Safety Analysis of Vehicle Platoon under V2V2I Communication

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Introduction

Intelligent road transportation system[1]
Vehicle-to-Vehicle interaction(V2V)
Organizing the interaction among vehicles
Information is interchanged and decisions are made on a “local” basis

Micro perspective: [2],[3]
Vehicle-to-Infrastructure interaction(V2I)
The infrastructure plays a coordination role by gathering "global" information and then suggesting certain behaviors on platoon
Macro perspective: [4],[5],[6]

Vehicle-to-Vehicle-Infrastructure(V2V2I) interaction
The hybrid system of the V2V and V2I
Vehicles are controlled by “Local” and “global” information

Middle perspective?

Problem Description

Intelligent vehicle(V4)[4]
A vehicle equipped with control systems that can sense the environment around the vehicles and that result in a more efficient vehicle operation by assisting the driver or by taking complete control of the vehicle

Considering autonomous vehicle with communication facility

Situation
- Platoon of Homogeneous IV in an one lane highway
- Infrastructure that controls vehicle platoon
- Each vehicles can communicate any others in platoon to reference their information
- It doesn’t matter whether communicating or not
- Communication is time invariant

Macro perspective

Middle perspective?

Purpose of Research

Middle perspective
Optimal vehicle control of platoon under V2V2I communication → need to consider collision avoidance constraints

Lack of safety analysis
Micro perspective
Analysis of String Stability is usually interpreted in frequency domain. It is difficult to consider safety which belongs to time domain
Macro perspective
Don’t consider a vehicle unit, but a platoon unit or traffic flow
Making an assumption of collision avoidance

Objective
- Safety analysis of vehicle platoon under V2V2I communication
- Simulation (Analysis of String Stability)
- (Optimal Control of vehicle platoon)

Modeling

Vehicle platoon model

x(k+1) = [I dt] x(k) + [0]

v(k+1) = [0 1] v(k) + [0]

a(k) = [0 1] a(k)

Input

ith vehicle’s input

u_i(k) = k_i r_i(k) + k_i s_i(k)

e_xi(k) = \sum_j a_j (x_j(k) - x_i(k)) + s_x

e_v(k) = \sum_j a_j (v_j(k) - v_i(k)) + s_v

Assumption

\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} a_i = \cdots = \sum_{i=1}^{n} a_i
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Modeling

There exists some $L_k$ such that

\[ e_i(k) = L_k x_i(k) + x_{i_j}(k) + x_{i_j}(k) \]

\[ \sum_{i,j} e_i(k) = 0 \]

State equation

\[ \begin{bmatrix} \dot{x}(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} I & J \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} 0 \\ L_k J \end{bmatrix} \]

Reference relative position

Time-invariant constant inter-vehicular distance

\[ x_k = dL_k J = \{1, 2, 3, \ldots, L_k \} = \sum j \alpha_j \]

Safety Analysis

Collision avoidance constraints

\[ D_x(k) \geq 0 \quad k = 1, 2, \ldots \]

Respecting step $k$, Assuming $D_x(k) \geq 0$

\[ \begin{bmatrix} \alpha_i(k) & \alpha_i(k) & \alpha_i(k) \end{bmatrix} \begin{bmatrix} \alpha_i(k) & \alpha_i(k) & \alpha_i(k) \end{bmatrix} \]

State term

\[ \begin{bmatrix} dL_k J \end{bmatrix} \geq 0 \quad \text{distance term} \]

\[ D_x(k+1) \]

Assuming $D_x(k+1) > 0$, infrastructure can control collision performance of $D_x(k+2)$ if $D_{L_k J} \not= 0$

Need to check a condition of $L_k$ that satisfy

For some $i$, $\{D_{L_k J} \} = 0$

\[ \begin{bmatrix} dL_k J \end{bmatrix} = 0 \]

\[ \begin{bmatrix} \alpha_i(k) & \alpha_i(k) & \alpha_i(k) \end{bmatrix} \]

Theorem 1

If $L_k$ satisfies the following condition:

There exists some $i$ such that

\[ \alpha_i(k) - \alpha_{i_j}(k) - \alpha_{i_j}(k) - \alpha_{i_j}(k) \]

with $\alpha_i = 0, j < 0$

Then $\{D_{L_k J} \} = 0$

Sketch of Proof

\[ \{D_{L_k J} \} = \sum \alpha_i - \sum \alpha_i - \sum \alpha_i = \alpha_i \]

\[ \{D_{L_k J} \} = 0 \]

Corollary 1

If $L_k$ satisfies the following conditions:

There exists some $i$ such that

\[ \alpha_i = 0 \]

\[ \alpha_i = \alpha_{i_j} \]

Then $\{D_{L_k J} \} = 0$

\[ \sum \alpha_i = 0 \]

Safety Analysis

Assume $D_{L_k J} \not= 0$

If $L_k$ satisfies the following conditions:

There exists some $i$ such that

\[ \alpha_i = 0 \]

\[ \sum \alpha_i = 0 \]

Then $D_{L_k J} < 0$

Sketch of proof

Lemma 1

If $L_k$ satisfies the following condition:

\[ \sum \alpha_i > \sum \alpha_i \]

Then $D_{L_k J} < 0$

Proof

\[ \sum \alpha_i - \sum \alpha_i > 0 \]
Safety Analysis

Collision avoidance constraints

\[ Dx(k) > 0 \quad k = 1, 2, \cdots \]

Assumption \( Dx(k) > 0, \quad D(k+1) > 0 \)

Infrastructure can control platoon safety by state and \( d \)

Communication structure

If \( \{ DL_i J_i \} = 0 \),
infrastructure can’t make safe between \( i \)th and \((i+1)\)th vehicles

\[ \text{Theorem 1, Corollary 1} \]

