# Survey on Various Approaches to Power Networks

Akihiro Nomura
FL10-26-3
8th, February, 2011



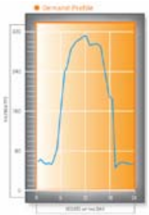
## Introduction : Power Networks



Power Network

A network which connects generations to loads

- Integrating renewable energy
  - sustainability, available worldwide
- Efficiency
  - minimize energy loss (Optimal power flow)
- Demand response
  - manage customers consumption of electricity in response to supply condition
  - help reduce peak-energy demand and adopt demand to fluctuating generations



DoE, Smart Grid Intro, 2008



## Preliminaries [5]

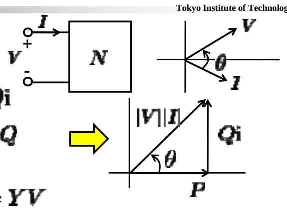


Voltage [V]: $V$ Current [A]: $I$

Power [W]: $q = VI^*$

$= |V||I|e^{\theta i} = P + Qi$

active power : $P$ reactive power : $Q$



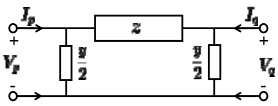

Impedance [Ω]: $Z$

Admittance [S]: $Y$ $Y = Z^{-1}$, $I = YV$

⇨ $\mathbf{I} = \mathbf{Y}_{\mathrm{bus}}\mathbf{V}$

$\mathbf{Y}_{\mathrm{bus}}$ : bus admittance matrix

- symmetric
- $Y_{ii}$ = the sum of admittance to the $i$th node
- $Y_{ij}$ = the negative of admittance between node $i$ and $j$




$$\begin{bmatrix} I_p \\ I_q \end{bmatrix} = \begin{bmatrix} (\frac{1}{z}+\frac{y}{2}) & -(\frac{1}{z}) \\ -(\frac{1}{z}) & (\frac{1}{z}+\frac{y}{2}) \end{bmatrix} \begin{bmatrix} V_p \\ V_q \end{bmatrix}$$



## Integration of Renewable Energy [1]



Renewable energy's output fluctuates widely and randomly

⇨ It is difficult to integrate them

⇨ Energy storage

$\mathcal{N} = \mathcal{G} \cup \mathcal{D}$ { $\mathcal{G}$: generator, $\mathcal{D}$: demand }

Power flow $i \neq j \in \mathcal{N}$ Line capacities

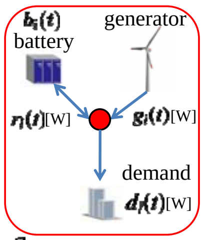
$V_i V_j Y_{ij}(\theta_i(t) - \theta_j(t)) \leq \bar{q}_{ij}(t) \cdots (1)$

$q_i(t) = \sum_{j \in \mathcal{N}} V_i V_j Y_{ij}(\theta_i(t) - \theta_j(t)) \cdots (2)$

$\theta_i$ : voltage phase at node $i$

Demand $i \in \mathcal{D}$

$q_i(t) = -d_i(t) \cdots (3)$




Generator $i \in \mathcal{G}$

$q_i(t) = g_i(t) + r_i(t) \cdots (4)$

$g_i(t) \geq 0 \cdots (5)$

Battery level $i \in \mathcal{G}$

$b_i(t) = b_i(t-1) - r_i(t) \cdots (6)$

$0 \leq b_i(t) \leq B_i \cdots (7)$



## Optimal Power Flow with Energy Storage [1]



$\min_{\theta, q, r, g, b} \sum_{t=1}^{T} \sum_{i \in \mathcal{G}} [c_i(g_i(t), t) + h_i(b_i(t), r_i(t))]$

generation cost battery cost

s.t. (1), (2), (3), (4), (5), (6), (7)

Restriction

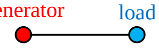
$c_i(g_i(t), t) := \frac{1}{2}\gamma_i(t) g_i^2(t)$

$h_i(b_i, r_i) = h_i(b_i)$

⇨ Optimal solutions $g^*(t)$, $b^*(t)$

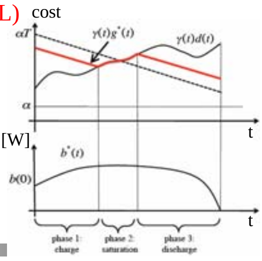
- Single generator and single load (SGSL)

generator load



Assumption

Demand $d(t)$ doesn't decrease too rapidly




⇨ $g^*(t)$ cross $d(t)$ at once, from above

$b^*(t)$ : charge → discharge

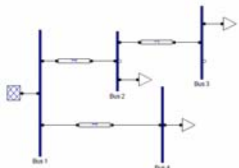


## Efficiency - Minimizing Power Loss [2]



Means for minimizing power loss

- exploitation of capacitor banks
- network reconfiguration
- installation of distributed generation units



⇨ how the power loss is related the network topology?

