Tokyo Institute of Technology

Design of the Robust Controller for an Omni-directional Mobile Robot

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Introduction

Omni-Directional Mobile Robot

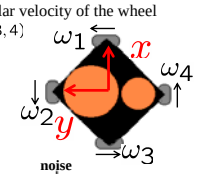
movement for x : wheel 2 and 4

movement for y : wheel 1 and 3

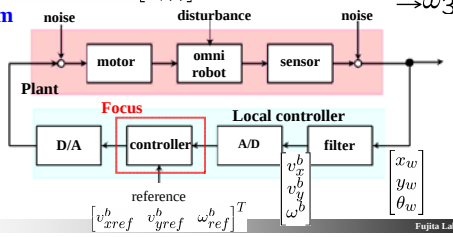
d[m] : distance between center of robot and one of the wheel
r[m] : radius of the wheel
ω_m[rad/s] : angular velocity of the wheel $(m = 1, 2, 3, 4)$

movement for diagonal direction: combination of movement for x and y

$$\begin{bmatrix} v_x^b(t) \\ v_y^b(t) \\ \omega_i^b(t) \end{bmatrix} = r \begin{bmatrix} 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ \frac{1}{4}d & \frac{1}{4}d & \frac{1}{4}d & \frac{1}{4}d \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \\ \omega_4(t) \end{bmatrix}$$



Block Diagram



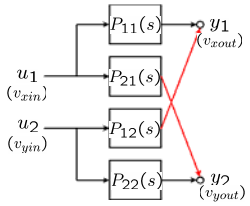
Outline

- Introduction
- Problem setting
- Identification
- Robust controller design
- Summary

Problem Setting

The Conventional Feedback Controller

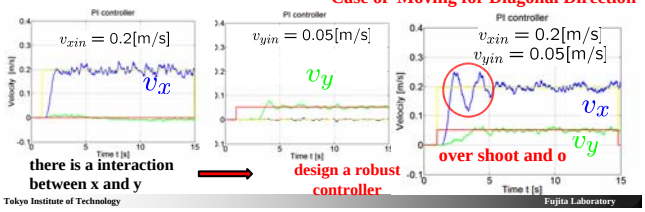
$$K_{PI}(s) = \begin{bmatrix} 0.68\frac{0.56s+1}{0.56s} & 0 \\ 0 & 0.68\frac{0.56s+1}{0.56s} \end{bmatrix}$$



PI controller :
not considered mutual interaction
In fact,
it should give a bad influence .

Step Response

Case of Moving for Diagonal Direction

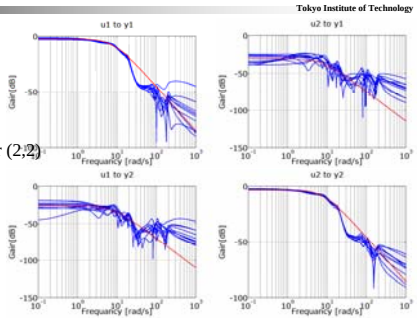


there is a interaction between x and y ⇒ **design a robust controller**

Plant Model Identification

Gain Diagram

- (1,1) similar to (2,2), and (1,2) similar to (2,1)
- the gain of (1,2) or (2,1) is smaller than that of (1,1) or (2,2)



Nominal Model (without time-delay)

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

quadratic approximation

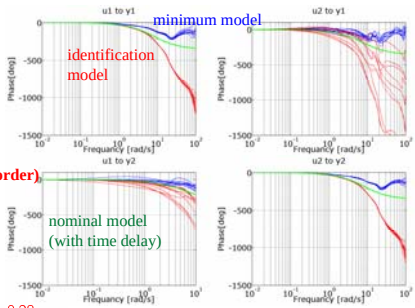
$$P_{11} = 0.75\frac{1}{\frac{1}{6}s+1}\frac{1}{\frac{1}{12}s+1} \quad P_{12} = 0.03\frac{1}{\frac{1}{6}s+1}\frac{1}{\frac{1}{10}s+1}$$

$$P_{22} = 0.05\frac{1}{\frac{1}{6}s+1}\frac{1}{\frac{1}{10}s+1} \quad P_{22} = 0.7\frac{1}{\frac{1}{6}s+1}\frac{1}{\frac{1}{12}s+1}$$

