Optimal Power Dispatch of Power Networks with Potential Games

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Backgrounds

Smart Grid

- Two-way energy management system between supply and demand side
- Use of renewable energy (ex. solar, wind)
  - Environmental conservation
  - Alternative energy of conventional energy
  - Decentralization of energy

Outputs depend largely on weather

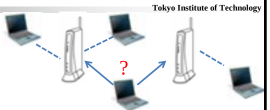
⇒ It is more difficult to satisfy supply-demand balance

⇒ Centralized and Distributed generations(renewable, battery) need to allocate electricity to customers in a coordinated way

No design policy of optimal power dispatch…

Objective (Game-theoretic Control)

Game-theoretic approach



- Agents are self-interested

⇒ Non-cooperative game

- The solution to the problem = the equilibrium of the game

Applications : resource allocation, sensor coverage

Advantages

- robustness to failures and environmental disturbances
- guarantee global convergence
- improved scalability

⇒ Potential game has
- Design policies of objective function
- Learning algorithms

Objective of this work

To apply potential game to optimal power dispatch problem of power network

Definition of Game

- Player set $N = \{1, \cdots, n\}$
- Collection of action sets $\mathcal{A} = \mathcal{A}_1 \times \ldots \times \mathcal{A}_n$

  Agent i's action set : $\mathcal{A}_i$

  Agent i's action : $a_i \in \mathcal{A}_i$
- Collection of objective function $U = \{U_1, \cdots, U_n\}$

  Agent i's objective function $U_i : \mathcal{A} \to \mathbb{R}$

  Every agent chooses a_i to maximize U_i

⇒ Game $G =< N, \mathcal{A}, U >$

Nash equilibrium

A pure Nash equilibrium is an action $a^* \in \mathcal{A}$ such that $\forall i \in N$

$$U_i(a_i^*, a_{-i}^*) = \max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i}^*)$$

$$(a_{-i} = (a_1, a_2, \cdots, a_{i-1}, a_{i+1}, \cdots, a_n))$$

(Ex.) Payoff Matrix

Player 2 (columns), Player 1 (rows); NE marked on (2,2) and (4,4)

	A	B
A	(2,2)	(1,0)
B	(0,1)	(4,4)

NE

Potential Game

- Potential function (global objective function) $\phi : \mathcal{A} \to \mathbb{R}$

  ϕ : max ⇒ Objective of a group is achieved

Potential game

A game $G =< N, \mathcal{A}, U >$ is a potential game if there is a potential function ϕ such that $\forall i \in N$, $\forall a_{-i} \in \mathcal{A}_{-i}$ and $\forall a_i', a_i'' \in \mathcal{A}_i$,

$$U_i(a_i', a_{-i}) - U_i(a_i'', a_{-i}) = \phi(a_i', a_{-i}) - \phi(a_i'', a_{-i})$$

Key property

- the guaranteed existence of a Nash equilibrium
- Local maxima of ϕ are Nash equlibra

(Ex.) Payoff

	A	B
A	(2,2)	(1,0)
B	(0,1)	(4,4)

Potential

	A	B
A	2	0
B	0	3

Global objective

⇒ Application of appropriate learning algorithms

maximize ϕ

Outline

- Introduction
- Optimal Power Dispatch Problem
- Optimal Power Dispatch Game
- Conclusions

Power Networks

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◆ Node: $\mathcal{V} = \mathcal{B} \cup \mathcal{R} \cup \mathcal{D}$

Battery: $\mathcal{B} = \{B_i | i = 1, \cdots, n_B\}$

Renewable energy:

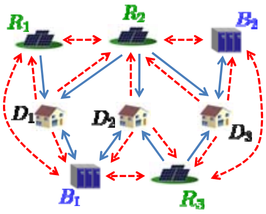
$\mathcal{R} = \{R_i | i = 1, \cdots, n_R\}$

