Safety Analysis of Vehicle Platoon under V2V2I Communication

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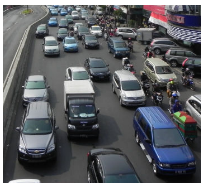
Background

Highway congestion

Highway congestion is imposing an intolerable burden on urban residents

Congestion occurs when vehicle's velocity variation propagates to following vehicles

It is difficult for human drivers to recognize tiny changing of the precede vehicle's velocity



Approaches

There are various approaches to improve congestion
They can be classified as
macro perspective and micro perspective

Macro perspective: On-ramp control, Transportation Network
Micro perspective: Vehicle Platoon Control

P. Varaiya, "Smart Cars on Smart Roads: Problems of Control", *IEEE Transactions on Automatic Control*, Vol. 38, No. 2, Feb. 1993

Introduction

Intelligent road transportation system[1]

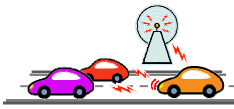
Vehicle-to-Vehicle interaction(V2V)
- Organizing the interaction among vehicles
- Information is interchanged and decisions are made on a "local" basis
- Micro perspective: [2],[3]

Vehicle-to-Infrastructure interaction(V2I)
- The infrastructure plays a coordination role by gathering "global" information and then suggesting certain behaviors on platoon
- Macro perspective: [4],[5],[6]

Vehicle-to-Vehicle-Infrastructure(V2V2I) interaction

The hybrid system of the V2V and V2I
Vehicles are controlled by "Local" and "global" information

Middle perspective?



Purpose of Research

Middle perspective

Optimal vehicle control of platoon under V2V2I communication
→need to consider collision avoidance constraints

Lack of safety analysis

Micro perspective
- Analysis of String Stability is usually interpreted in frequency domain
- It is difficult to consider safety which belongs to time domain

Macro perspective
- Don't consider a vehicle unit, but a platoon unit or traffic flow
- Making an assumption of collision avoidance

Objective
- Safety analysis of vehicle platoon under V2V2I communication
- Simulation (Analysis of String Stability)
- (Optimal Control of vehicle platoon)

Problem Description

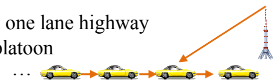
Intelligent vehicle(IV)[4]

A vehicle equipped with control systems that can sense the environment around the vehicles and that result in a more efficient vehicle operation by assisting the driver or by taking complete control of the vehicle

⇨ Considering autonomous vehicle with communication facility

Situation

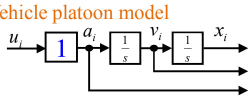
- Platoon of n Homogeneous IV in an one lane highway
- Infrastructure that controls vehicle platoon

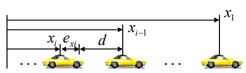


- Each vehicles can communicate any others in platoon to reference their information
- It doesn't matter whether communicating or not
- Communication is time invariant

Modeling

Vehicle platoon model





$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} I & tI \\ 0 & I \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u(k)$$

$$x(k) = [x_1(k), \cdots, x_n(k)]^T$$
$$v(k) = [v_1(k), \cdots, v_n(k)]^T$$
$$u(k) = [u_1(k), \cdots, u_n(k)]^T$$

t:Sampling time
x_{di} :Constant desired spacing

Input

ith vehicle's input

$$u_i(k) = k_p t e_{xi}(k) + k_v t e_{vi}(k)$$

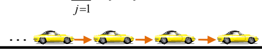
$$e_{xi}(k) = \sum_{j=1}^{n} \alpha_{ij}(x_j(k) - x_i(k)) + x_{di}$$

$$e_{vi}(k) = \sum_{j=1}^{n} \alpha_{ij}(v_j(k) - v_i(k))$$

Weighted communication state

$$\begin{cases} 0 < \alpha_{ij} \le 1 & \text{Reference } x_j, v_j \\ \alpha_{ij} = 0 & \text{No communication} \\ \alpha_{ii} = -1 & \end{cases}$$

$$\sum_{j=1}^{i-1} \alpha_{ij} + \sum_{j=i+1}^{n} \alpha_{ij} = 1 \quad \sum_{j=1}^{n} \alpha_{ij} = 0$$



