Survey of Synchronization Part II: Synchronization on $SO(3)$

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FL11-14-1

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Synchronization on $SO(3)$

Motivation

- Analysis of group behaviors such as schools of fish, flocking of birds, etc…
- Application to mobile sensor networks, formation control, etc…

Related Works

[1] T. R. Smith, H. Hanssmann and N. E. Leonard, "Orientation Control of Multiple Underwater Vehicles with Symmetry-breaking Potentials," *Proc. of the 40th IEEE CDC*, pp. 4598-4603, 2001.
[2] S. Nair and N. E. Leonard, "Stable Synchronization of Rigid Body Networks," Networks and Heterogeneous Media, Vol. 2, No. 4, pp. 595-624, 2007.

Energy shaping ($\phi(R)$) for two agents [1] or multiple agents (string) [2]
Partial Stability: stability for the shortest axis

[3] W. Ren, "Distributed Attitude Consensus among Multiple Networked Spacecraft," *Proc. of the 2006 ACC*, pp. 1760-1765, 2006.

Extensions of the results of [1,2]: not for the shortest axis

[4] L. Scardovi, N. E. Leonard and R. Sepulchre, "Stabilization of Collective Motion in Three Dimensions: A Consensus Approach," *Proc. of the 46th IEEE CDC*, pp. 2931-2936, 2007.

3D ver. of identical steered particles (all-to-all communication)

[5] A. Sarlette, R. Sepulchre and N. E. Leonard, "Cooperative Attitude Synchronization in Satellite Swarms: A Consensus Approach," *Proc. of the 17th IFAC Symp. Automatic Control in Aero Space*, 2007.
[6] A. Sarlette, R. Sepulchre and N. E. Leonard, "Autonomous Rigid Body Attitude Synchronization," *Automatica*, Vol. 45, No. 2, pp. 572-577, 2009.

Extensions of the results of [1,2,3]

[(1,2,3),5,6] ➡ Appendix

Consensus Approach:
bidirectional, connected → directed, time-varying (almost global stability)

H. Bai *et al.*

Related Works

[7] H. Bai, M. Arcak and J. T. Wen, "A Decentralized Design for Group Alignment and Synchronous Rotation without Inertial Frame Information," *Proc. of the 46th IEEE CDC*, pp. 2552-2557, 2007.
[8] H. Bai, M. Arcak and J. T. Wen, "Rigid Body Attitude Coordination without Inertial Frame Information," *Automatica*, Vol. 44, No. 3, pp. 3170-3175, 2008.
[9] H. Bai, M. Arcak and J. T. Wen, "Adaptive Motion Coordination: Using Relative Velocity Feedback to Track a Reference Velocity," *Automatica*, Vol. 45, No. 4, pp. 1020-1025, 2009.

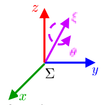
[10] H. Bai, M. Arcak and J. T. Wen, *Cooperative Control Design: A Systematic, Passivity-based Approach*, Springer, 2011

(Chapter 5)

In this seminar,

introduce results of [10] in detail, along with our results, because [10] is much used as reference for our future works

Representations of Attitude on $SO(3)$



Euler Angles (Rotations about the Rotated Coordinate Axes)

$$R = R_{z,\phi}R_{y,\theta}R_{z,\psi} \quad R = \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}$$

Roll, Pitch, Yaw Angles (Rotations about the Principal Coordinate Axes)

$$R = R_{z,\phi}R_{y,\theta}R_{x,\psi} \quad R = \begin{bmatrix} c_\phi c_\theta & -s_\phi c_\psi + c_\phi s_\theta s_\psi & s_\phi s_\psi + c_\phi s_\theta c_\psi \\ s_\phi c_\theta & c_\phi c_\psi + s_\phi s_\theta s_\psi & -c_\phi s_\psi + s_\phi s_\theta c_\psi \\ -s_\theta c_\psi & c_\theta s_\psi & c_\theta c_\psi \end{bmatrix}$$

Exponential Coordination

$$R = e^{\hat{\xi}\theta} \qquad \xi \in \mathcal{R}^3 : \text{Axis} \ (\|\xi\| = 1), \quad \theta \in \mathcal{R} : \text{Angle} \ (|\theta| < \pi) \qquad (v_\theta = 1 - \cos\theta)$$

$$e^{\hat{\xi}\theta} = \begin{bmatrix} \xi_x^2 v_\theta + c_\theta & \xi_x\xi_y v_\theta - \xi_z s_\theta & \xi_x\xi_z v_\theta + \xi_y s_\theta \\ \xi_x\xi_y v_\theta + \xi_z s_\theta & \xi_y^2 v_\theta + c_\theta & \xi_y\xi_z v_\theta - \xi_x s_\theta \\ \xi_x\xi_z v_\theta - \xi_y s_\theta & \xi_y\xi_z v_\theta + \xi_x s_\theta & \xi_z^2 v_\theta + c_\theta \end{bmatrix}$$

$$e^{\hat{\xi}\theta} = I_3 + \hat{\xi}s_\theta + \hat{\xi}^2(1 - c_\theta)$$

$$\xi = \frac{1}{2s_\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

$$\theta = \cos^{-1}\left(\frac{\mathrm{tr}(R) - 1}{2}\right)$$

$$(\mathrm{tr}(R) = 1 + 2\cos\theta)$$

Quaternion (4 parameters for no singularity)

$$q = \begin{bmatrix} q_0 \\ q_v \end{bmatrix} \in \mathcal{R}^4 \quad q_0 \in \mathcal{R}, \ q_v \in \mathcal{R}^3 \qquad (\text{like } \theta \in \mathcal{R}, \ \xi \in \mathcal{R}^3)$$

