Distributed Pose Estimation of Sensors and Targets in Visual Sensor Networks

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Visual Sensor Networks

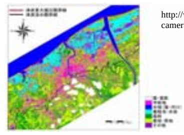
Visual Sensor Networks

A network consisting of spatially distributed smart cameras


Smart camera
http://www.image-acquire.com/ni-1742-smart-camera/

Application

- Environmental monitoring
- Surveillance
- Target tracking


http://www1.gsi.go.jp/geowww/EODAS/banda_ache/banda_ache.html

Pose estimation

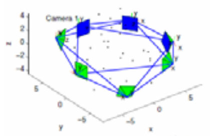
To estimate the position and orientation of the sensors and targets in a distributed fashion.

Distributed Pose Estimation

Distributed pose estimation

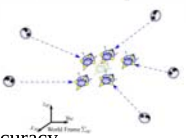
Distributed localization [5]

- Distributed pose estimation of the camera sensors
- Static estimation



Cooperative estimation [12]

- Dynamic estimation of the target motion
- Averaging performance for improving accuracy



Objective

Distributed simultaneous estimation of the sensors and target

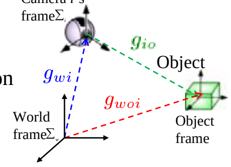
[5] R. Tron and R. Vidal, "Distributed Image-based 3-D Localization of Camera Sensor Networks," *Proc. of the 48th IEEE Conference on Decision and Control*, pp. 901-908, 2009.
[12] T. Hatanaka and M. Fujita, "Cooperative Estimation of 3D Target Motion via Networked Visual Motion Observers," *IEEE Transactions on Automatic Control*, 2011. (submitted)

Relative Rigid Body Motion

Pose of camera i : $g_{wi} = (p_{wi}, R_{wi}) \in SE(3)$

$$g_{wi} = \begin{bmatrix} R_{wi} & p_{wi} \\ 0_{1\times 3} & 1 \end{bmatrix} \quad p_{wi} \in \mathbb{R}^3 \text{ : Position} \quad R_{wi} \in SO(3) \text{: Orientation}$$

Pose of object : $g_{woi} = (p_{woi}, R_{woi})$



Body Velocity

$$\hat{V}^b_{wi} = g^{-1}_{wi}\dot{g}_{wi} = \begin{bmatrix} \hat{\omega}^b_{wi} & v^b_{wi} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{4\times 4}$$

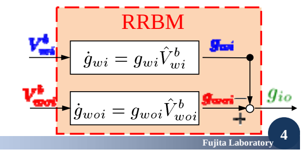
$v^b_{wi} \in \mathbb{R}^3$: Linear velocity $\omega^b_{wi} \in \mathbb{R}^3$: Angular velocity

Pose of object relative to camera i:

$$g_{io} = g^{-1}_{wi} g_{woi} = (p_{io}, R_{io})$$

Relative Rigid Body Motion (RRBM)

$$\dot{g}_{io} = -\hat{V}^b_{wi} g_{io} + g_{io}\hat{V}^b_{woi}$$



Visual Measurement

Visual Measurement

$$f_i = \begin{bmatrix} f^T_{i1} & f^T_{i2} & \cdots & f^T_{im} \end{bmatrix}^T \quad m \geq 4$$

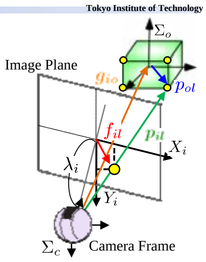
Perspective projection

$$f_{il} = \frac{\lambda_i}{z_{il}}\begin{bmatrix} x_{il} \\ y_{il} \end{bmatrix} \quad \lambda_i \text{ :focal length}$$

$$p_{il} = g_{io}p_{ol} = \begin{bmatrix} x_{il} & y_{il} & z_{il} \end{bmatrix}^T$$

: Position of feature points relative to camera frame

p_{ol} $l = 1, \cdots, m$: Position of feature points relative to object frame



The camera i can estimate the relative pose g_{io} from the visual measurements by Visual Motion Observer [8]

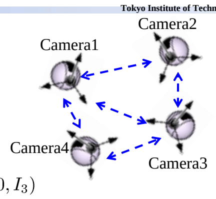
Estimate relative pose: $\bar{g}_{io} = (\bar{p}_{io}, \bar{R}_{io})$

Communication and Camera Settings

Communication graph $G = (\mathcal{V}, \mathcal{E})$

$(j, i) \in \mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ ⇒ i can get j's info.