If \( DLJ_i \neq 0 \),

There exists \( L_i \) that \( d \) has an only lower bound

\[ \text{Theorem 2} \]

Simulation

Comparative Verification

Precede following

\[ L_i = \begin{bmatrix} 1 & -1 \\ 1/2 & 0 & 0 \end{bmatrix} \]

Leader and Precede following

\[ L_i = \begin{bmatrix} 1/2 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 0 & -1 \end{bmatrix} \]

Simulation Settings

Parameter

\[ k_v = 2, k_x = 2 \]

\[ d = 25[m] \quad \text{Time head way} \]

\[ 80[km/h]: 1.13[s] \]

\[ 30[km/h]: 3.00[s] \]

Speed profile

80[km/h] → 22.2[m/s]

30[km/h] → 8.3[m/s]

Simulation Result (-0.4[G])

Precede following Velocity

Leader and Precede following Velocity

Relative distance

Safe

Simulation Result (-0.8[G])

Precede following Velocity

Leader and Precede following Velocity

Relative distance

Collision

Simulation Result

Leader and Precede following

\[ \frac{d}{dx} DL_i J_i = - \frac{D}{Dx} \left\{ x(k) + 2v(k) + k_v^L, I, x(k) + k_v^L, I, v(k) \right\} \]

\[ \frac{d}{dx} DL_i J_i = - \frac{D}{Dx} \left\{ x(k) + 2v(k) + k_v^L, I, x(k) + k_v^L, I, v(k) \right\} \]

\[ \text{dmin} - 0.4[G] \]

\[ \text{dmin} - 0.8[G] \]

\[ \text{dmin has much luxury} \]

Simulation Result (-0.8[G])

Relative distance

Collision

dmin feedback vehicle control

\[ \begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} \begin{bmatrix} I + k_v^L, L_x \\ I + k_v^L, L_x \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} = \begin{bmatrix} 0 \\ L_x \end{bmatrix} \]

\[ \begin{bmatrix} d_{\text{min}} \end{bmatrix} \]

\[ d_{\text{min}} \]

\[ \begin{bmatrix} d_{\text{min}} \end{bmatrix} \]

\[ d_{\text{min}} > d \]

\[ d_{\text{min}} < d \]
String Stability

Definition [1]
Consider a string of N dynamic systems the error signals $\epsilon(t)$ depends on the disturbances $d(t)$ in the following manner:

$$\epsilon(t) = H_{i-1}(s)d(t) \quad e, d \in \mathbb{R}^n \quad H_{i-1}(s): \mathbb{R}^n \rightarrow \mathbb{R}^n \quad (\ast)$$

The system $(\ast)$ is L₁ string stable if for any $\epsilon > 0$ there exist $\delta > 0$ such that

$$\|H_{i-1}\|_1 < \delta \Rightarrow \|\epsilon(t)\|_1 < \epsilon$$

Assumption
- LTI SISO plant/controller
- Each loop has relative degree
- Homogeneous loop

Deformation

$$\|H_{i-1}(s)\|_1 < \gamma \quad \gamma = \frac{\epsilon}{d} \quad \epsilon > 0$$

From [3], if

$$\|d(t)\|_1 < \gamma$$

then

$$\|\epsilon(t)\|_1 < \epsilon$$

The perturbation doesn’t propagate to following vehicles.
String Stability of Precede Following

Sufficiently Condition of String Stability

Analysis of String Stability $k_p = 2, k_v = 2$

The largest range of achieving String Stability

Simulation Result (-0.8 G) $d = 5 (m)$

Constant d vehicle control

$\begin{align*}
L_p &= \begin{bmatrix} 1 & -1 \\ 0 & 1 & -1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & -1 \\
\end{bmatrix} \\
\dot{x}_i &= \begin{bmatrix} x_{i+1} - x_i \\ x_{i-1} - x_i \\
\vdots & \vdots & \ddots & \vdots \\ x_2 - x_1 \\
\end{bmatrix}
\end{align*}$

$\begin{align*}
\dot{x}_i &= \begin{bmatrix} k_p x_{i+1} + k_v x_{i-1} \\ k_p x_i + k_v x_{i-1} \\
\vdots & \vdots & \ddots & \vdots \\ k_p x_i + k_v x_2 \\
\end{bmatrix}
\end{align*}$

$\begin{align*}
x_i &= \frac{1}{2} \begin{bmatrix} k_p & 0 & \cdots & 0 \\ -k_p & k_p & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\ -k_p & \cdots & -k_p & k_p \\
\end{bmatrix} x_i + \frac{1}{2} \begin{bmatrix} k_v & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & k_v \\
\end{bmatrix} x_{i-1} + \begin{bmatrix} k_v & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & k_v \\
\end{bmatrix} x_2
\end{align*}$

$\begin{align*}
x_i &= \frac{1}{2} \begin{bmatrix} k_p & 0 & \cdots & 0 \\ -k_p & k_p & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\ -k_p & \cdots & -k_p & k_p \\
\end{bmatrix} x_i + \frac{1}{2} \begin{bmatrix} k_v & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & k_v \\
\end{bmatrix} x_{i-1} + \begin{bmatrix} k_v & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & k_v \\
\end{bmatrix} x_2
\end{align*}$

String Stability from leader vehicle

Constant $d$ vehicle control

$\begin{align*}
\text{Velocity} \\
\text{Velocity}
\end{align*}$

$\begin{align*}
\text{Velocity} \\
\text{Velocity}
\end{align*}$

$\begin{align*}
\text{Velocity} \\
\text{Velocity}
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$\begin{align*}
\text{Velocity} \\
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\text{Velocity} \\
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