Unit Quaternion

Unit Quaternion

$$q = \begin{bmatrix} q_0 \\ q_v \end{bmatrix} \in \mathcal{R}^4 \qquad \|q\|_2 = 1$$

$$R = e^{\hat{\xi}\theta} \ ➡ \ q_0 = \cos(\theta/2), \ q_v = \xi\sin(\theta/2) \qquad (|\theta| < \pi)$$

$$(q_0, q_v) \ ➡ \ \theta = 2\cos^{-1} q_0, \ \xi = \begin{cases} \frac{q_v}{\sin(\theta/2)} & \theta \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Rodriguez Formula

$$R = I_3 + 2q_0\hat{q}_v + 2\hat{q}_v^2 \iff e^{\hat{\xi}\theta} = I_3 + \hat{\xi}s_\theta + \hat{\xi}^2(1 - c_\theta)$$

Quaternion Product $\quad p, q$: quaternion

$$p \circ q = \begin{bmatrix} q_0 p_0 - q_v^T p_v \\ \hat{p}_v q_v + p_0 q_v + q_0 p_v \end{bmatrix} \iff R_p R_q$$

Kinematics

$$\frac{dq_0}{dt} = -\frac{1}{2}(\omega^b)^T q_v, \ \frac{dq_v}{dt} = \frac{1}{2}q_0\omega^b + \frac{1}{2}\hat{\omega}^b q_v \iff \frac{dR}{dt} = \hat{\omega}^b R$$

Passivity of Unit Quaternion

Kinematics

$$\frac{dq_0}{dt} = -\frac{1}{2}(\omega^b)^T q_v, \ \frac{dq_v}{dt} = \frac{1}{2}q_0\omega^b + \frac{1}{2}\hat{\omega}^b q_v \iff \frac{dR}{dt} = \hat{\omega}^b R$$

The unit quaternion kinematics is *passive* from ω^b to q_v

Proof: $V_q = (q_0 - 1)^2 + q_v^T q_v \geq 0$

$$➡ \ \dot{V}_q = 2(q_0 - 1)\dot{q}_0 + 2q_v^T\dot{q}_v = -(q_0 - 1)(\omega^b)^T q_v + q_v^T(q_0\omega^b + \hat{\omega}^b q_v) = (\omega^b)^T q_v$$

$$q_v = \xi\sin(\theta/2), \ V_q = 2(1 - \cos(\theta/2)) = \mathrm{tr}(I_3 - R(\theta/2))$$

➡ $\dot{e}^{\hat{\xi}\theta} = \hat{\omega}^b e^{\hat{\xi}\theta}$ is *passive* from ω^b to $\xi\sin\theta$

$$V_e = \frac{1}{2}\mathrm{tr}(I_3 - e^{\hat{\xi}\theta})$$

Preliminary: Incidence Matrix

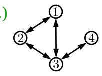
Graph Laplacian

$L = D - A$

D : degree matrix

A : adjacency matrix

Ex.)



$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

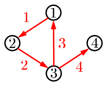
Incidence Matrix $B \in \mathcal{R}^{N \times e}$

for Oriented Bidirectional Graph

with N Vertices and M Links

$$B_{ik} = \begin{cases} 1 & \text{if link } k \text{ is incoming to vertex } i \quad (k \in \mathcal{N}_i^+) \\ -1 & \text{if link } k \text{ is outcoming from vertex } i \quad (k \in \mathcal{N}_i^-) \\ 0 & \text{otherwise} \end{cases}$$

Ex.)



Oriented Bidirectional Graph

$$B = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (N \times M) \quad \Rightarrow \quad \boxed{BB^T = L}$$

We consider only bidirectional graphs and Incidence Matrix B

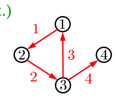
Kinematics of the Attitude Error

Relative Attitude for Link k

$$\tilde{R}^k = R_i^T R_j \quad \text{if } k \in \mathcal{N}_i^+ \text{ and } k \in \mathcal{N}_j^-$$

Ex.) $\tilde{R}^1 = R_2^T R_1, \ \tilde{R}^2 = R_3^T R_2$, etc..

Ex.)



Relative Angular Velocity for Link k

$$\tilde{\omega}^k = \omega_{ij}^b \quad \text{if } k \in \mathcal{N}_i^+ \text{ and } k \in \mathcal{N}_j^-$$

Ex.) $\tilde{\omega}^1 = \omega_{21}^b$, etc..