Neighbor set: $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$



Set of cameras

- Camera1 frame = World frame $g_{w1} = (0, I_3)$
- Known camera pose set: $i \in \mathcal{V}_k$

  $\bar{g}_{woi} = g_{wi}\bar{g}_{io}$ is known but may be different due to noise

  Average of the pose of the object (only $i \in \mathcal{V}_k$)

  $$\bar{g}^* = (\bar{p}^*, \bar{R}^*) := \arg \min_{g\in SE(3)} \sum_{i\in\mathcal{V}_k} \psi(g^{-1}\bar{g}_{woi})$$

  $$\psi(g) := \frac{1}{2}\|I_4 - g\|^2_F = \frac{1}{2}\|p\|^2 + \phi(R) \qquad \phi(R) := \frac{1}{2}\|I_3 - R\|^2_F = \mathrm{tr}(I_3 - R)$$

- Unknown camera pose set: $i \in \mathcal{V}_u$ $\mathcal{V}_u = \mathcal{V}\backslash\mathcal{V}_k$ Set $g_{io} = \bar{g}_{io}$

Problem Settings

Objective

Estimation of pose of the camera g_{wi} $i \in \mathcal{V}_u$ and object g_{woi} $i \in \mathcal{V}$

- g_{woi} is close to the known pose $g_{woi} = g_{wi}\bar{g}_{io}$
- Pose of object g_{woi} are close to the known average $\bar{g}^*$

Known camera pose set: $i \in \mathcal{V}_k$

- Estimation of g_{woi} $\quad g_{io} = g_{wi}^{-1} g_{woi}$

Unknown camera pose set: $i \in \mathcal{V}_u$

- Estimation of g_{woi} $\quad g_{wi} = g_{woi}\bar{g}_{io}^{-1}$ $\quad g_{io} = \bar{g}_{io}$

⇨ We consider only the estimates of g_{woi}

We first focus on orientation part R_{woi}

Update Procedure of the Estimates

Update Equations

$$\dot{R}_{woi} = \begin{cases} R_{woi}\mathrm{sk}(k_e R_{woi}^T \bar{R}_{woi}) + k_s \sum_{j\in\mathcal{N}_i} \mathrm{sk}(R_{woi}^T R_{woj}) & i \in \mathcal{V}_k \\ R_{woi}\mathrm{sk}(k_s \sum_{j\in\mathcal{N}_i} R_{woi}^T R_{woj}) & i \in \mathcal{V}_u \end{cases} \quad (1)$$

Gradient descent of $\phi(\bar{R}_{io}^T R_{io})$

Attitude synchronization [10]

$$\bar{R}_{woi} = R_{wi}\bar{R}_{io} \qquad \mathrm{sk}(M) = \frac{1}{2}(M - M^T)$$

$$\phi(R) := \frac{1}{2}\|I_3 - R\|_F^2 = \mathrm{tr}(I_3 - R)$$

[10] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in *SE(3)*," *IEEE Transactions on Control Systems Technology*, Vol. 17, No. 5, pp.1119–1134, 2009.

Assumptions

Assumption 1 (Communication Graph)

The communication graph $G = (\mathcal{V}, \mathcal{E})$ is fixed, balanced and strongly connected.