Problem setting

- $n$ buses
- bus $n$ : only generator bus, $V_n = V_0$, power $V_n I_n^*$
- power $P_k + Q_k i$ is required at load buses $k \in \{1, 2, \ldots, n-1\}$

$P_k$ : active power

$Q_k$ : active power

Minimizing active power loss

$\min \ \mathrm{Re}\{V_n I_n^*\} - \sum_{k=1}^{n-1} P_k$

s.t. $V_k I_k^* = -P_k - Q_k i$

$|V_n| = V_0$, $I = YV$

⇨ $P_{\mathrm{loss}}$ : the minimum of active power loss

The minimum power loss $Q_{\mathrm{loss}}$ is similar

## Dual Problem [2]

Lagrangian

$$L(\lambda,\bar\lambda,\mu,V)=\sum_{k=1}^{n-1}(\lambda_k+1)(\mathrm{Re}\{V_kI_k^*\}+P_k)+\sum_{k=1}^{n-1}\bar\lambda_k(\mathrm{Im}\{V_kI_k^*\}+Q_k)+\mu(V_nV_n^*-V_0^2)+\mathrm{Re}\{V_nI_n^*\}-\sum_{k=1}^{n-1}P_k$$

Lagrange multiplier $\lambda,\ \bar\lambda,\ \mu$

Definitions ($k=1,2,\ldots,n-1$)

$Y_k := e_ke_k^*Y$ (standard basis vector)

$$\mathbf{Y}_k := \frac{1}{2}\begin{bmatrix}\mathrm{Re}\{Y_k+Y_k^T\} & \mathrm{Im}\{Y_k^T-Y_k\}\\ \mathrm{Im}\{Y_k-Y_k^T\} & \mathrm{Re}\{Y_k+Y_k^T\}\end{bmatrix}$$

$$\mathbf{Y} := \begin{bmatrix}\mathrm{Re}\{Y\} & 0\\ 0 & \mathrm{Re}\{Y\}\end{bmatrix}\quad \bar{\mathbf{Y}}_k := \frac{1}{2}\begin{bmatrix}\mathrm{Im}\{Y_k+Y_k^T\} & \mathrm{Re}\{Y_k-Y_k^T\}\\ \mathrm{Re}\{Y_k^T-Y_k\} & \mathrm{Im}\{Y_k+Y_k^T\}\end{bmatrix}$$

$M\in\mathbb{R}^{2n\times 2n}$ : diagonal matrix whose entries are zero, except for its ($n$,$n$) and ($2n$,$2n$) entries are 1





## Dual Problem [2]

$W := [\ \mathrm{Re}\{V\}^T\ \ \mathrm{Im}\{V\}^T\ ]^T$

⇨ $L(\lambda,\bar\lambda,\mu,V)=\sum_{k=1}^{n-1}\lambda_kP_k+\sum_{k=1}^{n-1}\bar\lambda_kQ_k-\mu V_0^2+W^T\Phi(\lambda,\bar\lambda,\mu)W$

$$\left(\Phi(\lambda,\bar\lambda,\mu)=\sum_{k=1}^{n-1}\lambda_k\mathbf{Y}_k-\sum_{k=1}^{n-1}\bar\lambda_k\bar{\mathbf{Y}}_k+\mathbf{Y}+\mu M\right)$$

objective function (dual problem) : $\max_{\lambda,\bar\lambda,\mu}\min_V L(\lambda,\bar\lambda,\mu,V)$

$\Phi(\lambda,\bar\lambda,\mu)\succeq 0$ ⇨ $\min W^T\Phi(\lambda,\bar\lambda,\mu)W=0$

otherwise ⇨ $-\infty$

Dual problem

$$\max \sum_{k=1}^{n-1}\lambda_kP_k+\sum_{k=1}^{n-1}\bar\lambda_kQ_k-\mu V_0^2\quad \text{s.t. } \Phi(\lambda,\bar\lambda,\mu)\succeq 0$$





## LMI Optimization Problem and Duality Gap [2]

LMI Optimization Problem

$\lambda := [\lambda_1\cdots\lambda_{n-1}]\in\mathbb{R}^{n-1},\ \ \bar\lambda := [\bar\lambda_1\cdots\bar\lambda_{n-1}]\in\mathbb{R}^{n-1}$