Relative Angular Velocities for All Links

$\omega^b = \begin{bmatrix} \omega_1^b \\ \vdots \\ \omega_M^b \end{bmatrix}$

$$\tilde{\omega} = \bar{B}^T \omega^b \ (1) \quad \bar{B}_{ik} = \begin{cases} -I_3 & k \in \mathcal{N}_i^+ \\ (\tilde{R}^k)^T & k \in \mathcal{N}_i^- \\ 0 & \text{otherwise} \end{cases}$$

Ex.) $\tilde{\omega}^1 = \omega_{21}^b = R_{21}\omega_1^b - \omega_2^b$

Kinematics of Relative Attitudes

$$\frac{d\tilde{R}^k}{dt} = (\tilde{\omega}^k)^\wedge \tilde{R}^k$$

Dynamics of the Attitude

$$\tilde{R}^k \Rightarrow q^k = \begin{bmatrix} q_0^k \\ q_v^k \end{bmatrix} \qquad \tilde{R}^k = I_3 + 2q_0^k \hat{q}_v^k + 2(\hat{q}_v^k)^2$$

Kinematics of Relative Attitudes (Quaternion)

$$\frac{dq_0^k}{dt} = -\frac{1}{2}(\tilde{\omega}^k)^T q_v^k, \ (2) \quad \frac{dq_v^k}{dt} = \frac{1}{2} q_0^k \tilde{\omega}^k + \frac{1}{2}(\tilde{\omega}^k)^\wedge q_v^k \ (3) \quad \Leftrightarrow \quad \frac{d\tilde{R}^k}{dt} = (\tilde{\omega}^k)^\wedge \tilde{R}^k$$

Dynamics of Attitudes

$$I_i \dot{\omega}_i^b + \hat{\omega}_i^b I_i \omega_i^b = \tau_i \quad (4)$$

$I_i \in \mathcal{R}^{3\times 3}$: Inertia Matrix

$\tau_i \in \mathcal{R}^3$: Torque Input

Objective: Attitude Synchronization

(i) $\lim_{t\to\infty} R_i^T R_j = I_3 \ \forall i, j \in \{1, \cdots, N\}$

(ii) $\lim_{t\to\infty} \|\omega_i^b - \omega^d(t)\|_2 = 0 \ \forall i \in \{1, \cdots, N\}$

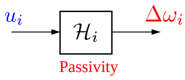
$\omega^d(t) \in \mathcal{R}^3$: Desired Velocity

(i) $R_i^T R_j = I_3 \Leftrightarrow q_v^k = 0, \ q_0^k = \sqrt{1 - \|q_v^k\|^2} = 1$

Passivity of Attitude Dynamics

Angular Velocity Error

$$\Delta\omega_i := \omega_i^b - \omega^d$$



Torque Input

$$\tau_i = I_i \dot{\omega}^d + \hat{\omega}^d I_i \omega_i^b - f_i \Delta\omega_i + u_i \quad (5) \qquad (f_i > 0)$$

Velocity Feedback ($-f_i \Delta\omega_i$), External Feedback for Attitude Synch. (u_i)

Error Dynamics (5) → (4)

$$\mathcal{H}_i : I_i \Delta\dot{\omega}_i + (\Delta\omega_i)^\wedge I_i \omega_i^b = -f_i \Delta\omega_i + u_i \quad (6)$$

Assumption

ω^d: known

(6) is *strictly passive* from u_i to $\Delta\omega_i$

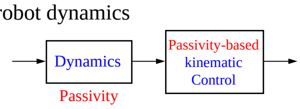
Proof: $V_i = \frac{1}{2}(\Delta\omega_i)^T I_i \Delta\omega_i \geq 0$: Kinetic Energy

$$\Rightarrow \dot{V}_i = (\Delta\omega_i)^T I_i \Delta\dot{\omega}_i$$
$$= -\underbrace{(\Delta\omega_i)^T (\Delta\omega_i)^\wedge I_i \omega_i^b}_{=0} - (\Delta\omega_i)^T f_i \Delta\omega_i + (\Delta\omega_i)^T u_i$$
$$= \underbrace{-f_i \|\Delta\omega_i\|_2^2}_{\text{negative definite}} + u_i^T \Delta\omega_i$$

Comment (1): Dynamics

[11]: We use a similar approach for robot dynamics

[10]: Bidirectional Connected Graph



In [10],

"The work in [12] considered kinematic control of attitude synchronization. Since the agent kinematics are relative degree one, the attitude synchronization can be achieved with strongly connected graphs."

Perhaps, it is because of the choice of the Lyapunov function candidate (individual energy) based on [13] (discuss afterward)

We probably can include dynamics by simply adding the kinetic energy, but agents may have to use their own velocities (Future Work 1)

[11] M. Fujita, H. Kawai and M. W. Spong, *IEEE Trans. on Control System Technology*, Vol. 15, No. 1, pp. 40-52, 2007.

[12] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in *SE*(3)," *IEEE Trans. on Control System Technology*, Vol. 17, No. 5, pp. 1119-1134, 2009.