Demand: $\mathcal{D} = \{D_i | i = 1, \cdots, n_D\}$

◆ Link: $\mathcal{E} = \mathcal{V} \times \mathcal{V}$

Energy flow: $\mathcal{E}_P$ ⟶

Information flow: $\mathcal{E}_I$ - - - ->



⇨ Graph: $G = (\mathcal{V}, \mathcal{E})$

Battery dynamics:

$$\Sigma_B: b_i(k) = b_i(k-1) - \sum_{j \in \mathcal{N}_{DB,i}} u_{bij}(k), i \in \mathcal{B}$$

battery level

$u_{bij} < 0$: charge

$\mathcal{N}_{DB,i}$: set of D_j can be supplied by B_i

Power Networks

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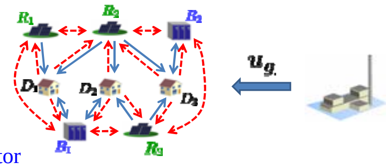

Generator

It is difficult to satisfy supply-demand balance by only renewable energy and battery.

Generator adjust supply-demand balance:

$$\Rightarrow d_j(k) = u_{gj}(k) + \sum_{i \in \mathcal{N}_{RD,j}} u_{rij}(k) + \sum_{i \in \mathcal{N}_{BD,j}} u_{bij}(k)$$

Supply of generator — Supply of renewable energy and battery

$\mathcal{N}_{RD,j}$: set of D_j can be supplied by R_i

$\mathcal{N}_{BD,j}$: set of D_j can be supplied by B_i

Optimal Power Dispatch Problem

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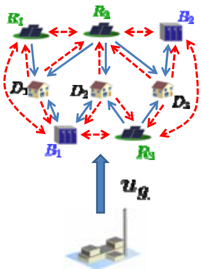
Assessment functions

➢ Maximize supply from renewable energy:

$$J_R = \sum_{i \in \mathcal{R}} w_{R,i} \left(r_i(k) - \sum_{j \in \mathcal{N}_{DR,i}} u_{rij}(k) \right)^2$$

➢ Remaining battery level = desired level:

$$J_B = \sum_{i \in \mathcal{B}} w_{B,i} (b_{i,ref} - b_i(k))^2$$



➢ Minimize electricity prices (fuel cost):

$$J_D = \sum_{j \in \mathcal{D}} w_{D,j} u_{gj}^2(k)$$

$$= \sum_{j \in \mathcal{D}} w_{D,j} \left(d_j(k) - \left(\sum_{i \in \mathcal{N}_{RD,j}} u_{rij}(k) + \sum_{i \in \mathcal{N}_{BD,j}} u_{bij}(k) \right) \right)^2$$

Optimal Power Dispatch Problem

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Constraint conditions

➢ Supply of renewable energy: $u_{rij}(k) \geq 0 \quad \sum_{j \in \mathcal{N}_{DR,i}} u_{rij} \leq r_i$

➢ Supply of renewable energy: $u_{gj}(k) > 0$ ⟵ running cost

$$d_j(k) = u_{gj}(k) + \sum_{i \in \mathcal{N}_{RD,j}} u_{rij}(k) + \sum_{i \in \mathcal{N}_{BD,j}} u_{bij}(k)$$

$$\Rightarrow \sum_{i \in \mathcal{N}_{RD,j}} u_{rij}(k) + \sum_{i \in \mathcal{N}_{BD,j}} u_{bij}(k) < d_j(k)$$

➢ Battery capacity: $b_i(k) \in [0, B_{i,max}]$

➢ Battery's charging and discharge rate:

$$\left| \sum_{j \in \mathcal{N}_{DB,i}} u_{bij}(k) \right| \leq \delta b_i$$

Objective: minimize each assessment function under constraint conditions

Outline

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- Introduction
- Optimal Power Dispatch Problem
- Optimal Power Dispatch Game
- Conclusions

Optimal Power Dispatch Game

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- Player set $N = \{1, \cdots, n\}$

  Renewable energy, Battery
- Collection of action sets $\mathcal{A} = \mathcal{A}_1 \times \ldots \times \mathcal{A}_n$

  Supply of renewable energy and battery: u_r , u_b
- Collection of objective function $U = \{U_1, \cdots, U_n\}$
- Feasible action sets $R(a) = R_1(a) \times \cdots \times R_n(a)$, $R_i(a) \subseteq \mathcal{A}_i$

