge \underbrace{\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\phi(\bar{R}^{*T}R_{woi}) - \phi(\bar{R}^{*T}R_{woj})}_{=0 \text{ Under Assumption1}} + \sigma_i\phi(R_{woi}^T R_{woj})$$

$$\begin{aligned}\dot{U}_R &\le -\frac{k_e}{2}\sum_{i\in\mathcal{V}_k}\mathrm{tr}(\Phi_1) - \frac{k_s}{2}\sum_{i\in\mathcal{V}}\mathrm{tr}(\Phi_2) \\ &\le -k_e\sum_{i\in\mathcal{V}_k}(\phi(\bar{R}^{*T}R_{woi}) - \phi(\bar{R}^{*T}\bar{R}_{woi}) + \sigma_i\phi(R_{woi}^T\bar{R}_{woi})) \\ &\quad -k_s\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\sigma_i\phi(R_{woi}^T R_{woj})\end{aligned}$$

Proof of Lemma2

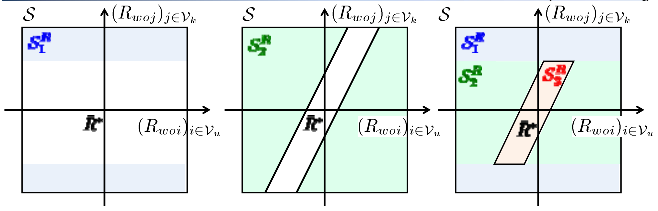
Lemma4

Suppose $\phi(\bar{R}^{*T}R_{woi}) < \beta := 1 - \sqrt{2(\phi(\bar{R}^{*T}\bar{R}_{woh}) + c)}$ holds true.

Then we have $\underbrace{\lambda_{\min}(\mathrm{sym}(\bar{R}^{*T}R_{woi}))}_{=\sigma_i} \ge \beta := 1 - \sqrt{2(\phi(\bar{R}^{*T}\bar{R}_{woh}) + c)}$

$$\begin{aligned}\dot{U}_R &\le k_e\rho_R - k_e\sum_{i\in\mathcal{V}_k}(\phi(\bar{R}^{*T}R_{woi}) + \beta\phi(R_{woi}^T\bar{R}_{woi})) \\ &\quad -k_s\beta\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\phi(R_{woi}^T R_{woj}) \qquad \rho_R := \sum_{i\in\mathcal{V}_k}\phi(\bar{R}^{*T}\bar{R}_{woi})\end{aligned} \tag{2}$$

Proof of Lemma2



Consider 3 sets $k = k_e/k_s$

$$\mathcal{S}_1^R = \{(R_{woi}) \in \mathcal{S} | \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) > \rho_R\}$$

$$\mathcal{S}_2^R = \{(R_{woi}) \in \mathcal{S} | \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \phi(R_{woi}^T R_{woj}) > \frac{k\rho_R}{\beta}\}$$

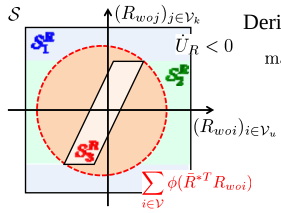
$$\mathcal{S}_3^R = \{(R_{woi}) \in \mathcal{S} \backslash (\mathcal{S}_1^R \cup \mathcal{S}_2^R)\}$$

$\mathcal{S}_1^R, \mathcal{S}_2^R$: $\dot{U}_R < 0$

The trajectories of estimates ultimately converge to the set $\mathcal{S}_3^R$

(Ultimate Boundedness)

Proof of Lemma2



Derive the region of $\mathcal{S}_3^R$

$$\max \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) \quad (2)$$

$$\text{s.t.} \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) \le \rho_R \quad (R_{woi}) \notin \mathcal{S}_1^R$$

$$\sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \phi(R_{woi}^T R_{woj}) \le \frac{k\rho_R}{\beta} \quad (R_{woi}) \notin \mathcal{S}_2^R$$

Rewrite maximization problem

$$(2) < \quad \max \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) \quad (3) \quad \text{s.t.} \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) \le \rho_R, \quad \phi(R_{woi}^T R_{woj}) \le \frac{k\rho_R}{\beta} \quad \forall i \in \mathcal{V}, \forall j \in \mathcal{N}_i$$

$$< \quad \max \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) \quad (4) \quad \text{s.t.} \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) \le \rho_R, \quad \phi(R_{woi}^T R_{woj}) \le \frac{\text{diam}(G) k\rho_R}{\beta} \quad \forall i, j \in \mathcal{V}$$