[13] N. Chopra and M. W. Spong, "Passivity-based Control of Multi-agent Systems," in *Advances in*

Attitude Synchronization Law

Attitude Synchronization Law

$$u_i = \sum_{l \in \mathcal{N}_i^+} q_v^l - \sum_{p \in \mathcal{N}_i^-} q_v^p \quad (7) \quad \Rightarrow \quad u = (B \otimes I_3) q_v \quad (8)$$

$u = \begin{bmatrix} u_1 \\ \vdots \\ u_M \end{bmatrix}$ $q_v = \begin{bmatrix} q_v^1 \\ \vdots \\ q_v^M \end{bmatrix}$

Difference from [12]

(7): $u_i = \sum \xi_{ij} \sin(\theta_{ij}/2) + \sum \xi_{ij} \sin(\theta_{ij}/2)$

j: agents for incoming links $\quad j$: agents for outcoming links

(7) uses the some control scheme as our works but,…

$R : (q_0, q_v)$ $\Leftrightarrow$ $R^T : (q_0, -q_v)$

[12]: $u_i = \sum_{j \in \mathcal{N}_i} \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee = \sum_{j \in \mathcal{N}_i} \xi_{ij} \sin\theta_{ij}$

(7) uses a half of the rotation angle: $\theta_{ij}/2$

In [10], there is no $\pi/2$ problem

[12] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in *SE*(3)," *IEEE Trans. on Control System Technology*, Vol. 17, No. 5, pp. 1119-1134, 2009.

Comment (2): $\pi/2$ Problem

In [10], there is no $\pi/2$ problem, probably because it considers only relative dynamics and Lyapunov functions based on relative attitudes

[10] proves synchronization by using only error energies and vector calc.

In [12], however, we use individual energies and matrix calc. such as

$$-\mathrm{tr}\left((e^{\hat{\xi}\theta_i}+e^{-\hat{\xi}\theta_i})(I_3-e^{-\hat{\xi}\theta_i}e^{\hat{\xi}\theta_j})\right) \le -\lambda_{min}(e^{\hat{\xi}\theta_i}+e^{-\hat{\xi}\theta_i})\mathrm{tr}(I_3-e^{-\hat{\xi}\theta_i}e^{\hat{\xi}\theta_j}) \quad |\theta_i|<\pi/2$$

$\phi(e^{\hat{\xi}\theta_{ij}})$ Trade off ? $\phi(e^{\hat{\xi}\theta_i})$

[10]: bidirectional, connected, no $\pi/2$ problem ⟷ [12]: directional, strongly connected, $\pi/2$ problem — Question!

Future Work 2: proof for [12] by using error energies $\phi(e^{\hat{\xi}\theta_{ij}})$ (perturbation)

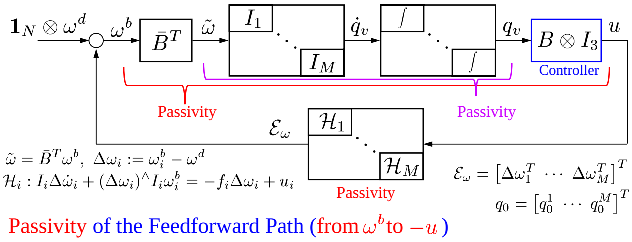
Future Work 3: proof for [12] by using unit quaternion q

On the other hand, in G. Heppeler's thesis, $\pi/2$ problem is escaped by using $\theta/2$ in 2D and [10] can prove synchronization by using $\theta/2$

Future Work 4: proof for [12] by using $\theta/2$ (quaternion?)

Difficulty: matrix calc. $e^{\hat{\xi}\theta_{ij}/2} \neq e^{-\hat{\xi}\theta_i/2}e^{\hat{\xi}\theta_j/2}$?

Closed-loop System



$\tilde{\omega} = \bar{B}^T\omega^b,\ \Delta\omega_i := \omega_i^b - \omega^d$

$\mathcal{H}_i : I_i\Delta\dot{\omega}_i + (\Delta\omega_i)^\wedge I_i\omega_i^b = -f_i\Delta\omega_i + u_i$

$\mathcal{E}_\omega = [\Delta\omega_1^T \ \cdots \ \Delta\omega_M^T]^T$

$q_0 = [q_0^1 \ \cdots \ q_0^M]^T$

Passivity of the Feedforward Path (from ω^b to $-u$)

Proof: $V_u = (q_0 - \mathbf{1}_M)^T(q_0 - \mathbf{1}_M) + q_v^T q_v \ge 0$: Attitude Error Energy

$\dot{V}_u = 2(q_0 - \mathbf{1}_M)^T\dot{q}_0 + 2q_v^T\dot{q}_v$

$= \tilde{\omega}^T q_v$ (substitute (2), (3))

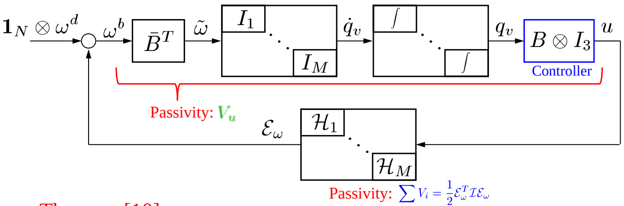
$= (\omega^b)^T\bar{B}q_v$

$= (\omega^b)^T(-(B\otimes I_3)q_v)$ (direct calc.)