  Constraint conditions
- State $X = \{X_1, \cdots, X_n\}$

  Battery level b
- Transition of states $P : \mathcal{A} \times X \rightarrow X$

  Battery dynamics: $b_i(k) = b_i(k-1) - \sum_{j \in \mathcal{N}_{DB,i}} u_{bij}(k)$

⇨ Strategic game $\Gamma = \{N, \mathcal{A}, U, R, X, P\}$

Optimal Power Dispatch Game

Design of each agent's objective function

Potential game at state x

A game is a potential game at state x if there is a potential function ϕ such that $\forall i \in N$, $\forall a_{-i} \in \mathcal{A}_{-i}$ and $\forall a_i' \in \mathcal{A}_i$,

$$U_i(a_i', a_{-i}, x) - U_i(a, x) = \phi(a_i', a_{-i}, x) - \phi(a, x)$$

$$(a_{-i} = (a_1, a_2, \cdots, a_{i-1}, a_{i+1}, \cdots, a_n))$$

⇨ At each state x, optimal power dispatch game has Nash equilibriums

Nash equilibrium at state x

A Nash equilibrium is an action $a^* \in \mathcal{A}$ such that $\exists x$,

$$U_i(a_i^*, a_{-i}^*, x) = \max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i}^*, x), \quad \forall i \in N$$

Optimal Power Dispatch Game

Agent : Renewable energy, Battery Action: u_r , u_b

Potential function (global objective)

$$\phi(u_r, u_b, b) = -q_D J_D - q_R J_R - q_B J_B$$

Each agent's objective function: q_D, q_R, q_B : weight

Renewable energy $\mathcal{R}_i \in \mathcal{R}$:

$$U_{Ri} = -\sum_{j \in N_{DR,i}} w_{D,j} \left(d_j(k) - \left(\sum_{l \in N_{RD,j}} u_{rlj}(k) + \sum_{l \in N_{BD,j}} u_{blj}(k) \right) \right)^2 - w_{R,i} \left(r_i(k) - \sum_{j \in N_{DR,i}} u_{rij}(k) \right)^2$$

Battery $\mathcal{B}_i \in \mathcal{B}$:

$$U_{Bi} = -\sum_{j \in N_{DB,i}} w_{D,j} \left(d_j(k) - \left(\sum_{l \in N_{RD,j}} u_{rlj}(k) + \sum_{l \in N_{BD,j}} u_{blj}(k) \right) \right)^2 - w_{B,i} (b_{i,ref} - b_i(k))^2$$

Optimal Power Dispatch Game: Example

Assessment functions

$J_D = (3 - u_r - u_b)^2$ $(d = 3)$

$J_R = (2 - u_r)^2$ $(r = 2)$

$J_B = (1 - (b - u_b))^2$ $(b_{ref} = 1)$

Potential function

$\phi = -10 J_D - J_R - J_B$

Utility

R : $U_R = -10 J_D - J_R$

B : $U_B = -10 J_D - J_B$

Actions

R : $u_r = \{0, 1, 2\}$

B : $u_b = \{-1, 0, 1\}$

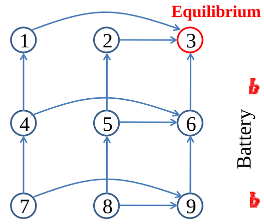
Payoff matrix

	Battery	Renewable energy 0	1	2
$b = 0$	1			
	0	(-91,-94)	(-41,-41)	(-11,-10)
	-1	(-160,-164)	(-90,-91)	(-40,-40)
$b = 1$	1	(-41,-44)	(-11,-11)	(-1,0)
	0	(-90,-94)	(-40,-41)	(-10,-10)
	-1	(-161,-164)	(-91,-91)	(-41,-40)
$b = 2$ (B_{max})	1	(-40,-44)	(-10,-11)	(0,0)
	0	(-91,-94)	(-41,-41)	(-11,-10)
	-1			