Assumption $\alpha_{ii} = \alpha_{i2} = \cdots = \alpha_{in}$

Modeling

There exists some L_g such that

$e_x(k) = L_g x(k) + x_d$

$e_v(k) = L_g v(k)$

ex) $e_{x1}(k) = -x_1(k) + x_3(k)$, $e_{x2}(k) = x_1(k) - x_2(k)$, $e_{x3}(k) = 1/2(x_1(k) - x_3(k)) + 1/2(x_2(k) - x_3(k))$ ⇨ $e_x(k) = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 1/2 & 1/2 & -1 \end{bmatrix} x(k)$

$u(k) = k'_p e_x(k) + k'_v e_v(k)$

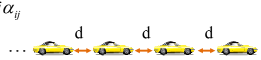
$k'_p = k_p t \quad k'_v = k_v t$

$$L_g = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ & & \ddots & \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nn} \end{bmatrix} = \begin{bmatrix} -1 & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & -1 & \cdots & \alpha_{2n} \\ & & \ddots & \\ \alpha_{n1} & \alpha_{n2} & \cdots & -1 \end{bmatrix}$$

Reference relative position

Time-invariant constant inter-vehicular distance

$x_d = dL_g J \quad J = [1,2,3,\cdots]^T \quad \{L_g J\}_i = \sum_{j=1}^{n} j\alpha_{ij}$



State equation

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} I & tI \\ k'_p L_g & I + k'_v L_g \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + k'_p \begin{bmatrix} 0 \\ L_g J \end{bmatrix} d$$

Reference from infrastructure

Safety Analysis

Collision avoidance constraints

$Dx(k) > \mathbf{0} \quad k = 1,2,\cdots$

$$D = \begin{bmatrix} \ddots & & & \\ & 1 & -1 & \\ & & & \ddots \end{bmatrix}$$

Respecting step k, Assuming $Dx(k) > \mathbf{0}$

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} I & tI \\ k'_p L_g & k'_v L_g + I \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + k'_p \begin{bmatrix} 0 \\ dL_g J \end{bmatrix} = \begin{bmatrix} x(k) + tv(k) \\ k'_p L_g x(k) + k'_v L_g v(k) + k'_p dL_g J \end{bmatrix}$$

$$\begin{bmatrix} x(k+2) \\ v(k+2) \end{bmatrix} = \begin{bmatrix} x(k) + 2tv(k) + t(k'_p L_g x(k) + k'_v L_g v(k)) + k'_p tdL_g J \\ k'_p L_g(x(k) + tv(k)) + k'_v L_g(k'_p L_g x(k) + k'_v L_g v(k) + k'_p dL_g J) + k'_p dL_g J \end{bmatrix}$$

⇨ $Dx(k+1)$ depend on $x(k), v(k)$

$Dx(k+2)$ depend on $x(k), v(k), d$

— State term

— Inter-vehicular distance term

Assuming $Dx(k+1) > \mathbf{0}$, infrastructure can control collision performance of $Dx(k+2)$ if $DL_g J \neq \mathbf{0}$

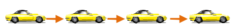
Need to check a condition of L_g that satisfy

For some i, $\{DL_g J\}_i = 0$

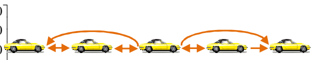
Safety Analysis

Ex)

$$DL_g J = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & & \\ & 1 & -1 & \\ & & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = D \begin{bmatrix} 0 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



$$DL_g J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1/2 & -1 & 1/2 & & \\ 1/4 & 1/4 & -1 & 1/4 & 1/4 \\ & & 1/2 & -1 & 1/2 \\ & & & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = D \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$



$\{DL_g J\}_i = 0$ occurs when

- relatively equivalent communication graph $\begin{bmatrix} 1 & -1 & & \\ & 1 & -1 & \\ & & 1 & -1 \end{bmatrix}$
- bidirectional communication graph $\begin{bmatrix} 1/2 & -1 & 1/2 & \\ & 1/2 & -1 & 1/2 \end{bmatrix}$

Safety Analysis

Theorem 1

If L_g satisfies the following condition:

There exists some i such that

$$\begin{bmatrix} \alpha_{i(i-1)} - \alpha_{i(i+1)} & \alpha_{i(i-2)} - \alpha_{i(i+2)} & \cdots \end{bmatrix} = \begin{bmatrix} \alpha_{(i+1)i} - \alpha_{(i+1)(i+2)} & \alpha_{(i+1)(i-1)} - \alpha_{(i+1)(i+3)} & \cdots \end{bmatrix} \quad (*)$$

with $\alpha_{ij} = 0, j < 0$.