$= (\omega^b)^T(-u)$ (substitute (8))

$(q_0^k-1)^T(q_0^k-1)+(q_v^k)^Tq_v^k$
$= 2(1-\cos(\theta^k/2))$
$= 2\mathrm{tr}(I_3 - e^{\hat{\xi}(\theta^k/2)})$
$= 2\phi(e^{\hat{\xi}(\theta^k/2)})$

Attitude Synchronization



Theorem [10]

If the graph is acyclic, then $q_v \to 0$ and $\mathcal{E}_\omega \to 0$ as $t \to 0$

Proof: $V = V_u + \frac{1}{2}\mathcal{E}_\omega^T\mathcal{I}\mathcal{E}_\omega$: Sum of the Energies

$\mathcal{I} = \mathrm{diag}\{I_1,\cdots,I_N\}\otimes I_3$

$\mathcal{F} = \mathrm{diag}\{f_1,\cdots,f_N\}\otimes I_3$

$\dot{V} = -(\omega^b)^T u + \mathcal{E}_\omega^T u - \mathcal{E}_\omega^T\mathcal{F}\mathcal{E}_\omega$

$= -(\mathbf{1}_N\otimes\omega^d)^T u - \mathcal{E}_\omega^T\mathcal{F}\mathcal{E}_\omega$

$= -(\mathbf{1}_N\otimes\omega^d)^T(B\otimes I_3)q_v - \mathcal{E}_\omega^T\mathcal{F}\mathcal{E}_\omega$

$= -\mathcal{E}_\omega^T\mathcal{F}\mathcal{E}_\omega \le 0$

=0

B : sum of column elements is zero

Comment (3): Proof Techniques

Theorem [10]

If the graph is acyclic, then $q_v \to 0$ and $\mathcal{E}_\omega \to 0$ as $t \to 0$

Proof: $\dot{V} = -\mathcal{E}_\omega^T\mathcal{F}\mathcal{E}_\omega \le 0$

- the equilibrium $\{\mathcal{E}_\omega, q_0, q_v\} = \{0, \mathbf{1}_M, 0\}$ is stable
- all signals $\{\mathcal{E}_\omega, u, q_0, q_v\}$ are bounded
- $\dot{\mathcal{E}}_\omega$ is also bounded and $\mathcal{E}_\omega^T\mathcal{F}\mathcal{E}_\omega$ is uniformly continuous
- $\mathcal{E}_\omega \to 0$, i.e. $\|\omega_i^b - \omega^d(t)\|_2 \to 0$ from Barbalat's Lemma (Appendix)

By similar methods, $u \to 0$ which means q_v converges to the null space of $B\otimes I_3$

If the graph is acyclic (B is full column rank), then $q_v \to 0$ (like nonsingular matrices)

In [10], the negative definite part is related to only the velocity error part

However, [10] proves that attitude errors converge to 0 by using Barbalat's Lemma boundation

I would like to learn to use this technique

Ex.) $\dot{U} \le -\sum\sum\lambda_{min}(e^{-\hat{\xi}\theta_i}+e^{-\hat{\xi}\theta_i})\mathrm{tr}(I_3 - e^{-\hat{\xi}\theta_i}e^{\hat{\xi}\theta_j}) \le 0$

Can we weaken?

Future Work 4: study and training of the proof technique

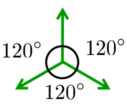
Comment (5): $\pi/2$ Problem 2

[10]: no $\pi/2$ problem ➡ Cyclic graph is not good

The null space of $B\otimes I_3$ (p. 6)

$u_i = \sum\xi_{ij}\sin(\theta_{ij}/2)$ ➡

Relative Information



Agents do not rotate

Indeed, we also find the same situation for not synchronization, but we have not found other situations yet in strongly connected graphs

If we overcome $\pi/2$ problem for directed, strongly connected graphs, we may be able to the same augment

i.e. cyclic graphs are not good or almost globally stable except for some unstable equilibria

Adaptive Design for Desired Angular Velocities

So far, the desired velocity $\omega^d(t)$ is available to each agent

➡ Only the leader possesses $\omega^d(t)$ information and the remaining agents estimate it

In Hatanaka's work [14], all agent know the same desired body velocity $\omega^d(t)$, so this work can be a good reference

Assumption:

$$\omega^d(t) = \sum_{j=1}^{r}\phi^j(t)\beta^j$$

$\phi^j(t)\in\mathcal{R}$: base functions available to each agent

$\beta^j\in\mathcal{R}^3$: column vectors available only to the leader

That is, each agent does not know the axis of the desired angular velocity but know the value like an absolute one

Estimate of Unknown β^j : $\bar{\beta}^j$

Estimate of the Desired Velocity

$$\bar{\omega}_i^d(t) = \sum_{j=1}^{r}\phi^j(t)\bar{\beta}_i^j = (\Phi(t)^T\otimes I_3)\bar{\beta}_i$$

known: $\Phi(t) = \begin{bmatrix}\phi^1(t)\\ \vdots \\ \phi^r(t)\end{bmatrix}$ estimate: $\bar{\beta}_i = \begin{bmatrix}\bar{\beta}_i^1\\ \vdots \\ \bar{\beta}_i^r\end{bmatrix}$

[14] T. Hatanaka, "3D上の群れ問題再考," *Technical Report*, 2011.