Proof of Lemma2

$$\max \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) \quad (3) \quad \text{s.t.} \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) \le \rho_R, \quad \phi(R_{woi}^T R_{woj}) \le \frac{\text{diam}(G) k\rho_R}{\beta} \quad \forall i, j \in \mathcal{V}$$

$$= \quad \max \rho_R + \sum_{i\in\mathcal{V}_u} \phi(\bar{R}^{*T} R_{woi}) \quad (4) \quad \text{s.t.} \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) = \rho_R, \quad \phi(R_{woi}^T R_{wol_k}) \le \frac{\text{diam}(G) k\rho_R}{\beta}$$

$$l_k := \arg\max_{j\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woj}) \quad \forall i \in \mathcal{V}$$

Function $\phi(R)$ is Frobenius Norm (Sum of squares of its elements)

$$\phi(\bar{R}^{*T} R_{woi}) := \frac{1}{2} \|\bar{R}^* - R_{woi}\|_F^2 \qquad \|A\|_F^2 = \sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2$$

Consider each element x_i : element of $\bar{R}^* - R_{woi}$

$$(4) < \quad \max \sum_{i\in\mathcal{V}_u} x_i^2 + \rho_R \quad (5)$$

$$\text{s.t.} \sum_{i\in\mathcal{V}_k} x_i^2 = \rho_R, \quad (x_{l_k} - x_i)^2 \le \frac{\text{diam}(G) k\rho_R}{\beta} \quad \forall i \in \mathcal{V}$$

Proof of Lemma2

$$\max \sum_{i\in\mathcal{V}_u} x_i^2 + \rho_R \quad (5)$$

$$\text{s.t.} \sum_{i\in\mathcal{V}_k} x_i^2 = \rho_R, \quad (x_{l_k} - x_i)^2 \le \epsilon'_R, \quad \epsilon'_R = \frac{\text{diam}(G) k\rho_R}{\beta} \quad \forall i \in \mathcal{V}$$

$\forall i \in \mathcal{V}_k \quad x_{l_k} \ge x_i$

$$x_{l_k} - \sqrt{\epsilon'_R} \le x_i$$

$$|\mathcal{V}_k|(x_{l_k} - \sqrt{\epsilon'_R})^2 \le \sum_{i\in\mathcal{V}_k} x_i^2 = \rho_R$$

$\forall i \in \mathcal{V}_u \quad x_{l_k} \le x_i$

$$x_i \le x_{l_k} + \sqrt{\epsilon'_R}$$

$$\sum_{i\in\mathcal{V}_u} x_i^2 \le |\mathcal{V}_u|(x_{l_k} + \sqrt{\epsilon'_R})^2 \quad (7)$$

$$\Rightarrow \quad x_{l_k} \le \sqrt{\frac{\rho_R}{|\mathcal{V}_k|}} + \sqrt{\epsilon'_R} \quad (6)$$

Substitute (6) to (7)

$$\sum_{i\in\mathcal{V}_u} x_i^2 \le |\mathcal{V}_u|(\sqrt{\frac{\rho_R}{|\mathcal{V}_k|}} + 2\sqrt{\epsilon'_R})^2 = \alpha_R \rho_R$$

$$\boldsymbol{\alpha_R = |\mathcal{V}_u|(\frac{1}{\sqrt{|\mathcal{V}_k|}} + 2\sqrt{\frac{\text{diam}(G) k}{\beta}})^2}$$

Proof of Lemma2

The trajectories of estimates ultimately converge to the set satisfying

$$\sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) \le (1 + \alpha_R)\rho_R \qquad \boldsymbol{\alpha_R = |\mathcal{V}_u|(\frac{1}{\sqrt{|\mathcal{V}_k|}} + 2\sqrt{\frac{\text{diam}(G) k}{\beta}})^2}$$

If the gain $k = k_e/k_s$ is sufficiently small

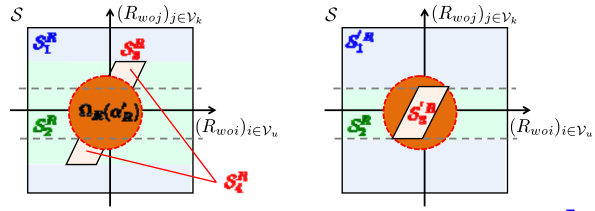
$$k = k_e/k_s \to 0 \qquad \boldsymbol{\alpha_R \to \frac{|\mathcal{V}_u|}{|\mathcal{V}_k|}}$$

