Optimal Inter-Vehicular Distance Control with Constraints in Vehicle Networks

Takuto Takagi
FL11-11-1
15th, July, 2011

Background
Highway congestion is imposing an intolerable burden on urban residents.

Concentration occurs when the vehicle's velocity variation propagates to following vehicles.

It is difficult for human drivers to recognize the tiny change in the preceding vehicle's velocity.

Approaches
There are various approaches to improve congestion.

They can be classified as:
- Macro perspective and micro perspective.

Macro perspective: On-ramp control, Transportation Network.
Micro perspective: Vehicle Platooning Control.

P. Varaiya, "Smart Cars on Smart Roads: Problems of Control."

Review
Considering n vehicles in an one lane highway.

Optimization Problem
\[
\begin{align*}
\text{max} & \quad f(x) = ax^2 + bx + c \\
\text{s.t.} & \quad g_i(x) = 0, \quad i = 1, \ldots, m
\end{align*}
\]

Proposition 4-1
Consider the finite horizon optimization problem (1) under the dynamics (2).

The optimal control at time k, for k=0,1,...,N-1 is

The optimal solution is found by solving a sequence of linear programs.

New Approach
Considering n vehicles in an one lane highway.

Optimization Problem
\[
\begin{align*}
\text{max} & \quad f(x) = ax^2 + bx + c \\
\text{s.t.} & \quad g_i(x) = 0, \quad i = 1, \ldots, m
\end{align*}
\]

Parameter Settings
Vehicle Dynamics
\[
q_{k+1} = A_k q_k + B_k S_k q_k = [x_k, v_k]^T
\]

Communication graph
Considering the simplest graph

Feedback the preceding vehicle information

Input (Inter-vehicular distance)
Time-invariant constant inter-vehicular distance
\[
S_j = M \cdot [0, M, M, \ldots]^T
\]

Considering the case of one input

Active Set
Example

\[
\begin{align*}
\text{min} & \quad f(x) = ax^2 + bx + c \\
\text{strictly convex} & \quad g_i(x) = 0, \quad i = 1, \ldots, m
\end{align*}
\]

Meaning
Active set:
- It determines which constraints influence the final result of optimization.
- In solving the linear programming problem, the active set gives the hyperplanes that intersect at the solution point.

If we know active set in advance, we can make the constraints low-dimensional.
Constraints analysis

Deformation the collision avoidance constraints

\[ D_x \geq 0 \]

\[ D_{x1} \geq 0, D_{x2} \geq 0, \ldots, D_{xn} \geq 0 \]

\[ D = \begin{bmatrix} 1 & -1 \end{bmatrix} \]

**Assumption**

Constraints

\[ D_{x1} \geq 0, D_{x2} \geq 0, D_{x3}(M) \geq 0, \ldots, D_{x5}(M) \geq 0 \]

**Constraints analysis**

Assumption

Constraints

System is stable, there is no disturbance, assume \( D_{x0} \geq 0, D_{x1} \geq 0 \)

\[ x_{s1} - x_{s2} = M \]

\[ \Phi = \Phi_{s1} \]

Future works

- Analyzing the relationship between graph laplacian and constraints
- Analyzing the relationship between gain and string stability
Appendix

Consideration about Graph Laplacian

Relative communication structure

\[ \begin{bmatrix}
 1 & -1 & 0 \\
 1 & -1 & 0 \\
 1 & -1 & 1
\end{bmatrix} \]

It would be better for vehicles to feedback the absolute information (reference) such as a leader.

Control input

\[ u_t = K_x(s)(x-x_t) + K_v(v-v_t) \]

Assumption

(A5) \( K_x, K_v \) are constant gain which satisfy (A1)

There exists some \( L \) such that

\[ u = K_x(\dot{x} - S) + K_v \]

Assumption

(A6) \( L \) is time invariant

Platoon model

\[ q = A^*q + B^*u \]

\[ q = [x^T, v^T]^T \]

Platoon model

\[ q = A^*q + B^*u \]

String Stability

Definition[1]

Consider a string of N dynamic systems: the error signals \( e(t) \) depends on the disturbances \( d(t) \) in the following manner:

\[ e(t) = H_{ij}(s)d(t) \quad e, d \in \mathbb{R}^n \quad H_{ij}(s): \mathbb{R}^n \rightarrow \mathbb{R}^n \] (∗)

The system (*) is L sliding mode if given any \( \varepsilon > 0 \) there exist a \( \delta > 0 \) such that

\[ \|e\| < \delta \Rightarrow \|\dot{e}\| < \varepsilon \]

Assumption

• LTI SISO plant/controller
• Each loop has relative degree
• Homogeneous loop

Deformation

\[ \begin{align*}
  \|e\| < \delta \Rightarrow \|\dot{e}\| < \varepsilon \\
  \|\dot{e}\| = \sup_{t \geq 0} \|\dot{e}\|\] \\
  \|H_{ij}(s)\| < \gamma \\
\end{align*} \]

Each vehicle has a different communication graph from the one of precedent vehicle.
From [3], if \( \|r(t)\| < \gamma \) such that
\[
\left\| H_{i,d} (s) \right\| < \gamma, \forall N
\]
sufficient condition:
\[
\left\| \frac{\partial}{\partial s} r_{i}(s) \right\| \leq \frac{1}{\gamma}
\]
The perturbation doesn’t propagate to following vehicles.