Update Law and Comment (4): Velocity Observer

Modified Torque Input

$\tau_1 = I_1\dot{\omega}^d + \hat{\omega}^d I_1 \omega_1^b - f_1 \Delta\omega_1 + u_1$ (9) (the same as (5))

$\tau_i = I_i \dot{\bar{\omega}}_i^d + \hat{\bar{\omega}}_i^d I_i \omega_i^b - f_i \Delta\bar{\omega}_i + u_i$ (10)

Velocity Error

$\Delta\bar{\omega}_i = \omega_i - \bar{\omega}_i^d$

Attitude Synchronization Law: the Same

$$u_i = \sum_{l\in\mathcal{N}_i^+} q_v^l - \sum_{p\in\mathcal{N}_i^-} q_v^p \quad (7)$$

Update Law for the Parameter $\bar{\beta}_i$

$\dot{\bar{\beta}}_i = K_i(\Phi(t)\otimes I_3)u_i$ (u_i: sum of attitude errors)

$K_i = K_i^T > 0$: gain matrix

➡ $\Delta\bar{\omega}_i = \omega_i - \bar{\omega}_i^d$ ($\bar{\omega}_i^d$: $\int u_i d\tau$)

$$\bar{\omega}_i^d(t) = \sum_{j=1}^{r} \phi^j(t)\bar{\beta}_i^j = (\Phi(t)^T \otimes I_3)\bar{\beta}_i$$

"P Control" → "PI Control" : similar to Hatanaka's work [15]

Future Work 5: apply the scheme to our approach (only the leader knows ω^d)

Future Work 6: visual feedback attitude synch. with velocity observers

[15] T. Hatanaka and M. Fujita, "Cooperative Estimation of 3D Target Object Motion via Networked Visual Motion Observers," *Proc. of the 50th CDC-ECC*, 2011. (to appear)

Passivity of the Closed-loop System

Estimate Error

$\tilde{\beta}_i = \bar{\beta}_i - \beta$

Estimate Error Dynamics

$\dot{\tilde{\beta}}_i = K_i(\Phi(t)\otimes I_3)u_i$ (11)

$\beta = \begin{bmatrix}\beta^1\\ \vdots \\ \beta^r\end{bmatrix}$

(11), is *passive* from $(\Phi(t)\otimes I_3)u_i$ to $\tilde{\beta}_i$

$V_{\beta i} = \frac{1}{2}\tilde{\beta}_i^T K_i^{-1}\tilde{\beta}_i$

➡ $\dot{V}_{\beta i} = \tilde{\beta}_i^T(\Phi(t)\otimes I_3)u_i$

Error Dynamics (9), (10) → (3)

$I_1\Delta\dot{\omega}_1 + (\Delta\omega_1)^\wedge I_1 \omega_1^b = -f_1\Delta\omega_1 + u_1$ (12)

$I_i\Delta\dot{\bar{\omega}}_i + (\Delta\bar{\omega}_i)^\wedge I_i \omega_i^b = -f_i\Delta\bar{\omega}_i + u_i$ (13)

$\mathcal{E}'_\omega = \begin{bmatrix}\Delta\omega_1\\ \Delta\bar{\omega}_2\\ \vdots \\ \Delta\bar{\omega}_M\end{bmatrix}$

(12), (13) is *strictly passive* from u to $\mathcal{E}'_\omega$

$V_\omega = \frac{1}{2}(\mathcal{E}'_\omega)^T \mathcal{I}\mathcal{E}'_\omega$ ➡ $\dot{V}_\omega = -(\mathcal{E}'_\omega)^T\mathcal{F}\mathcal{E}'_\omega + (\mathcal{E}'_\omega)^T u$

Control schemes are all based on passivity properties

➡ Lyapunov Function Candidate

$V_a = V_u + V_\omega + V_\beta$ (attitude, velocity, estimate errors)

$$V_\beta = \sum_{i=2}^{N} V_{\beta i}$$

Attitude Synchronization with Adaptive Design

Theorem [10]

If the graph is acyclic, then $q_v \to 0$ and $\omega_i^b \to \omega^d$ as $t \to 0$

Proof: $V_a = V_u + V_\omega + V_\beta$

$$\text{➡ } \dot{V}_a = -(\omega^b)^T u + (\mathcal{E}'_\omega)^T u + \sum_{i=2}^{N}\tilde{\beta}_i^T(\Phi(t)\otimes I_3)u_i - (\mathcal{E}'_\omega)^T\mathcal{F}\mathcal{E}'_\omega$$

=0 (similar to Theorem 1)

$(\Phi(t)^T\otimes I_3)\tilde{\beta}_i = \bar{\omega}_i^d - \omega^d$

$$= -(\mathcal{E}'_\omega)^T\mathcal{F}\mathcal{E}'_\omega \le 0$$

The same approach as the proof of theorem 1 (Barbalat's Lemma)

➡ $\mathcal{E}'_\omega \to 0,$ i.e. $\Delta\omega_1 \to 0,\ \Delta\bar{\omega}_i \to 0$

If the graph is acyclic (B is full column rank), then $q_v \to 0$

Moreover, Barbalat's Lemma gives $\dot{q}_v \to 0$

➡ $\tilde{\omega} \to 0$ from (2), (3)

➡ ω^b converges to the null space of $B^T \otimes I_3$ from (1)