Given any $\alpha_R > |\mathcal{V}_u|/|\mathcal{V}_k|$

$$\Rightarrow \quad \frac{1}{|\mathcal{V}|} \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) < (1 + \alpha_R)\frac{1}{|\mathcal{V}|}\rho_R < (1 + \alpha_R)\frac{1}{|\mathcal{V}_k|}\rho_R \quad (\because |\mathcal{V}| > |\mathcal{V}_k|)$$

holds.

Proof of Theorem1



Idea: Expand the region of $\mathcal{S}_1^R$

Consider the set $\mathcal{S}_4^R = \{(R_{woi}) \in \mathcal{S} \backslash (\mathcal{S}_1 \cup \mathcal{S}_2 \cup \Omega_R(\alpha'_R))\}$

It is sufficient to prove that $\dot{U}_R < 0$ in the region $\mathcal{S}_4^R$

From Lemma2

$$U_R \le \sum_{i\in\mathcal{V}_k} k_e(\phi(\bar{R}^{*T} \bar{R}_{woi}) - \phi(\bar{R}^{*T} R_{woi}) - \beta(1-\epsilon)\phi(R_{woi}^T \bar{R}_{woi})) - a_R$$

$$a_R := \beta \sum_{i\in\mathcal{V}_k} k_e \epsilon \phi(R_{woi}^T \bar{R}_{woi}) + \beta \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} k_s \phi(R_{woi}^T R_{woj}) \quad (2)$$

Proof of Theorem1

$$\phi(R_{woi}^T\bar{R}_{woi}) \geq \delta\phi(R_{woj^*}^T\bar{R}_{woi}) - \frac{\delta}{1-\delta}\phi(R_{woj^*}^T R_{woi}) \qquad \delta\in(0,1) \qquad (3)$$

Let j^* the node satisfying $j^* = \arg\min_{i_0} D(i_0)$

$G_T^* = (\mathcal{V}, \mathcal{E}_T^*) \in \mathcal{T}(j^*)$ the graph satisfying $G_T^* = \arg\min_{G_T\in\mathcal{T}(j^*)} \tilde{D}(G_T)$

$$\sum_{i\in\mathcal{V}_k}\phi(R_{woj^*}^T R_{woi}) \leq \sum_{i\in\mathcal{V}} d_{G_T^*}(i) \sum_{l\in\{0,\cdots,d_{G_T^*}(i)-1\}} \phi(R_{wov_l(i)}^T R_{wov_{l+1}(i)})$$

Upper-bounded by $\tilde{D}(G_T^*) = W$

$$\sum_{i\in\mathcal{V}_k}\phi(R_{woj^*}^T R_{woi}) \leq W \sum_{E=(v^1,v^2)\in\mathcal{E}_T} \phi(R_{wov^1}^T R_{wov^2}) \leq W \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\phi(R_{woi}^T R_{woj})$$

Since $(R_{woi})_{i\in\mathcal{V}} \in \mathcal{S}_4^R$ $\quad (R_{woi})_{i\in\mathcal{V}} \notin \mathcal{S}_2^R$ holds true

$$\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\phi(R_{woi}^T R_{woj}) \leq \frac{k\rho_R}{\beta}$$

$$\Rightarrow \sum_{i\in\mathcal{V}_k}\phi(R_{woj^*}^T R_{woi}) \leq \frac{kW\rho_R}{\beta} \qquad (4)$$

Proof of Theorem1

From the definition of average

$$\sum_{i\in\mathcal{V}_k}\phi(R_{woj^*}^T\bar{R}_{woi}) \geq \sum_{i\in\mathcal{V}_k}\phi(\bar{R}^{*T}\bar{R}_{woi}) = \rho_R \qquad (5)$$

Substitute (3), (4), (5) to (2)

$$\dot{U}_R \leq k_e\{-\sum_{i\in\mathcal{V}_k}(\phi(\bar{R}^{*T}R_{woi})) + (1-(1-\epsilon)(\delta\beta - \frac{kW\delta}{1-\delta}))\rho_R\} - a_R$$