Assumption 2 (Object Pose)

- The object is static. $\bar{V}_{woi}^b = 0$
- There exists a pair $(i, j) \in \mathcal{V} \times \mathcal{V}$ such that $\bar{R}_{woi} \neq \bar{R}_{woj}$
- $\bar{R}^{*T} R_{woi} > 0 \quad \forall i \in \mathcal{V}$ holds true

  The relative angle between is smaller than $\pi/2$

Main Result

Theorem 1

Suppose the estimates R_{woi} are updated according to the update equation (1). Under the assumption 1 and 2, if the initial estimates satisfy $R_{woi}^T \bar{R}^* > 0$, then there exists a finite $T > 0$ such that

$$\sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) \leq (1+\epsilon) \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} \bar{R}_{woi})$$

Error between average and estimates

Error between average and measurements

$$\epsilon = |\mathcal{V}_u|\left(\frac{1}{\sqrt{|\mathcal{V}_k|}} + 2\sqrt{\frac{\mathrm{diam}(G)}{k\beta}}\right)^2 \qquad \beta > 0$$

$|\mathcal{V}_k|$: number of known pose camera

for sufficient large $k = k_s / k_e > 0$

$|\mathcal{V}_u|$: number of unknown pose camera

Sketch of proof

Energy function $U_R := \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi})$

Conclusion

Conclusion

- Problem settings
- Orientation estimation and averaging

Future Works

- Simulations and experiments
- Position estimation

References

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[3] J. Aspnes, T. Eren, D. K. Goldenberg, A. S. Morse, W. Whiteley, Y. R. Yang, B. D. O. Anderson and P. N. Belhumeur, "A Theory of Network Localization," *IEEE Transactions on Mobile Computing*, Vol. 5, No. 12, pp. 1663-1678, 2006.

[4] G. Piovan, I. Shames, B. Fidan, F. Bullo and B. D. O. Anderson, "On Frame and Orientation Localization for Relative Sensing Networks," Automatica, 2011. (submitted)

[5] R. Tron and R. Vidal, "Distributed Image-based 3-D Localization of Camera Sensor Networks," *Proc. of the 48th IEEE Conference on Decision and Control*, pp. 901-908, 2009.

[6] R. Tron, R. Vidal and A. Terzis, "Distributed Pose Averaging in Camera Sensor Networks via Consensus on SE(3)," *Proc. of the International Conference on Distributed Smart Cameras*, 2008.

References

[7] T. Hatanaka, T. Nishi and M. Fujita, “Passivity-based Cooperative Estimation Algorithm for Networked Visual Motion Observers,” *Proc. of the SICE Annual Conference 2011*, 2011. (submitted)

[8] M. Fujita, H. Kawai and M. W. Spong, “Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis,” *IEEE Transactions on Control Systems Technology*, Vol.15, No. 1, pp. 40-52, 2007.

[9] P. A. Absil, R. Mahony and R. Sepulchre, “Optimization Algorithms on Matrix Manifolds,” Princeton Press, 2008.

[10] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, “Passivity-based Attitude Synchronization in *SE(3),” IEEE Transactions on Control Systems Technology*, Vol. 17, No. 5, pp.1119–1134, 2009.

[11] T. Hatanaka and M. Fujita, “Passivity-based Cooperative Estimation of 3D Target Motion for Visual Sensor Networks: Analysis on Averaging Performance,*” Proc. of the 2011 American Control Conference*, pp. 3399-3404, 2011.

[12] T. Hatanaka and M. Fujita, “Cooperative Estimation of 3D Target Motion via Networked Visual Motion Observers,” *IEEE Transactions on Automatic Control*, 2011. (submitted)

Appendix

Lemma1

Lemma 1 (Positively Invariance)

Under assumption 1and 2, if $\bar{R}^{*T}R_{woi} > 0$ holds at the initial time, Then for any positive scalar *c*, there exists a finite time $\tau(c)$ such that

$$\phi(\bar{R}^{*T}R_{woi}) \leq \phi(\bar{R}^{*T}\bar{R}_{woh}) + c \qquad \forall t \geq \tau(c) \qquad i \in \mathcal{V}$$

$$h := \arg\max_j \phi(\bar{R}^{*T}\bar{R}_{woj})$$

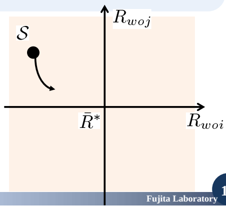
Proof

Energy function: $U_R := \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T}R_{woi})$