Time Delay Identification

Phase Diagram

identify the time delay from phase diagram



Time Delay : 0.28 [s]

Pade approximation (1st order)

$$\frac{-\frac{0.28}{2}s+1}{\frac{0.28}{2}s+1}$$

$$P_{11} = 0.75\frac{1}{\frac{1}{6}s+1}\frac{1}{\frac{1}{12}s+1}\frac{-\frac{0.28}{2}s+1}{\frac{0.28}{2}s+1} \quad P_{12} = 0.03\frac{1}{\frac{1}{6}s+1}\frac{1}{\frac{1}{10}s+1}\frac{-\frac{0.28}{2}s+1}{\frac{0.28}{2}s+1}$$

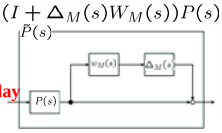
$$P_{22} = 0.05\frac{1}{\frac{1}{6}s+1}\frac{1}{\frac{1}{10}s+1}\frac{-\frac{0.28}{2}s+1}{\frac{0.28}{2}s+1} \quad P_{22} = 0.7\frac{1}{\frac{1}{6}s+1}\frac{1}{\frac{1}{12}s+1}\frac{-\frac{0.28}{2}s+1}{\frac{0.28}{2}s+1}$$

Generalized Plant

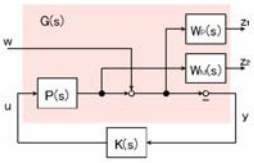
Construction of Generalized Plant

Multiplicative (Output) Uncertainty

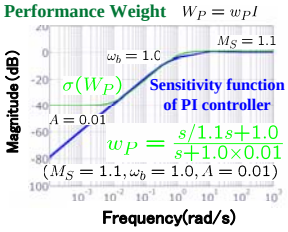
$$\tilde{P}(s) = (I + \Delta_M(s)W_M(s))P(s)$$



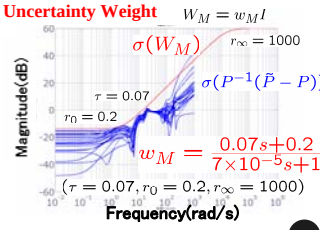
without time-delay



Performance Weight $W_P = w_P I$



Uncertainty Weight $W_M = w_M I$

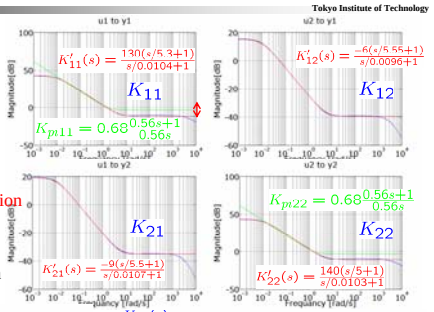


Approximation of the Robust Controller 1

γ-Iteration

$$\|F_l(G,K)\|_\infty < \gamma$$

$$\longrightarrow \quad \gamma = 2$$



Robust Controller 1

8th order (unstable zero)

incapable of implementation due to high frequency

2nd order ignore the high frequency area

$$K_{11}(s) = \frac{1139(s+1.429\times10^4)(s+1.381\times10^4)}{(s+1.371\times10^4)(s+1.384\times10^4)(s+4028)(s+3556)} \times \frac{(s+3649)(s+12)(s+10)^2(s+6)}{(s+16.2)(s+10.09)(s+9.907)(s+0.009988)}$$

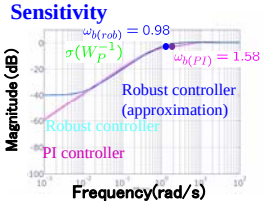
$$K_{12}(s) = \frac{13.10(s+1.567\times10^4)(s+1.429\times10^4)}{(s+1.371\times10^4)(s+1.384\times10^4)(s+4028)(s+3556)} \times \frac{(s-10^4)(s+12)(s+11.99)(s+10)(s+6)}{(s+16.2)(s+10.09)(s+9.907)(s+0.009988)}$$

$$K_{21}(s) = \frac{13.1007(s+1.429\times10^4)(s+12)^2}{(s+1.371\times10^4)(s+1.384\times10^4)(s+4028)(s+3556)} \times \frac{(s+10)(s+6)(s^2+2.945\times10^4+2.609\times10^0)}{(s+16.2)(s+10.09)(s+9.907)(s+0.009988)}$$