Optimal Power Dispatch Game: Example

Best response

$$a_i = \arg\max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i}, b)$$



	Battery	Renewable energy 0	1	2
$b = 0$	1			
	0	(1) (-91,-94)	(2) (-41,-41)	(3) (-11,-10)
	-1	(-160,-164)	(-90,-91)	(-40,-40)
$b = 1$	1	(-41,-44)	(-11,-11)	(-1,0)
	0	(4) (-90,-94)	(5) (-40,-41)	(6) (-10,-10)
	-1	(-161,-164)	(-91,-91)	(-41,-40)
$b = 2$	1	(-40,-44)	(-10,-11)	(0,0)
	0	(7) (-91,-94)	(8) (-41,-41)	(9) (-11,-10)
	-1			

Equilibrium

$[u_r^*, u_b^*, b^*]$ ⇨ $[u_r^*, 0, b^*]$?

Convergence Analysis: Renewable energy's action

$$U_{Ri} = -\sum_{j \in N_{DR,i}} w_{D,j} \left(d_j(k) - \left(\sum_{l \in N_{RD,j}} u_{rlj}(k) + \sum_{l \in N_{BD,j}} u_{blj}(k) \right) \right)^2 - w_{R,i} \left(r_i(k) - \sum_{j \in N_{DR,i}} u_{rij}(k) \right)^2$$

- Renewable energy's action is not affected by battery state.

$$\frac{\partial U_{Ri}}{\partial u_{rij}} = 2 \sum_{j \in N_{DR,i}} w_{D,j} \left(d_j - \left(\sum_{l \in N_{RD,j}} u_{rlj} + \sum_{l \in N_{BD,j}} u_{blj} \right) \right) + 2 w_{R,i} \left(r_i - \sum_{j \in N_{DR,i}} u_{rij} \right)$$

$$\frac{\partial U_{Ri}}{\partial u_{rij}} > 0 \quad (\because \text{constraint conditions})$$

- Renewable energy takes action to increase his supply.
- Renewable energy's action has upper limit: $\sum_{j \in N_{DR,i}} u_{rij} \le r_i$

⇨ Renewable energy $\mathcal{R}$'s action change is finite.

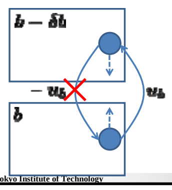
Convergence Analysis: Battery's action

Assumption

Battery $\mathcal{B}_i$'s action set: $A_{Bi} = \{-\delta b_i, 0, \delta b_i\}$, $\left| \sum_{j \in N_{DB,i}} u_{bij} \right| = \delta b_i$

$$U_{Bi} = -\sum_{j \in N_{DB,i}} w_{D,j} \left(d_j(k) - \left(\sum_{l \in N_{RD,j}} u_{rlj}(k) + \sum_{l \in N_{BD,j}} u_{blj}(k) \right) \right)^2 - w_{B,i} (b_{i,ref} - b_i(k))^2$$

- Battery's action is not affected by other battery's state.
- Battery doesn't take action to restore battery's state.



proof

Assumption:

$$U_{Bi}(0, b_i) < U_{Bi}(u_{bij}, b_i - \delta b_i) \quad \cdots ①$$

$$U_{Bi}(0, b_i - \delta b_i) < U_{Bi}(-u_{bij}, b_i) \quad \cdots ②$$

Convergence Analysis: Battery's action

$$\Delta = d_j - \sum_{i\in\mathcal{N}_{BD,j}} u_{rij} \qquad \Theta = b_{i,ref} - b_i$$

⇨ $$U_{Bi} = -\sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}} u_{blj}\right)^2 - w_{B,i}\Theta^2$$

①: $$-\sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}} u_{blj}\right)^2 - w_{B,i}(\Theta+\delta b_i)^2$$

$$> -\sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}\setminus\{i\}} u_{blj}\right)^2 - w_{B,i}\Theta^2 \quad \cdots③$$

②: $$-\sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}\setminus\{i\}} u_{blj} + u_{bij}\right)^2 - w_{B,i}\Theta^2$$

$$> -\sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}\setminus\{i\}} u_{blj}\right)^2 - w_{B,i}(\Theta+\delta b_i)^2 \quad \cdots④$$