$\mu\in\mathbb{R}$

$$\max\ f(\lambda,\bar\lambda,\mu) := \sum_{k=1}^{n-1}\lambda_kP_k+\sum_{k=1}^{n-1}\bar\lambda_kQ_k-\mu V_0^2$$

$$\text{s.t. } \Phi(\lambda,\bar\lambda,\mu) := \sum_{k=1}^{n-1}\lambda_k\mathbf{Y}_k-\sum_{k=1}^{n-1}\bar\lambda_k\bar{\mathbf{Y}}_k+\mathbf{Y}+\mu M\succeq 0$$

⇨ $P_{\min}$ : the optimal value of $f(\lambda,\bar\lambda,\mu)$

weak duality theorem ⇨ $P_{\mathrm{loss}}\geq P_{\min}$

$f(\lambda,\bar\lambda,\mu)$ depends only on the load profile

$\Phi(\lambda,\bar\lambda,\mu)$ depends only on the topology of the network

$\mathrm{rank}\Phi(\lambda,\bar\lambda,\mu)\geq 2n-2$ ⇨ $P_{\mathrm{loss}}=P_{\min}$ zero duality gap!





## Market Models for Demand Response [3]

Market model (for matching supply or shaping demand)

- non-cooperative game
  - customer, utility company : selfish
    each objectives : maximize own revenue
  - with unique equilibrium which satisfies social welfare
    ∴ Each player maximizes its payoff
    ⇨ maximizes social welfare
- distributed demand response scheme
  - the utility and customers jointly determine price and supply
  - based on gradient algorithm • iterative

$q_i^{k+1}(t)=[q_i^k(t)+\gamma(U_i'(q_i^k(t),t)-p^k(t))]^{+i}$

customer —demand→ utility ⇨ equilibrium

utility —price→ customer $p^k(t)=C'(Q^k(t),t)$

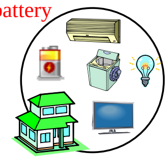




## Demand Response Based on Utility Maximization [4]

Households which operate different appliances and battery

⇨ Each customer $i$ operates a set $A_i$ of appliance



- Energy storage

$$b_i(t)=\sum_{\tau=1}^{t}r_i(\tau)+b_i(0),\quad 0\leq b_i(t)\leq B_i$$

- Customer's objective : maximize own benefit

$$\max_{q_i,r_i}\sum_{a\in A_i}U_{i,a}(q_{i,a})-D_i(r_i)-\sum_t p(t)Q_i(t)$$

($D_i(r_i)$: battery cost)

$q_{i,a}$ : power demanded by customer i for appliance $a$

Total demand : $Q_i(t)=\sum_{a\in A_i}q_{i,a}(t)+r_i(t)$

$q_i := (q_{i,a},\forall a\in A_i)$

$q := (q_i,\forall i)$

$p$ : price $U_{i,a}$: utility

- Utility's objective : maximize the social welfare

$$\max_{q,r}\sum_i\left(\sum_{a\in A_i}U_{i,a}(q_{i,a})-D_i(r_i)\right)-\sum_t C\left(\sum_i Q_i(t)\right)$$

(running cost)





## Demand Response Based on Utility Maximization [4]

- Distributed algorithm
  - The utility and customers jointly compute optimal prices and demand schedule
  - based on gradient algorithm • iterative

$p^k(t)=C'\left(\sum_i Q_i^k(t)\right)$

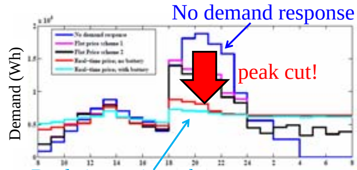
utility —price→ customer; customer —demand→ utility

- Detailed appliance model (Utility and constraints)
  1. air conditioner, refrigerator
  2. plug-in hybrid electric vehicle, cloth washer
  3. lightning
  4. entertainment

$q_{i,a}^{k+1}(t)=q_{i,a}^k(t)+\gamma\left(\frac{\partial U_{i,a}(q_i^k)}{\partial q_{i,a}^k(t)}-p^k(t)\right)$

$r_i^{k+1}(t)=r_i^k(t)-\gamma\left(\frac{\partial D_i(r_i^k)}{\partial r_i^k(t)}+p^k(t)\right)$

real time pricing
battery
⇨ reduce the peak load and variation in demand



## Summary

### Renewable Energy

- Optimal power flow with energy storage

⇨ Unrealistic…

### Efficiency

- Relationship between power loss and network topology

### Demand Response

- Market models
- Real-time pricing + battery
- Different appliance model

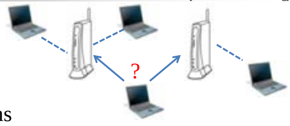
⇨ It is difficult for utility to know each customers' utility?
Modeling of utility and cost function is an active research issue





