



Optimal Power Dispatch of Power Networks with Potential Games



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Backgrounds

Smart Grid : electricity network using digital technology

- Real time management of supply and demand
- Energy efficiency
- Increasing reliability
- Use of Renewable energy (solar, wind, etc)
 - reduction greenhouse gases
 - their output fluctuate widely and randomly
 - instability!

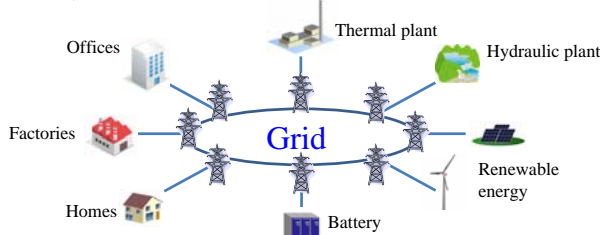


Conventional power plants and renewable energy need to generate electricity in a coordinated way



Objective (Optimal Power Dispatch)

Electric system — Reliability + Economic potential



- property of generators(thermal, solar, etc)
- network structure

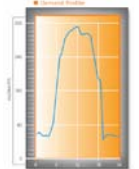
➡ **Optimal power dispatch for customers**
determine each generator's supply such that **minimizes fuel cost** and satisfies **supply-demand balance**



Objective (Related Work)

Related work : Analysis of power network

- Optimal power flow with battery[1, 2]
 - introduction of **battery**
 - static → **dynamic optimization**
 - analysis optimal generation schedule
- Demand response[3]
 - manage customer consumption in response to supply condition
 - non-cooperative game** (customers and utility)
 - decision-making process of price and power dispatch



No design policy of optimal power dispatch
➡ **Game theoretic approach**

DoE, Smart Grid Intro, 2008



Objective (Game-theoretic Control)

Game-theoretic approach

- Agents are self-interested
➡ **Non-cooperative game**
- The solution to the problem = the equilibrium of the game



Applications : resource allocation, sensor coverage

Advantages

- robustness to failures and environmental disturbances
- guarantee global convergence
- improved scalability

➡ **Potential game** has

Design policies of objective function Learning algorithms

Objective of this work
To apply potential game to optimal power dispatch problem of power network



Definition of Game

- Player set $N = \{1, \dots, n\}$
 - Collection of action sets $\mathcal{A} = \mathcal{A}_1 \times \dots \times \mathcal{A}_n$
Agent i 's action set : \mathcal{A}_i
Agent i 's action : $\mathbf{a}_i \in \mathcal{A}_i$
 - Collection of objective function $U = \{U_1, \dots, U_n\}$
Agent i 's objective function $U_i : \mathcal{A} \rightarrow \mathbb{R}$
Every agent chooses \mathbf{a}_i to maximize U_i
- ➡ **Game $G = \langle N, \mathcal{A}, U \rangle$**

Nash equilibrium

A pure Nash equilibrium is an action $\mathbf{a}^* \in \mathcal{A}$ such that $\forall i \in N$

$$U_i(\mathbf{a}_i^*, \mathbf{a}_{-i}^*) = \max_{\mathbf{a}_i \in \mathcal{A}_i} U_i(\mathbf{a}_i, \mathbf{a}_{-i}^*)$$

$$(\mathbf{a}_{-i} = (\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{i-1}, \mathbf{a}_{i+1}, \dots, \mathbf{a}_n))$$

(Ex.) Payoff Matrix

| | | | |
|----------|---|----------|-------|
| | | Player 2 | |
| | | A | B |
| Player 1 | A | (2,2) | (1,0) |
| | B | (0,1) | (4,4) |
| | | NE | |



Potential Game

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- Potential function (global objective function) $\phi : \mathcal{A} \rightarrow \mathbb{R}$

$\phi : \max$ \Rightarrow Objective of a group is achieved

Potential game

A game $G = \langle N, \mathcal{A}, U \rangle$ is a potential game if there is a potential function ϕ such that $\forall i \in N, \forall a_{-i} \in \mathcal{A}_{-i}$ and $\forall a'_i, a''_i \in \mathcal{A}_i$,

$$U_i(a'_i, a_{-i}) - U_i(a''_i, a_{-i}) = \phi(a'_i, a_{-i}) - \phi(a''_i, a_{-i})$$

Key property

- the guaranteed existence of a Nash equilibrium
- Local maxima of ϕ are Nash equilibria

\Rightarrow Application of appropriate learning algorithms

Optimal Distributed Inhomogeneous Synchronous Learning (ODISL) [6]

- Convergence to optimal Nash equilibrium with probability 1

(Ex.) Payoff Potential

| | A | B | | A | B |
|---|-------|-------|---|---|---|
| A | (2,2) | (1,0) | A | 2 | 0 |
| B | (0,1) | (4,4) | B | 0 | 3 |

Global objective



Scenarios

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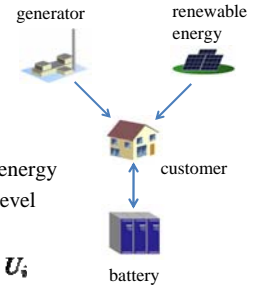
Problem setting

\triangleright Design of potential game

- Agent : generator, renewable energy, battery, customer
- Potential function ϕ

- minimize supply from generator
- maximize supply from renewable energy
- remaining battery level = desired level
- demand = supply

\Rightarrow Each agent's objective function U_i



Numerical simulation

- \triangleright Learning algorithm : ODISL [6]
- \triangleright Compared with results of [2]



Reference

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- [1] K. M. Candy, S. H. Low, U. Topcu and H. Xu, "A Simple Optimal Power Flow Model with Energy Storage," Proc. of the 49th IEEE Conference on Decision and Control, pp.1051-1057, 2010
- [2] 星, "分散協調予測制御を用いたバッテリーを含むパワーネットワークの動的最適潮流計算," 東京工業大学修士論文, 2011
- [3] L. Chen, N. Li, S. H. Low, and J. C. Doyle, "On Two Market Model for Demand Response in Power Networks," Proc. of the first IEEE International Conference on Smart Grid Communications, pp.1708-1713, 2010
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- [5] R. Gopalakrishnan, "An Architectural View of Game Theoretic Control," California Institute of Technology Master's Thesis, 2010
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