Convergence Analysis: Battery's action

③, ④: $$2\sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}\setminus\{i\}} u_{blj}\right)^2$$

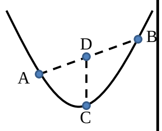
$$> \sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}} u_{blj}\right)^2 + \sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}\setminus\{i\}} u_{blj} + u_{bij}\right)^2$$

⇨ $$\sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}\setminus\{i\}} u_{blj}\right)^2$$

$$> \frac{\sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}} u_{blj}\right)^2 + \sum_{j\in\mathcal{N}_{DB,i}} w_{D,j}\left(\Delta - \sum_{l\in\mathcal{N}_{BD,j}\setminus\{i\}} u_{blj} + u_{bij}\right)^2}{2}$$

Contradiction! (∵ property of convex function)

∴ Battery doesn't take action to restore battery's state.



- Battery has constraint: $b_i(k) \in [0, B_{i,max}]$

$$\mathcal{A}_{Bi} = \{-\delta b_i, 0, \delta b_i\}$$

⇨ Battery $\mathcal{B}$'s action change is finite.

Convergence Analysis: Optimal Power Dispatch Game

- ϕ, U_{Ri}, U_{Bi}
- Best response: $a_i = \arg\max_{a_i\in\mathcal{A}_i} U_i(a_i, a_{-i}, b)$
- Renewable energy $\mathcal{R}$'s action change is finite.
- Battery $\mathcal{B}_i$'s action set: $\mathcal{A}_{Bi} = \{-\delta b_i, 0, \delta b_i\}$
- Battery $\mathcal{B}$'s action change is finite.

⇨ Optimal power dispatch game

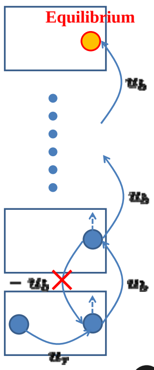
$$\Gamma = \{N, \mathcal{A}, U, R, X, P\}$$

has a stationary state (equilibrium).

Stationary state (equilibrium)

$$[u_r^*, u_b^*, b^*] \Rightarrow [u_r^*, 0, b^*]$$

$$\frac{\partial\phi}{\partial u_b}(u_r^*, 0, b^*) = 0$$



Conclusion

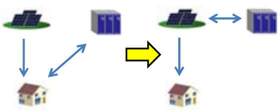
Summary

- ➢Definition of optimal power dispatch game
- ➢Analysis of optimal power dispatch game
  - Convergence to equilibrium

Future Works

- ➢Analysis of different configuration of power networks
- ➢Simulations

Check { $\mathcal{B}$'s and $\mathcal{R}$'s actions / equilibrium



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Appendix

Convergence Analysis: Battery's action

$$\Delta = d_j - \sum_{l \in N_{BD,j}} u_{rlj} \qquad \Theta = b_{i,ref} - b_i$$

$$\Rightarrow \quad U_{Bi} = -\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j}} u_{klj} \right)^2 - w_{B,i}\Theta^2$$

①: $$-\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j}} u_{klj} \right)^2 - w_{B,i} \left(\Theta + \sum_{l \in N_{BD,j}} u_{kij} \right)^2$$

$$> -\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2 - w_{B,i}\Theta^2 \quad \cdots ③$$

②: $$-\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} + u_{kij} \right)^2 - w_{B,i}\Theta^2$$

$$> -\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2 - w_{B,i} \left(\Theta + \sum_{l \in N_{BD,j}} u_{kij} \right)^2 \quad \cdots ④$$

Convergence Analysis: Battery's action

③: $$-\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j}} u_{klj} \right)^2 + \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2$$

$$> -w_{B,i}\Theta^2 + w_{B,i} \left(\Theta + \sum_{i \in N_{BD,j}} u_{kij} \right)^2 \quad \cdots ⑤$$