Then $\{DL_g J\}_i = 0$.

Sketch of Proof

$$\{L_g J\}_i = \sum_{j=1}^{n} j\alpha_{ij} = \sum_{j=1}^{n} j\alpha_{ij} - i\sum_{j=1}^{n} \alpha_{ij} = -(\alpha_{i(i-1)} - \alpha_{i(i+1)}) - 2(\alpha_{i(i-2)} - \alpha_{i(i+2)})\cdots$$

$$\{DL_g J\}_i = \sum_{j=1}^{n} j(\alpha_{ij} - \alpha_{(i+1)j}) - \sum_{j=1}^{n} i\alpha_{ij} - (i+1)\alpha_{(i+1)j} = 0 \quad (*)$$

$$L_g = \begin{bmatrix} -1 & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & -1 & \cdots & \alpha_{2n} \\ & & \ddots & \\ \alpha_{n1} & \alpha_{n2} & \cdots & -1 \end{bmatrix}$$

Corollary 1

If L_g satisfies the following conditions:

There exists some i such that

- $\alpha_{ij} = 0$ *with* $i < j$
- $\alpha_{ij} = \alpha_{(i+1)(j+1)}$, $\alpha_{in} = \alpha_{(i+1)1} = 0$.

Then $\{DL_g J\}_i = 0$.

Safety Analysis

Assume $DL_g J \neq \mathbf{0}$,

infrastructure can make the platoon safe if d satisfies

$$Dx(k+2) = D(x(k) + 2tv(k) + t(k'_p L_g x(k) + k'_v L_g v(k) + k'_p dL_g J)) > 0$$

$$\Leftrightarrow dDL_g J > -\frac{D}{k'_p t}\left(x(k) + 2tv(k) + k'_p tL_g x(k) + k'_v tL_g v(k)\right)$$

⇨ $\{DL_g J\}_i > 0$ d has an upper bound

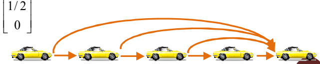
$\{DL_g J\}_i < 0$ d has a lower bound

If the upper bound exists, d is likely not to exist effective range

It is recommended to satisfy the condition $DL_g J < \mathbf{0}$

Ex)

$$DL_g J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & & & \\ 1/2 & 1/2 & -1 & & \\ 1/2 & & 1/2 & -1 & \\ 1/2 & & & 1/2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = D \begin{bmatrix} 0 \\ -1 \\ -3/2 \\ -2 \\ -5/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ 1/2 \\ 1/2 \\ 0 \end{bmatrix}$$



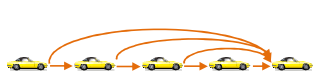
Safety Analysis

Theorem 2

If L_g satisfies the following conditions:

There exists some i such that

- $\alpha_{1j} = 0$
- $\sum_{j=1}^{n} j\alpha_{ij} = -\frac{i}{2}$



Then $DL_g J < \mathbf{0}$.

Sketch of proof

Lemma 1

If L_g satisfies the following condition:

$$\sum_{j=1}^{n} j\alpha_{1j} > \cdots > \sum_{j=1}^{n} j\alpha_{nj}$$

Then $DL_g J < \mathbf{0}$.

Proof

$$DL_g J = D \begin{bmatrix} \sum_{j=1}^{n} j\alpha_{1j} \\ \vdots \\ \sum_{j=1}^{n} j\alpha_{nj} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{n} j(\alpha_{1j} - \alpha_{2j}) \\ \vdots \\ \end{bmatrix} > \boldsymbol{0}$$

Lemma 1 ⇨ Theorem 2

Safety Analysis

Collision avoidance constraints

$Dx(k) > \mathbf{0} \quad k = 1, 2, \cdots$

Assumption $Dx(k) > \mathbf{0}$, $Dx(k+1) > \mathbf{0}$

Infrastructure can control platoon safety by state and d

Communication structure

If $\{DL_g J\}_i = 0$,

infrastructure can't make safe between ith and $(i+1)$th vehicles

⇨ Theorem 1, Corollary 1

If $DL_g J \neq \mathbf{0}$,

There exists L_g that d has an only lower bound

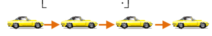
⇨ Theorem 2

Simulation

Comparative Verification

Precede following

$L_g = \begin{bmatrix} \ddots & & \\ & 1 \quad -1 & \\ & & \ddots \end{bmatrix}$



Leader and Precede following

$L_g = \begin{bmatrix} \vdots & & & \ddots & \ddots & \\ 1/2 & 0 & \cdots & 1/2 & -1 & 0 \\ 1/2 & 0 & \cdots & 0 & 1/2 & -1 \\ \vdots & & & & & \ddots \end{bmatrix}$