## Introduction : Game-Theoretic Control

### Resource allocation




- the distribution of resources among competing groups of agents
- arises in nearly all computer systems
  e.g. sensor coverage, wireless access point assignment
- needs to be solved in a distributed, decentralized manner

⇨ Game-theoretic approach

◆ non-cooperative game : players are self-interested

◆ the solution to the problem emerges as the equilibrium of the game

➢ Advantages

- robustness to failures and environmental disturbances
- minimal communication requirements
- improved scalability





## Resource Allocation Game

- Player set $N := \{1,\dots,n\}$ • Resource set $\mathcal{R} := \{r_1,\dots,r_m\}$
- Action set for agents $i \in N$ : $\mathcal{A}_i \subseteq 2^{\mathcal{R}}$
- Utility function $U_i : \mathcal{A} \to \mathbb{R}$ $\quad \mathcal{A} = \mathcal{A}_1 \times \dots \times \mathcal{A}_n$
- Action profile : $a = (a_1,\dots,a_n) \in \mathcal{A}$

$$a_{-i} = \{a_1,\dots,a_{i-1},a_{i+1},\dots,a_n\} \Rightarrow a = (a_i, a_{-i})$$

**Pure Nash equilibrium** $a^* \in \mathcal{A}$

$$U_i(a_i^*, a_{-i}^*) = \max_{a_i \in \mathcal{A}_i} U_i(a_i, a_{-i}^*), \ \forall i \in N$$

**Potential game**

$\exists \phi : \mathcal{A} \to \mathbb{R}$ such that $\forall i \in N, \forall a_{-i} \in \mathcal{A}_{-i}$ and $\forall a_i', a_i'' \in \mathcal{A}_i$,

$$U_i(a_i', a_{-i}) - U_i(a_i'', a_{-i}) = \phi(a_i', a_{-i}) - \phi(a_i'', a_{-i})$$

Potential game has at least one equilibrium





## Distributed Welfare Games (DWG)

- Welfare function : $W : \mathcal{A} \to \mathbb{R}$

◆ Linearly separable

set of agents allocated to resource $r$ in $a$

$$W(a) = \sum_{r \in \mathcal{R}} W_r(\{a\}_r), \quad \{a\}_r := \{i \in N : r \in a_i\}$$

local welfare function for resource $r$

◆ Submodular $\quad \forall X, Y \subseteq N$

$$W_r(X) + W_r(Y) \geq W_r(X \cup Y) + W_r(X \cap Y)$$

**Distributed Welfare Game**

Each player's utility : some triction of the welfare garnered at each resource the agent is using

$$\Rightarrow U_i(a_i, a_{-i}) = \sum_{r \in a_i} f_r(i, \{a\}_r)$$

$f_r$ : distribution rule at resource $r$

(1) $f_r(i, \{a\}_r) \geq 0$ (2) $i \notin \{a\}_r \Rightarrow f_r(i, \{a\}_r) = 0$

(3) $\Sigma_i f_r(i, \{a\}_r) \leq W_r(\{a\}_r)$



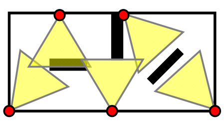


## Future Works

### Game-Theoretic Control

- Resource Allocation game
- Distributed Welfare Game

### Future Works



Applying game-theoretic control to

- Camera networks
  — Optimal placement of multiple visual ssensors
- Power networks
  — Optimal power flow with battery

⇨ Resource allocation problem





## References (Power Networks)


[1] K. M. Candy, S. H. Low, U. Topcu and H. Xu, "A Simple Optimal Power Flow Model with Energy Storage," In *IEEE Conference on Decision and Control*, 2010

[2] J. Lavaei and S. H. Low, "Relationship Between Power Loss and Network Topoligy in Power Systems," Proc. of *49th IEEE Conference on Decision and Control*, 2010

[3] L. Chen, N. Li, S. H. Low, and J. C. Doyle, "On Two Market Model for Demand Response in Power Networks," *IEEE SmartGridComm*, 2010

[4] N. Li, L. Chen, S. H. Low, "Optimal Demand Response Based on Utility Maximization in Power Networks," submitted to *IEEE Power Engineering Society Ganeral Meeting*, 2011