④: $$-w_{B,i}\Theta^2 + w_{B,i} \left(\Theta + \sum_{i \in N_{BD,j}} u_{kij} \right)^2$$

$$> -\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2 + \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} + u_{kij} \right)^2$$

$$\cdots ⑥$$

Convergence Analysis: Battery's action

⑤, ⑥: $$-\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j}} u_{klj} \right)^2 + \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2$$

$$> -\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2 + \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} + u_{kij} \right)^2$$

$$2 \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2$$

$$> \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j}} u_{klj} \right)^2 + \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} + u_{kij} \right)^2$$

Convergence Analysis: Battery's action

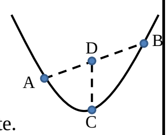
$$2 \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2$$

$$> \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j}} u_{klj} \right)^2 + \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} + u_{kij} \right)^2$$

$$\Rightarrow \quad \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} \right)^2$$

$$> \frac{\sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j}} u_{klj} \right)^2 + \sum_{j \in N_{DB,i}} w_{D,j} \left(\Delta - \sum_{l \in N_{BD,j} \setminus \{i\}} u_{klj} + u_{kij} \right)^2}{2}$$

Contradiction! (∵property of convex function)



∴ Battery doesn't take action to restore battery's state.

Architecture for Potential Games and Utility Designs

Utility Design
Wonderful Life
Shapley Value
Weighted Shapley Value

Potential Games

Gradient play
Log-linear learning
Learning Design

Cost sharing methodologies

Player set $S \subseteq N,\ i \in S$

- Wonderful Life Utility (WLU)

$$f_r(i,S) = W_r(S) - W_r(S \setminus \{i\})$$

- The Shapley Value (SV)

$$f_r(i,S) = \sum_{T \subseteq S \setminus \{i\}} \omega_T \left(W_r(S) - W_r(S \setminus \{i\}) \right)$$

$$\omega_T = \frac{|T|!(|S| - |T| - 1)!}{|S|!}$$

- The Weighted Shapley Value (WSV)

Distribution Rule	Existence of Equilibrium	Potential Game	Budget Balanced	Tractable	Informational Requirement
WLU	yes	yes	**no**	**yes**	Medium
SV, WSV	yes	yes	**yes**	**no**	**High**

State-Based Non-Cooperative Design

State-based games

state $x(t) \in X$

- Utility function $U_i(a_i, a_{-i}, x)$
- State transition function $P : A \times X \rightarrow \Delta(X)$: the set of probability distribution over X
- Each player selects an action to maximize his expected utility

$$\therefore\ a_i(t) \in \arg\max_{a_i(t) \in A_i} \mathbf{E}[U_i(a(t), x(t))]$$

- $x(t+1)$ is chosen randomly according to $P(a(t), x(t)) \in \Delta(X)$

State-based Nash Equilibrium $[a^*, x^*]$

for every x' in the support of $P(a^*, x^*)$,

$$U_i(a_i^*, a_{-i}^*, x') = \max_{a_i \in A_i} U_i(a_i, a_{-i}^*, x')$$

State-based Potential games

$\exists \phi : A \rightarrow \mathbb{R}$ such that $a_i' \in A_i$,

$$U_i(a_i', a_{-i}, x) - U_i(a, x) > 0 \Rightarrow \phi(a_i', a_{-i}) - \phi(a) > 0$$

Priority-Based Distribution Rule

state = priority ⇨ **Priority-based**

x_i^r : the priority of player i at resource r

- $x_i^r < x_j^r$: i has higher priority than j
- $x_i^r = 1$: top priority

State dynamics : first in first out (FIFO)

- Multiple players seek to join a resource simultaneously
  ⇨ The order of the entering players is randomly chosen
- $a(t) = a(t-1) \Rightarrow x(t+1) = x(t)$

Priority-based utility $U_i(a', x) = \mathbf{E}_{P(a',x)} V_i(a', x')$

Expectation which x' is chosen : $\mathbf{E}_{P(a',x)}$ $\quad \bar{x}_i^r := \{j \in N : x_j^r \le x_i^r\}$

Marginal contribution : $V_i(a, x) = \sum_{r \in a_i} (W^r(\bar{x}_i^r) - W^r(\bar{x}_i^r \setminus i))$

⇨

Distribution Rule	Budget Balanced	Tractable	PoS	PoA
Priority-Based	yes	yes	1	1/2