Simulation Settings

Parameter

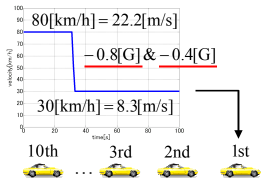
$k_p = 2, k_v = 2$

$d = 25[\text{m}]$ →Time head way

80[km/h]: 1.13[s]

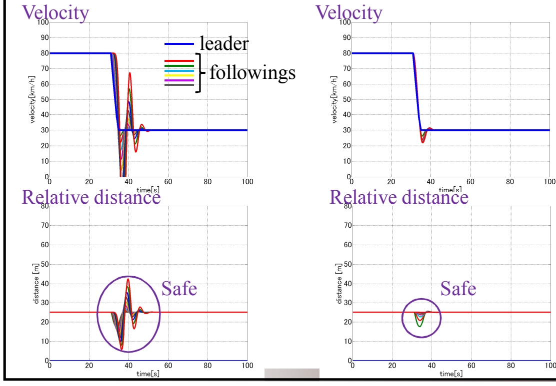
30[km/h]: 3.00[s]

Speed profile



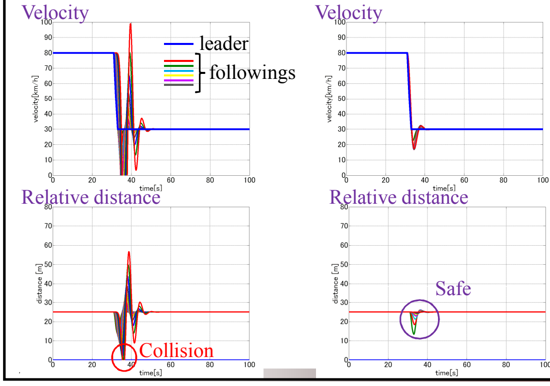
Simulation Result (-0.4[G])

Precede following — Leader and Precede following



Simulation Result (-0.8[G])

Precede following — Leader and Precede following



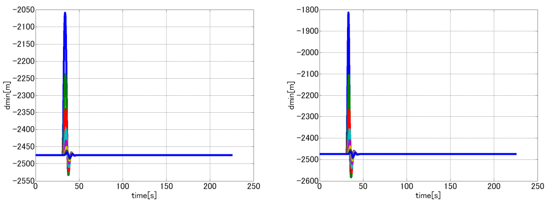
Simulation Result

Leader and Precede following

$dDL_g J > -\frac{D}{k'_p t}\left(x(k) + 2tv(k) + k'_p tL_g x(k) + k'_v tL_g v(k)\right)$

⇨ $d_{\min} DL_g J = -\frac{D}{k'_p t}\left(x(k) + 2tv(k) + k'_p tL_g x(k) + k'_v tL_g v(k)\right)$

dmin -0.4[G] — dmin -0.8[G]

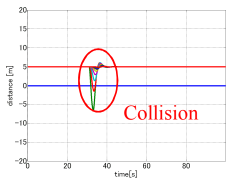


dmin has much luxury

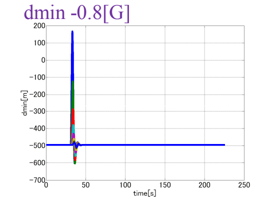
Simulation Result(-0.8[G])

$d = 25[\text{m}] \rightarrow 5[\text{m}]$

Relative distance



dmin -0.8[G]



dmin feedback vehicle control

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} I & tI \\ k'_p L_g & I + k'_v L_g \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + k'_p \begin{bmatrix} 0 \\ L_g J \end{bmatrix} d'_{\min}$$

$$d'_{\min} = \begin{cases} d_{\min} & d_{\min} > d \\ d & d_{\min} < d \end{cases}$$

Simulation Result(-0.8[G]) $d=5$[m]