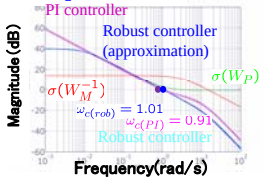
$$K_{22}(s) = \frac{1139(s+1.373\times10^4)(s+1.429\times10^4)(s+3932)}{(s+1.371\times10^4)(s+1.384\times10^4)(s+4028)(s+3556)} \times \frac{(s+12)(s+10)^2(s+6)}{(s+16.2)(s+10.09)(s+9.907)(s+0.009988)}$$

Analysis of Robust Controller 1

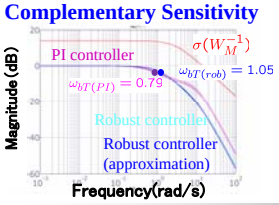
Sensitivity



Open Loop



Complementary Sensitivity

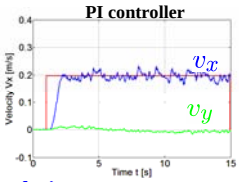


$$wP = \frac{s/1.1s+1.0}{s+1.0\times0.01}$$

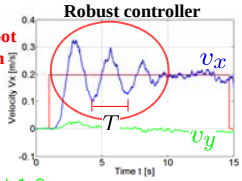
$$w_M = \frac{0.07s+0.2}{7\times10^{-5}s+1}$$

Implementation and Analysis

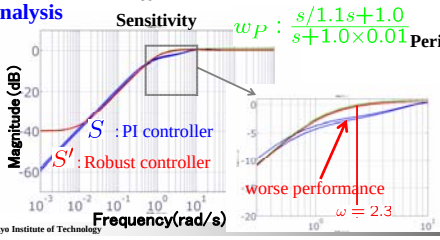
Step Response



large over shoot and oscillation



Analysis



Period of Damped Oscillati

$T = 2.71$[s]

$\omega = 2.32$[rad/s]

$(= 10^{0.365})$

need to reduce the sensitivity here

Redesign (Performance Weight)

Guide

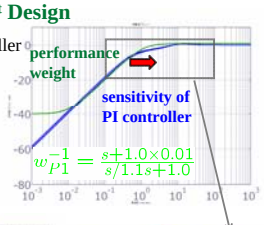
First, aim at designing comparable with PI controller

move to the performance weight to the right

larger peak

$\longrightarrow$ **Oscillate at other frequency**

1st Design



New Design

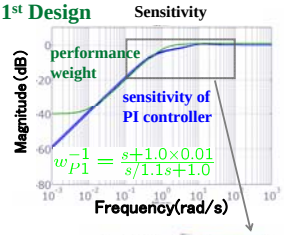
Redesign (Performance Weight)

Performance Weight

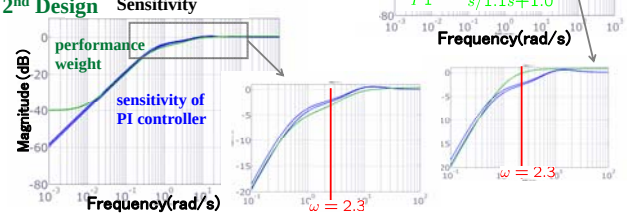
w_P : **2nd order**

$$w_{P2}^{-1} = \frac{0.01(s/0.009+1.0)(s/3+1)}{(s/0.55+1)(s/5.1+1)}$$

1st Design Sensitivity



2nd Design Sensitivity



Approximation and Sensitivity

Robust controller 2

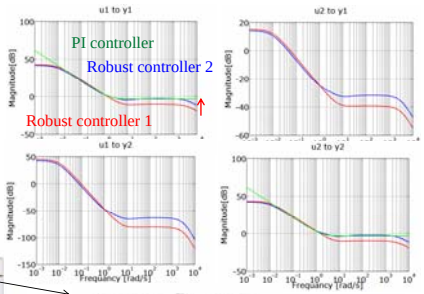
cubic approximation

$$K'_{11} = -\frac{115(s/1.5+1)(s/20+1)(s/4.4+1)}{(s/0.009+1)(s/30+1)(s/3.5+1)}$$