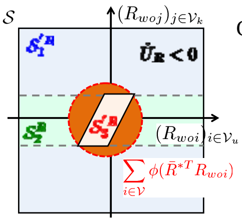
From the assumption $k \leq \frac{\beta}{W}$ $\quad \delta\beta - \frac{kW\delta}{1-\delta} \leq (\sqrt{\beta}-\sqrt{kW})^2$

$$\dot{U}_R \leq k_e\{-\sum_{i\in\mathcal{V}_k}(\phi(\bar{R}^{*T}R_{woi})) + (1-(1-\epsilon)(\sqrt{\beta}-\sqrt{kW})^2)\rho_R\} - a_R$$

$$= k_e\alpha_R'\rho_R - k_e\sum_{i\in\mathcal{V}_k}\phi(\bar{R}^{*T}R_{woi}) - a_R$$

$$\alpha_R' = 1-(1-\epsilon)(\sqrt{\beta}-\sqrt{kW})^2$$

Proof of Theorem1



Consider 3 sets

$$\mathcal{S}_1'^R = \{(R_{woi})\in\mathcal{S} | \sum_{i\in\mathcal{V}_k}\phi(\bar{R}^{*T}R_{woi}) > \alpha_R'\rho_R\}$$

$$\mathcal{S}_2^R = \{(R_{woi})\in\mathcal{S} | \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\phi(R_{woi}^T R_{woj}) > \frac{k\rho_R}{\beta}\}$$

$$\mathcal{S}_3'^R = \{(R_{woi})\in\mathcal{S}\backslash(\mathcal{S}_1'^R\cup\mathcal{S}_2^R)\}$$

$\mathcal{S}_1'^R, \mathcal{S}_2^R$: $\dot{U}_R<0$ The trajectories converge to the set $\mathcal{S}_3'^R$

Same as Lemma2, derive the region of $\mathcal{S}_3'^R$

$$\max \sum_{i\in\mathcal{V}}\phi(\bar{R}^{*T}R_{woi})$$

s.t. $\sum_{i\in\mathcal{V}_k}\phi(\bar{R}^{*T}R_{woi}) \leq \alpha_R'\rho_R$ $\quad (R_{woi})\notin\mathcal{S}_1^R$

$\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i}\phi(R_{woi}^T R_{woj}) \leq \frac{k\rho_R}{\beta}$ $\quad (R_{woi})\notin\mathcal{S}_2^R$

$\Rightarrow$ $\frac{1}{|\mathcal{V}|}\sum_{i\in\mathcal{V}}\phi(\bar{R}^{*T}R_{woi}) \leq (\alpha_R'+\alpha_R)\frac{1}{|\mathcal{V}_k|}\rho_R$ holds.

Conclusion

Conclusion

- Averaging performance analysis

Future Works

- Tracking performance analysis
- Experimental Verification

References

[1] T. Hatanaka and M. Fujita, “Cooperative Estimation of 3D Target Motion via Networked Visual Motion Observers,” *IEEE Transactions on Automatic Control*, 2011. (submitted)

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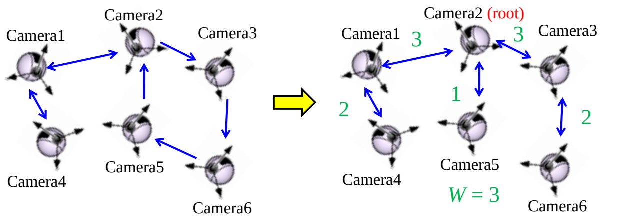
Appendix

Communication Graph



$W := \min_{i_0 \in \mathcal{V}} D(i_0) \quad D(i_0) := \min_{G_T \in \mathcal{T}(i_0)} \tilde{D}(G_T) \quad \tilde{D}(G_T) := \max_{E \in \mathcal{E}_T} \sum_{i \in \mathcal{V}} \delta_{G_T}(E; i) d_{G_T}(i)$

$\delta_{G_T}(E; i) = \begin{cases} 1 & \text{if } P_{G_T(i)} \text{ includes } E \in \mathcal{E}_T \\ 0 & \text{otherwise} \end{cases}$ $d_{G_T}(i)$: length of path $P_{G_T}(i)$