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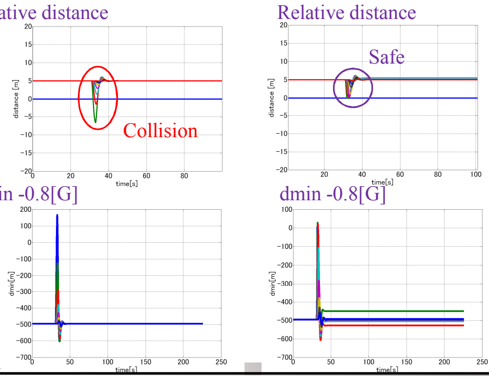
Constant d vehicle control | dmin feedback vehicle control

Relative distance | Relative distance



Summary

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Summary

- Introduction of the concept "V2V2I"
- Safety analysis of vehicle platoon under communication
  - -Communication graph that lacks of safety
  - -Communication graph that certainly exists safe inter-vehicular distance

Future works

- Relationship between string stability and safety
- Considering vehicle dynamics
- Weighted communication structure
- Optimal platoon control with safety inter-vehicular distance

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[2] X. Liu, S. S. Mahal, A. J. Goldsmith and J. K. Hedrick, "Effects of Communication Delay on String Stability in Vehicle Platoons," The IEEE 4th International Conference on Intelligent Transportation Systems, 2001
[3] G. Orosz, J. Moehlis and F. Bullo, "Robotic Reactions: Delay-induced patterns in autonomous vehicle systems," Physical Review, E811, 025204(R) pp. 1-4, 2010
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[6] G. Como, K. Savla, D. Acemoglu, M. A. Dahleh and E.Frazzoli, "On Robustness Analysis of Large-scale Transportation Networks," International Symposium on Mathematical Theory of Networks and Systems, Budapest, Hungary, 2399-2406, 2010

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Appendix

String Stability

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Definition[1]

Consider a string of N dynamic systems .the error signals $e(t)$ *depends on the disturbances* $d(t)$ *in the following manner:*

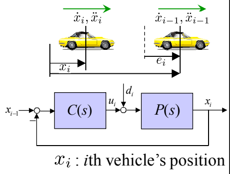
$$e(t)=H_{e,d}(s)d(t) \qquad e,d\in R^n \qquad H_{n,d}(s):R^N\to R^N \qquad (*)$$

The system (*) is L_2 *string stable if given any* $\varepsilon>0$ *there exist a* $\delta>0$ *such that*

$$\|d(\cdot)\|_2<\delta \Rightarrow \|e(\cdot)\|_2<\varepsilon$$

Assumption

- LTI SISO plant/controller
- Each loop has relative degree
- Homogeneous loop



Deformation

$$\|d(\cdot)\|_2<\delta \Rightarrow \|e(\cdot)\|_2<\varepsilon$$

$$\|G(s)\|_\infty=\sup_{d(t)}\frac{\|e(t)\|_2}{\|d(t)\|_2}[2]$$

$$\|H_{e,d}(s)\|_\infty<\gamma \qquad \gamma=\frac{\varepsilon}{\delta}$$

x_i : *i*th vehicle's position
u_i : input
d_i: disturbance
e_i : error

23

String Stability

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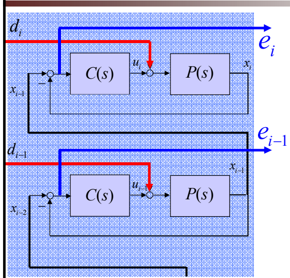


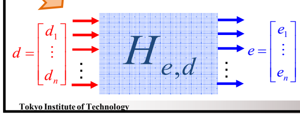
From [3],

If $\left\|\frac{e_i(s)}{e_{i-1}(s)}\right\|_\infty<1$, then $\exists\gamma>0$ such that

$$\|H_{e,d}(s)\|_\infty<\gamma, \forall N$$

sufficient condition : $\left\|\frac{e_i(s)}{e_{i-1}(s)}\right\|_\infty<1$

The perturbation doesn't propagate to following vehicles



24

String Stability of Precede Following

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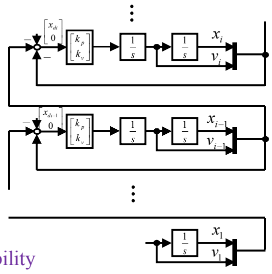
Precede following

$$L_g = \begin{bmatrix} \ddots & & \\ & 1 \quad -1 & \\ & & \ddots \end{bmatrix}$$



$$s^2 x_i = k_v s(x_{i-1} - x_i - x_{di}) + k_p(x_{i-1} - x_i - x_{di})$$

$$\Leftrightarrow \quad x_i = \frac{k_v s + k_p}{s^2 + k_v s + k_p}(x_{i-1} - x_{di})$$