[5] A. R. Bergen and V. Vittal, *Power System Analysis*, 2nd ed. Prentice Hall, 2000

## References (Game-Theoretic Control)



[6] J. R. Marden and A. Wierman, "Distributed Welfare Games," *Under submission*
[7] J. R. Marden and A. Wierman, "Distributed Welfare Games with Application to Sensor Coverage," Proc. of *47th IEEE Conference of Decision and Control* , pp.1708 - 1713, 2008
[8] R. Gopolakrishnan, "An Architectural View of Game Theoretic Control," *California Institute of Technology Master's Thesis*, 2010
[9] R. Gopolakrishnan, J. R. Marden and A. Wierman, "An Architectural View of Game Theoretic Control," Proc. of *ACM Sigmetrics*, 2010
[10] J. R. Marden and A. Wierman, "Overcoming Limitations of Game-Theoretic Distributed Control," Proc. of *48th IEEE Conference of Decision and Control*, pp. 6466 - 6471, 2009
[11] 岡田 章, "ゲーム理論," 有斐閣, 2000





# Appendix



## Dual Problem



**Primal Problem**

$$\max_{x_r} \sum_{r \in \mathcal{S}} U_r(x_r)$$

$$\text{s.t.} \quad \sum_{r: l \in r} x_r \le c_l, \ \forall l \in \mathcal{L}$$

$$x_r \ge 0, \ \forall r \in \mathcal{S}$$

Lagrange dual $p_l$ : Lagrange multiplier

$$D(p) = \max_{\{x_r > 0\}} \sum_r U_r(x_r) - \sum_l p_l \left( \sum_{s: l \in s} x_s - c_l \right)$$

Utility Constraints

⇒ **Dual Problem**

$$\min_{p \ge 0} D(p)$$



## Submodular



Coalition $\forall S, T \subseteq N$

Supermodular (convex game)

$$v(S) + v(T) \le v(S \cup T) + v(S \cap T)$$

⇒ $v(S \cup \{i\}) - v(S) \le v(T \cup \{i\}) - v(T), \ S \subseteq T, \ i \notin S$

Marginal contribution

The larger coalitions he joins, the higher marginal contribution he gets

Submodular

$$v(S) + v(T) \ge v(S \cup T) + v(S \cap T)$$

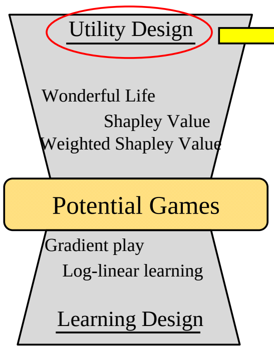
⇒ $v(S \cup \{i\}) - v(S) \ge v(T \cup \{i\}) - v(T), \ S \subseteq T, \ i \notin S$

The larger coalitions he joins, the lower marginal contribution he gets



## Distributed Welfare Games






'hourglass' architecture

Cost sharing methodologies

Player set $S \subseteq N$, $i \in S$

- Wonderful Life Utility (WLU)

$$f_r(i, S) = W_r(S) - W_r(S \setminus \{i\})$$

- The Shapley Value (SV)

$$f_r(i, S) = \sum_{T \subseteq S \setminus \{i\}} \omega_S (W_r(S) - W_r(S \setminus \{i\}))$$

$$\omega_S = \frac{|T|!(|S| - |T| - 1)!}{|S|!}$$

- The Weighted Shapley Value (WSV)



## Shapley Value



Coalition
- {A}, {B}, {C}
- {A, B}, {A, C}, {B, C}
- {A, B, C}

Characteristic function

$v(A) = 6$, $v(B) = 4$, $v(C) = 2$

$v(A, B) = 20$, $v(A, C) = 15$, $v(B, C) = 10$

$v(A, B, C) = 24$

Marginal contributions

| order | A | B | C |
|---|---|---|---|
| A B C | 6 | 14 | 4 |
| A C B | 6 | 9 | 9 |
| B A C | 16 | 4 | 4 |
| B C A | 14 | 4 | 6 |
| C A B | 13 | 9 | 2 |
| C B A | 14 | 8 | 2 |

Marginal contributions (A←B←C)

$A : v(A) - 0 = 6$

$B : v(A, B) - v(A) = 20 - 6 = 14$

$C : v(A, B, C) - v(A, B) = 24 - 20 = 4$

Shapley value

$$\phi_i(v) = \sum_{S: i \notin S} \frac{|S|!(|N| - |S| - 1)!}{|N|!} (v(S \cup \{i\}) - v(S))$$

$\phi_A = 11.5$
$\phi_B = 8$
$\phi_C = 4.5$