➡ $\omega_i^b \to \omega_1^b, \omega_1^b \to \omega^d$ i.e. $\|\omega_i^b - \omega^d\|_2 \to 0$

$\frac{dq_0^k}{dt} = -\frac{1}{2}(\tilde{\omega}^k)^T q_v^k$ (2)

$\frac{dq_v^k}{dt} = \frac{1}{2}q_0^k\tilde{\omega}^k + \frac{1}{2}(\tilde{\omega}^k)^\wedge q_v^k$ (3)

$\tilde{\omega} = \bar{B}^T\omega^b$ (1)

Summary of the Results of [9]

- Graph: bidirectional, acyclic, connected

  If the graph is cyclic, null space of $B\otimes I_3$ is not 0 ✕ $q_v \to 0$

- Control Law:

  $u_i = \sum \xi_{ij}\sin(\theta_{ij}/2) - \sum \xi_{ij}\sin(\theta_{ij}/2)$ (7)

  j: agents for incoming links, j: agents for outcoming links

- No $\pi/2$ Problem:

  perhaps because Lyapunov function is based on relative attitude

- Proof Technique: bounded, Barbalat's Lemma

- Adaptive Design for the Desired Angular Velocity

  $\omega^d(t) = \sum_{j=1}^{r}\phi^j(t)\beta^j$ $\bar{\omega}_i^d(t) = \sum_{j=1}^{r}\phi^j(t)\bar{\beta}_i^j = (\Phi(t)^T\otimes I_3)\bar{\beta}_i$ $\dot{\bar{\beta}}_i = K_i(\Phi(t)\otimes I_3)u_i$

  only the leader knows the information

  the approach is similar to Hatanaka's work

Future Works

Target Theme for the 51st CDC: Flocking (without Vision)

((1): consider dynamics)

(2): proof for [12] by using error energies $\phi(e^{\hat{\xi}\theta_{ij}})$ (perturbation)

(3): $\pi/2$ problem

Approach: $\phi(e^{\hat{\xi}\theta_{ij}})$ (perturbation), quaternion, $\xi\sin(\theta/2)$

(4): study and training of the proof techniques (bounded, Barbalat's Lemma)

is it possible to apply ?

(5): apply the adaptive design to our approach (only the leader knows ω^d)

((6): visual feedback attitude synch. with velocity observers)

(7): survey of flocking

(8): collision avoidance for flocking ([16])

Seminar Schedule

11/8: Survey of Flocking

11/22: Progress Report ?

[12] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in *SE*(3)," *IEEE Trans. on Control System Technology*, Vol. 17, No. 5, pp. 1119-1134, 2009.
[16] G. M. Atinc and D. M. Stipanovic, "Cooperative Collision-free Control of Lagrangian Multi-

Appendix

Barbalat's Lemma

Barbalat's Lemma

Let $f(t): \mathcal{R} \to \mathcal{R}$ be a uniformly continuous function on $[0, \infty)$. Suppose that $\lim_{t\to\infty} \int_0^t f(s)ds$ exists and is finite. Then

$$f(t) \to 0 \text{ as } t \to \infty.$$

[17] H. K. Khalil, *Nonlinear Systems, Third Edition*, Prentice Hall, 2002.

Theorem

Suppose $f(t,x)$ is piecewise continuous in t and locally Lipschitz in x, uniformly in t, on $[0,\infty) \times \mathcal{R}^n$. Furthermore, suppose $f(t,0)$ is uniformly bounded for all $t \geq 0$. Let $V: [0,\infty) \times \mathcal{R}^n \to \mathcal{R}$ be a continuously differentiable function such that

$$W_1(x) \leq V(t,x) \leq W_2(x)$$

$$\dot{V}(x) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x) \leq -W(x)$$

$\forall t \geq 0, \forall x \in W_1(x)$, where $W_1(x)$ and $W_2(x)$ are continuous positive definite functions and $W(x)$ is a continuous positive semidefinite function on $\mathcal{R}^n$. Then, all solutions of $\dot{x} = f(t,x)$ are bounded and satisfy

$$W(x(t)) \to 0 \text{ as } t \to \infty.$$

In the proof, $V(t,x) = V_u + \frac{1}{2}\mathcal{E}_\omega^T \mathcal{I} \mathcal{E}_\omega,\ W(x) = \mathcal{E}_\omega^T \mathcal{F} \mathcal{E}_\omega \quad (x = [q^T\ \mathcal{E}_\omega^T]^T)$

Barbalat's Lemma

Barbalat's Lemma

Let $f(t): \mathcal{R} \to \mathcal{R}$ be a uniformly continuous function on $[0, \infty)$. Suppose that $\lim_{t\to\infty} \int_0^t f(s)ds$ exists and is finite. Then

$$f(t) \to 0 \text{ as } t \to \infty.$$

Theorem [10]

Let $\xi(t): \mathcal{R} \to \mathcal{R}$ be a continuous function defined on $[0,\infty)$. If $\xi(t) \to 0$ as $t \to \infty$ and $\ddot{\xi}(t)$ is bounded, then $\dot{\xi}(t) \to 0$ as $t \to \infty$.