Sufficiently Condition of String Stability

$$\left\| \frac{e_{xi}}{e_{x(i-1)}} \right\|_\infty = \left\| \frac{k_v s + k_p}{s^2 + k_v s + k_p} \right\|_\infty < 1$$

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String Stability of Precede Following

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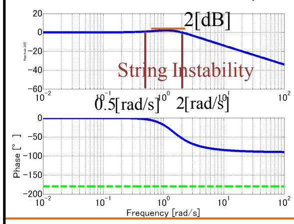
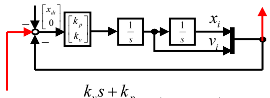
Analysis of String Stability $k_p = 2, k_v = 2$



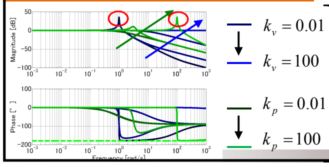


$$x_i = \frac{k_v s + k_p}{s^2 + k_v s + k_p}(x_{i-1} - x_{di})$$

kp: small

→ Neglecting position control

→ Increasing dangerousness of collision



The largest range of achieving Sting Stability

⇨ kv: Large, kp: Small $\quad \frac{k_v s + k_p}{s^2 + k_v s + k_p} \to \frac{k_v}{s + k_v}$

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String Stability of Leader and Precede Following

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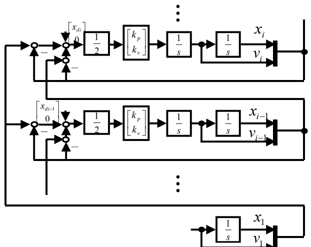
Leader and Precede following

$$L_g = \begin{bmatrix} \vdots & & \ddots & \ddots & & \\ 1 & 0 & \cdots & 1 & -1 & 0 \\ 1 & 0 & \cdots & 0 & 1 & -1 \\ \vdots & & & & & \ddots \end{bmatrix}$$



$$sx_i = \frac{1}{2}k_v s(x_{i-1} - x_i) + \frac{1}{2}k_p(x_{i-1} - x_i)$$
$$+ \frac{1}{2}k_v s(x_1 - x_i) + \frac{1}{2}k_p(x_1 - x_i) - k_p x_{di}$$

$$\Leftrightarrow \quad x_i = \frac{1}{2}\frac{k_v s + k_p}{s^2 + k_v s + k_p}x_1 + \frac{1}{2}\frac{k_v s + k_p}{s^2 + k_v s + k_p}x_{i-1} - \frac{k_v s + k_p}{s^2 + k_v s + k_p}x_{di}$$

$$x_i = \left(\sum_{j=1}^{i-2} \left(\frac{1}{2}\frac{k_v s + k_p}{s^2 + k_v s + k_p} \right)^{j-2} + \left(\frac{1}{2}\frac{k_v s + k_p}{s^2 + k_v s + k_p} \right)^{i-2} \frac{k_v s + k_p}{s^2 + k_v s + k_p} \right) x_1 - \frac{k_v s + k_p}{s^2 + k_v s + k_p}x_{di}$$

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String Stability of Leader and Precede Following

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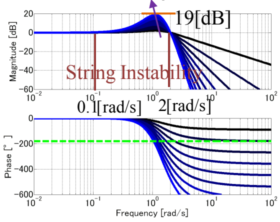
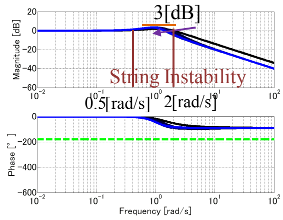
String Stability from leader vehicle

Precede following $k_p = 2, k_v = 2$ — Leader following $k_p = 2, k_v = 2$





2nd vehicle ↓ 10th vehicle

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Simulation Result(-0.8[G]) $d = 5[\mathrm{m}]$

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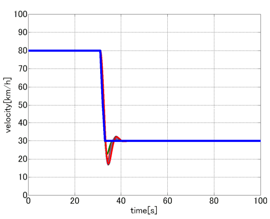
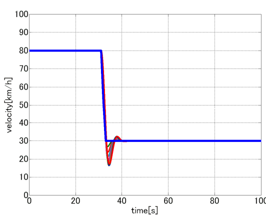
Constant d vehicle control — dmin feedback vehicle control

Velocity



Velocity



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