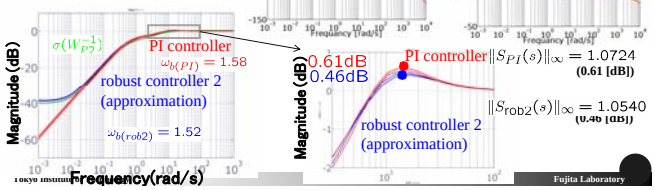
$$K'_{12} = \frac{-5(s/1.6+1)(s/25+1)(s/4.5+1)}{(s/0.009+1)(s/30+1)(s/3.5+1)}$$

$$K'_{21} = -\frac{8.5(s/1.4+1)(s/25+1)(s/5.1+1)}{(s/0.009+1)(s/30+1)(s/3.5+1)}$$

$$K'_{22} = -\frac{120(s/1.3+1)(s/25+1)(s/4.7+1)}{(s/0.009+1)(s/35+1)(s/3.5+1)}$$

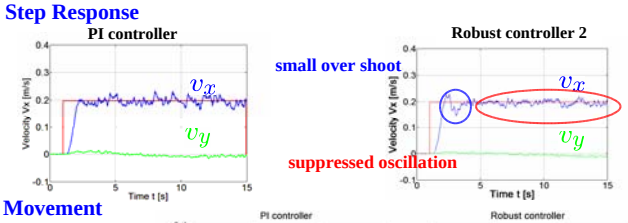


Sensitivity

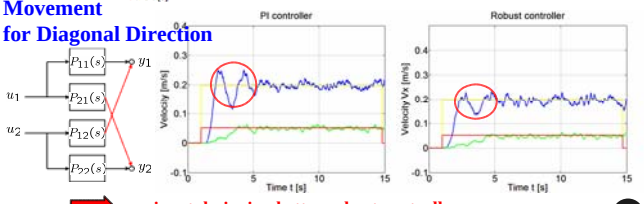


Implementation 2

Step Response

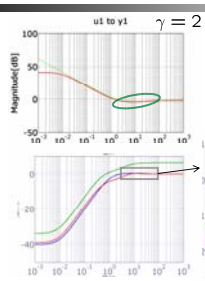


Movement for Diagonal Direction



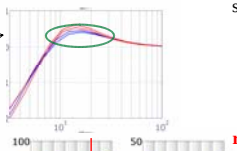
aim at designing better robust controller

Correlation of Controller with Sensitivity



neighborhood 10 [rad/s] :
- gain is lifting
- sensitivity is reducing

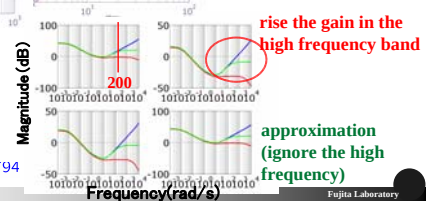
to reduce peak → make γ smaller



3rd Design

Robust controller 2
$\gamma = 2$

Robust controller 3
$\gamma = 0.9801$ $\gamma_{opt} = 0.9794$

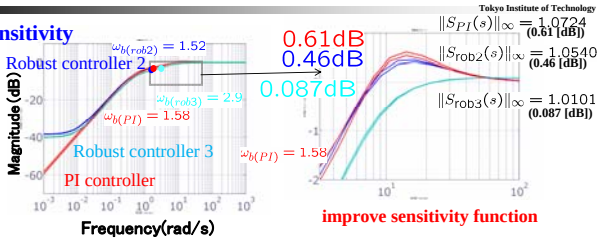


rise the gain in the high frequency band

approximation (ignore the high frequency)