$P_{G_T}(i)$: path from i_0 to i along G_T

$G_T = (\mathcal{V}, \mathcal{E}_T) \in \mathcal{T}(i_0)$ $\mathcal{T}(i_0)$: set of spanning tree over G_u with a root i_0

G_u: replace all the directed edges of G by the undirected ones

Proof of Theorem1

$$\phi(R_{woi}^T \bar{R}_{woi}) \geq \delta\phi(R_{woj^*}^T \bar{R}_{woi}) - \frac{\delta}{1-\delta}\phi(R_{woj^*}^T R_{woi}) \qquad \delta \in (0,1) \qquad (3)$$

$$\|x - y\|^2 \geq a\|x - z\|^2 - \frac{a}{1-a}\|y - z\|^2$$

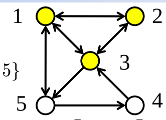
$$(1-a)\|x\|^2 + \frac{1}{1-a}\|y^2\| + \frac{a^2}{1-a}\|z\|^2 - 2x^T y + 2ax^T z - \frac{2a}{1-a}y^T z$$

$$= (1-a)\left(x - \frac{1}{1-a}y + \frac{a}{1-a}z\right)^2 \geq 0$$

Simulations

Camera Settings $i \in \mathcal{V} := \{1, \cdots, 5\}$

● $i \in \mathcal{V}_k = \{1, 2, 3\}$ ○ $i \in \mathcal{V}_u = \{4, 5\}$



$\mathrm{diam}(G) = 2$

$W = 1$

Target pose measurements

$$\bar{p}_{wo1} = \begin{bmatrix} -0.3 \\ 0.6 \\ 1.9 \end{bmatrix}, \bar{p}_{wo2} = \begin{bmatrix} -0.2 \\ 0.5 \\ 1.6 \end{bmatrix}, \bar{p}_{wo3} = \begin{bmatrix} -0.6 \\ 0.4 \\ 1.8 \end{bmatrix}$$

$$\bar{\xi\theta}_{wo1} = \begin{bmatrix} 0.3 \\ 0.2 \\ 0.3 \end{bmatrix}, \bar{\xi\theta}_{wo2} = \begin{bmatrix} 0.4 \\ 0.15 \\ 1.2 \end{bmatrix}, \bar{\xi\theta}_{wo3} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.15 \end{bmatrix}$$

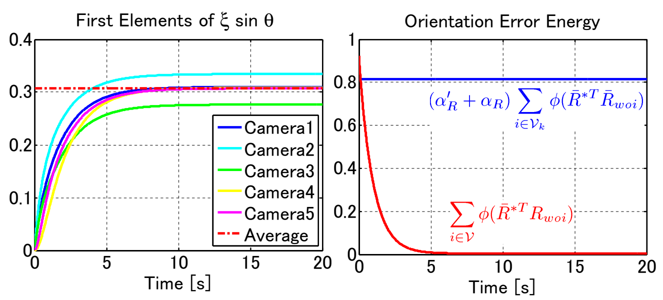
Average of target pose

$$\bar{p}^* = \begin{bmatrix} -0.3667 \\ 0.5000 \\ 1.7667 \end{bmatrix}, \bar{\xi\theta}^* = \begin{bmatrix} 0.3168 \\ 0.2002 \\ 0.2168 \end{bmatrix} \Rightarrow \beta = 0.7937$$

Initial Estimates $p_{woi}(0) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \quad R_{woi}(0) = I_3$

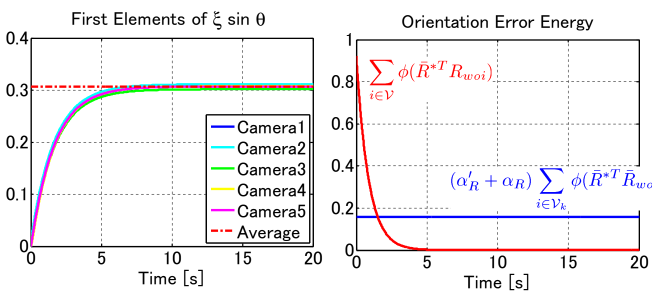
Simulation

$k_e = 1, k_s = 1 (k = 1)$ $\alpha'_R = 1 \quad \alpha_R = 28.1566$



Simulation

$k_e = 1, k_s = 10 (k = 0.1)$ $\alpha'_R = 0.6730 \quad \alpha_R = 5.0010$



Estimates are close to the average

Large k achieves a good averaging performance