Under assumption 2

$$\mathcal{S} = \{(R_{woi})_{i\in\mathcal{V}} | \bar{R}^{*T}R_{woi} > 0 \quad \forall i \in \mathcal{V}\}$$

is positively invariant



Proof of Theorem

Energy function

$$U_R := \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T}R_{woi})$$

Derivative of the energy function

$$\begin{aligned}
\dot{U}_R &= \sum_{i\in\mathcal{V}} e_R^T(\bar{R}^{*T}R_{woi})\omega_{woi} \qquad)^\vee \\
&= -\sum_{i\in\mathcal{V}_k} \mathrm{tr}(\bar{R}^{*T}R_{woi}\mathrm{sk}(k_e R_{woi}^T \bar{R}_{woi} + k_s \sum_{j\in\mathcal{N}_i} R_{woi}^T R_{woj})) \\
&\quad - \sum_{i\in\mathcal{V}_u}\sum_{j\in\mathcal{N}_i} \mathrm{tr}(\bar{R}^{*T}R_{woi}\mathrm{sk}(k_s R_{woi}^T R_{woj})) \\
&= -\frac{1}{2}k_e \sum_{i\in\mathcal{V}_k} \mathrm{tr}(\Phi_1) - \frac{1}{2}k_s \sum_{i\in\mathcal{V}} \mathrm{tr}(\Phi_2) \\
\Phi_1 &= \bar{R}^{*T}\bar{R}_{woi} - \bar{R}^{*T}R_{woi}\bar{R}_{woi}^T R_{woi} \\
\Phi_2 &= \sum_{j\in\mathcal{N}_i} (\bar{R}^{*T}R_{woj} - \bar{R}^{*T}R_{woi}R_{woj}^T R_{woi})
\end{aligned}$$

Proof of Theorem

Lemma2 $\forall R_1, R_2, R_3 \in SO(3)$ $\quad \mathrm{sym}(M) = \frac{1}{2}(M + M^T)$

$$\frac{1}{2}\mathrm{tr}(R_1^T R_2 - R_1^T R_3 R_2^T R_3) \geq \phi(R_1^T R_3) - \phi(R_1^T R_2) + \lambda_{\min}(\mathrm{sym}(R_1^T R_3))\phi(R_3^T R_2)$$

Under assumption1

$$\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} \phi(\bar{R}^{*T}R_{woi}) - \phi(\bar{R}^{*T}R_{woj}) = 0$$

$$\frac{1}{2}\sum_{i\in\mathcal{V}} \mathrm{tr}(\Phi_2) \geq \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} \sigma_i \phi(R_{woi}^T R_{woj}) \qquad \sigma_i = \lambda_{\min}(\mathrm{sym}(\bar{R}^{*T}R_{woi}))$$

$$\frac{1}{2}\sum_{i\in\mathcal{V}_k} \mathrm{tr}(\Phi_1) \geq \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T}R_{woi}) - \phi(\bar{R}^{*T}\bar{R}_{woi}) + \sigma_i \phi(R_{woi}^T \bar{R}_{woi})$$

$$\begin{aligned}
\dot{U}_R &\leq -k_e \sum_{i\in\mathcal{V}_k} (\phi(\bar{R}^{*T}R_{woi}) - \phi(\bar{R}^{*T}\bar{R}_{woi}) + \sigma_i \phi(R_{woi}^T \bar{R}_{woi})) \\
&\quad -k_s \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} \sigma_i \phi(R_{woi}^T R_{woj})
\end{aligned}$$