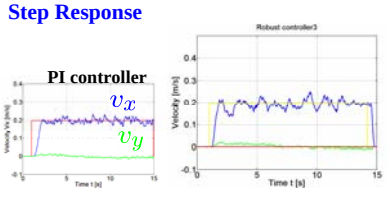
Robust Controller 3

Sensitivity



improve sensitivity function

Step Response



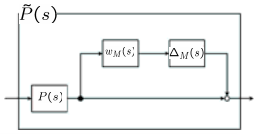
a lowered sensitivity, but step response (steady-state performance) is not good

an estimation of uncertainty weight might be improper

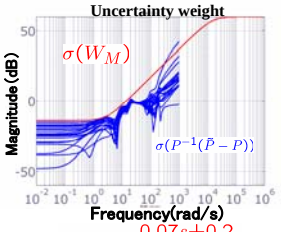
Uncertainty Weight Change

Nominal Model (with time-delay)

$$P_d(s) = \begin{bmatrix} 0.75\frac{1}{s/6+1}\frac{1}{s/12+1}\frac{1-\frac{0.28}{2}s}{1+\frac{0.28}{2}s} & 0.03\frac{1}{s/6+1}\frac{1}{s/10+1}\frac{1-\frac{0.28}{2}s}{1+\frac{0.28}{2}s} \\ 0.05\frac{1}{s/6+1}\frac{1}{s/10+1}\frac{1-\frac{0.28}{2}s}{1+\frac{0.28}{2}s} & 0.70\frac{1}{s/6+1}\frac{1}{s/12+1}\frac{1-\frac{0.28}{2}s}{1+\frac{0.28}{2}s} \end{bmatrix}$$



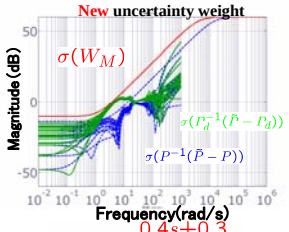
Uncertainty weight



$$w_M = \frac{0.07s+0.2}{7\times10^{-5}s+1}$$

$(\tau = 0.07, r_0 = 0.2, r_\infty = 1000)$

New uncertainty weight

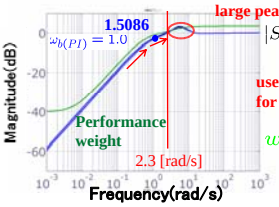


$$w_M = \frac{0.4s+0.3}{4\times10^{-4}s+1}$$

$(\tau = 0.4, r_0 = 0.3, r_\infty = 1000)$

Performance Weight

Sensitivity of PI controller



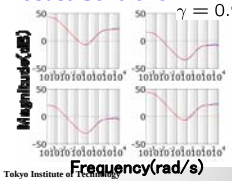
$|S_{PI}(s)\|_\infty = 1.5086$
(3.57 [dB])

use the 2nd order performance weight for low sensitivity around 2.3[rad/s]

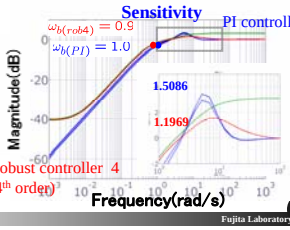
$$w_{P3}^{-1} = \frac{0.01(s/0.006+1.0)(s/0.6+1)}{(s/3+1)(s/4.5+1)}$$

Robust Controller 4

$\gamma = 0.9575$

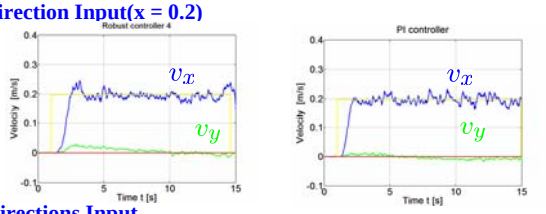


Sensitivity

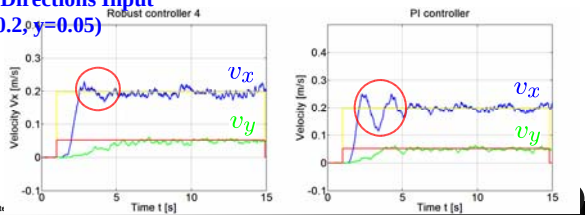


Step Response

One Direction Input(x = 0.2)



Two Directions Input
(x = 0.2, y=0.05)



Summary and Future Works

Summary

- Identification
- Design robust controller (2 Input 2Output)

Future Works

- Design robust controller (4 Input 3Output)