In the proof, $\xi(t) = \Delta\omega_i$

Since $\Delta\ddot{\omega}_i$ is continuous and uniformly bounded from (6) and $\Delta\omega_i \to 0$, we get $\Delta\dot{\omega}_i \to 0$ which implies from (6) that $u_i \to 0$

Other Works [2,3,4,5]

[5] A. Sarlette and R. Sepulchre and N. E. Leonard, "Autonomous Rigid Body Attitude Synchronization," *Automatica*, Vol. 45, No. 2, pp. 572-577, 2009.

Attitude Kinematics

$$R_k^T \frac{d}{dt} R_k = \hat{\omega}_k^b \quad (14)$$

Attitude Dynamics

$$J_k \dot{\omega}_k^b = (J_k \omega_k^b)^\wedge \omega_k^b + \tau_k \quad (15)$$

Objective: Attitude Synchronization

$$\lim_{t\to\infty} R_k^T R_j = I_3 \ {}^\forall k, j \in \{1, \cdots, N\}$$

Distance between R_k and R_j

$$d_{jk} := \sqrt{3 - \mathrm{tr}(R_k^T R_j)} \quad \left(= \sqrt{\mathrm{tr}(I_3 - R_k^T R_j)}\right)$$

Artificial Potential [1]

$$V = \frac{\sigma}{2} \sum_k \sum_{k \in \mathcal{N}_j} \mathrm{tr}(R_k^T R_j),\ \sigma < 0 \quad \Leftrightarrow \quad \frac{1}{2}\mathrm{tr}(I_3 - R_k)$$

Other Works [2,3]

Torque Input

$$\tau_k = \tau_k^{(P)} + \tau_k^{(D)} \qquad V = \frac{\sigma}{2} \sum_k \sum_{k \in \mathcal{N}_j} \mathrm{tr}(R_k^T R_j),\ \sigma < 0$$

Attitude Synchronization Law (Energy Shaping)

$$\tau_k^{(P)} = -[\mathrm{grad}_{Q_k}(V)]^\vee = -\sigma \sum_{j \in \mathcal{N}_k} (R_k^T R_j - R_j^T R_k)^\vee \quad \Leftrightarrow \quad K_k \mathrm{sk}(R_k^T R_j)^\vee$$

Total Energy

$$H = T + V \quad T = \frac{1}{2} \sum_k \omega_k^T J_k \omega_k \quad \Rightarrow \quad \frac{d}{dt} H = \sum_k \omega_k^T \tau_k^{(D)}$$

Asymptotic stability requires $\tau_k^{(D)}$ to decrease H

[2] exponential stabilizes when V contains an additional term aligning the short axis with a specific direction in inertial space (not autonomous)

[3] proposes the following torque input which deceases till $\omega_k = 0$

$$\tau_k^{(D)} = -\gamma \omega_k,\ \gamma > 0 \quad \Leftrightarrow \quad (5)$$

Sarlette, *et al.* [5]

Because of the manifold structure of *SO*(3), unlike in Euclidean spaces, the cost function could have local minima

[2,3]: fixed, bidirectional, connected graphs

⇒ Utilize the consensus algorithm which guarantees global stability

Auxiliary Variables: $Y_k \in \mathcal{R}^{3\times 3}$

Y_k : consensus ⇒ R_k tracks the projection of Y_k on *SO*(3)

Consensus Algorithm in Inertial Frame

$$\frac{d}{dt} Y_k = \beta \sum_{j \in \mathcal{N}_k} (Y_j - Y_k),\ \beta > 0$$

⇒ For the graph uniformly connected, consensus is achieved [18]

Consensus Algorithm in Body Frame: $X_k = R_k^T Y_k$

$$\frac{d}{dt} X_k = \beta \sum_{j \in \mathcal{N}_k} (R_k^T R_j X_j - X_k) - \hat{\omega}_k X_k$$

[18] L. Moreau, "Stability of Continuous-time Distributed Consensus Algorithms," *Proc. of the 43rd IEEE CDC*, pp. 3998-4003, 2004.

Sarlette, *et al.* [5]

Distance from Y_k to $R_k \in SO(3)$ in $\mathcal{R}^{3\times 3}$

$$\|Y_k - R_k\|_2^2 = \mathrm{tr}((Y_k - R_k)^T (Y_k - R_k)) = 3 + \mathrm{tr}(Y_k^T Y_k) - 2\mathrm{tr}(R_k^T Y_k)$$

Tracking Algorithm of R_k to the Projection of Y_k on *SO*(3)

$$R_k^T \dot{R}_k = -\sigma(R_k^T Y_k - Y_k^T R_k)$$ like attitude synchronization laws

Thus, [5] proposes the following attitude synchronization law

$$\tau_k = \underbrace{-\sigma(X_k - X_k^T)^\vee}_{\tau_k^{(P)}} \underbrace{- \gamma \omega_k}_{\tau_k^{(D)}} \quad (16)$$

$$\frac{d}{dt} X_k = \beta \sum_{j \in \mathcal{N}_k} (R_k^T R_j X_j - X_k) - \hat{\omega}_k X_k \quad (17)$$

Theorem [5]

For the graph uniformly connected, attitude synchronization with $\omega_k = 0\ {}^\forall k$ is almost globally stable for (14), (15) with controller (16), (17)