Proof of Theorem

Lemma3

Suppose $\phi(\bar{R}^{*T}R_{woi}) < \beta := 1 - \sqrt{2(\phi(\bar{R}^{*T}\bar{R}_{woh}) + c)}$ holds true.
Then we have

$$\lambda_{\min}(\mathrm{sym}(\bar{R}^{*T}R_{woi})) \geq \beta := 1 - \sqrt{2(\phi(\bar{R}^{*T}\bar{R}_{woh}) + c)}$$

$$\rho_R := \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T}\bar{R}_{woi})$$

$$\begin{aligned}
\dot{U}_R &\leq k_e \rho_R - k_e \sum_{i\in\mathcal{V}_k} (\phi(\bar{R}^{*T}R_{woi}) + \beta\phi(R_{woi}^T \bar{R}_{woi})) \\
&\quad -k_s \sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i} \beta\phi(R_{woi}^T R_{woj})
\end{aligned}$$

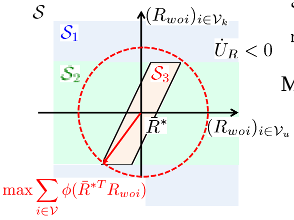
Proof of Theorem

Consider 3 sets $\mathcal{S} = \{(R_{woi})_{i\in\mathcal{V}} | \bar{R}^{*T} R_{woi} > 0 \quad \forall i \in \mathcal{V}\}$

$$\mathcal{S}_1 = \{(R_{woi})_{i\in\mathcal{V}} \in \mathcal{S} | \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) > \rho_R\}$$

$$\mathcal{S}_2 = \{(R_{woi})_{i\in\mathcal{V}} \in \mathcal{S}\backslash\mathcal{S}_1 | \sum_{i\in\mathcal{V}_k}\sum_{j\in\mathcal{N}_i} \phi(R_{woi}^T R_{woj}) > \frac{1}{\beta k}\rho_R\} \qquad k = k_s/k_e$$

$$\mathcal{S}_3 = \{(R_{woi})_{i\in\mathcal{V}} \in \mathcal{S}\backslash(\mathcal{S}_1 \cup \mathcal{S}_2)\}$$



$\mathcal{S}_1, \mathcal{S}_2$: $\dot{U}_R < 0$

$\max \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) > \rho_R$ ⇨ $\dot{U}_R < 0$

Maximization problem

$$\max \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi})$$

$$\text{s.t.} \begin{cases} \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) \le \rho_R \\ \sum_{i\in\mathcal{V}_k}\sum_{j\in\mathcal{N}_i} \phi(R_{woi}^T R_{woj}) \le \frac{1}{\beta k}\rho_R \end{cases}$$

Proof of Theorem

Maximization problem

$$\max \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi})$$

$$\text{s.t.} \begin{cases} \sum_{i\in\mathcal{V}_k} \phi(\bar{R}^{*T} R_{woi}) \le \rho_R \\ \sum_{i\in\mathcal{V}_k}\sum_{j\in\mathcal{N}_i} \phi(R_{woi}^T R_{woj}) \le \frac{1}{\beta k}\rho_R \end{cases}$$

Maximization value

$$\max \sum_{i\in\mathcal{V}} \phi(\bar{R}^{*T} R_{woi}) = (1 + \frac{|\mathcal{V}_u|}{|\mathcal{V}_k|})\rho_R + 4|\mathcal{V}_u|\sqrt{\frac{\rho_R \epsilon'}{|\mathcal{V}_k|}} + 4|\mathcal{V}_u|\epsilon' \qquad \epsilon' = \frac{\mathrm{diam}(G)\rho_R}{k\beta}$$

$\mathrm{diam}(G) = \min_{j\in\mathcal{V}} \sum_{i\in\mathcal{V}} l_{ij}^2$ $\quad l_{ij}$: Size of the shortest path from note *i* to *j* along the graph *G* whose edges are replaced by undirected ones

$|\mathcal{V}_k|$: number of known pose camera $\quad |\mathcal{V}_u|$: number of unknown pose camera

⇨ $1+\epsilon$ level averaging $\quad \epsilon = |\mathcal{V}_u|(\frac{1}{\sqrt{|\mathcal{V}_k|}} + 2\sqrt{\frac{\mathrm{diam(G)}}{k\beta}})^2$