Passivity-based Cooperative Estimation for Networked Visual Motion Observers

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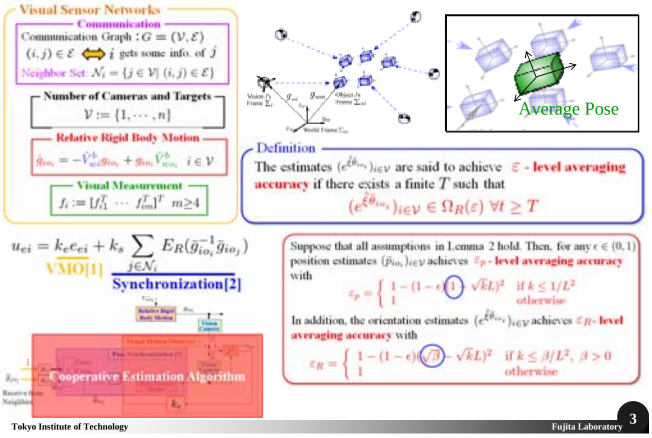
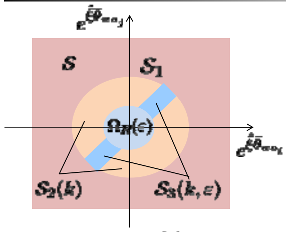
Outline

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE TCST*, Vol. 15, No. 1, pp. 40-52, 2007.

Review



Revisit to Proof



$$\mathcal{S} = \left\{ (e^{\hat\xi\bar\theta_{wo_i}})_{i\in\mathcal{V}} \,\middle|\, e^{-\hat\xi\bar\theta_{wo_i}^T} e^{\hat\xi\bar\theta_{wo_i}} > 0\ \forall i \in \mathcal{V} \right\}$$

$$\Omega_R(\varepsilon) := \left\{ (e^{\hat\xi\bar\theta_{wo_i}})_{i\in\mathcal{V}} \,\middle|\, \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta_{wo}^*} e^{\hat\xi\bar\theta_{wo_i}}) \le \varepsilon\rho_R \right\}$$

$$\mathcal{S}_1 = \mathcal{S} \cap \Omega_R(1)$$

$$\mathcal{S}_2(k) = \left\{ (e^{\hat\xi\bar\theta_{wo_i}}) \,\middle|\, \beta \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}}) \ge k\rho_R \right\}$$

$$\mathcal{S}_3(k,\varepsilon) = \mathcal{S}_1 \setminus (\mathcal{S}_2(k) \cup \Omega_R(\varepsilon))$$

After the time $\tau(c)$

$$\dot V \le -a_1 \ \text{If}\ (e^{\hat\xi\bar\theta_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}_1 \quad a_1 := \beta \sum_{i\in\mathcal{V}} \left(k_e \phi(e^{-\hat\xi\theta_{wo}^*} e^{\hat\xi\bar\theta_{wo_i}}) + k_s \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}}) \right)$$

$$\dot V \le -a_2 \ \text{If}\ (e^{\hat\xi\bar\theta_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}_2(k) \quad a_2 := k_s \sum_{i\in\mathcal{V}} \left(\phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) + \beta\phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}}) \right)$$

$$\dot V \le -a_3 \ \text{If}\ (e^{\hat\xi\bar\theta_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}_3(k,\varepsilon_R) \quad a_3 := \beta \sum_{i\in\mathcal{V}} \left(k_e \varepsilon \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_i}}) + k_s \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}}) \right)$$

$a_1 \sim a_3$ can be viewed as a measure of the convergence speed

Time Complexity

Definition

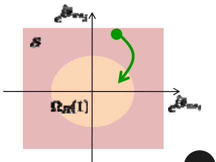
Given target orientations $(e^{\hat\xi\bar\theta_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}$ and initial estimates $(e^{\hat\xi\bar\theta_{io_i}})_{i\in\mathcal{V}} \in \mathcal{S}$, when the present estimation algorithm is applied, **the time complexity of ε – level averaging accuracy task** is defined by

$$T(\varepsilon) := \inf T \ \text{s.t.}\ (e^{\hat\xi\bar\theta_{wo_i}})_{i\in\mathcal{V}} \in \Omega_R(\varepsilon)\ \forall t \ge T$$

Inspired by Bullo et al. [4]

$$\Omega_R(\varepsilon) := \left\{ (e^{\hat\xi\bar\theta_{wo_i}})_{i\in\mathcal{V}} \,\middle|\, \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta_{wo}^*} e^{\hat\xi\bar\theta_{wo_i}}) \le \varepsilon\rho_R \right\}$$

$$\dot V \le -a_1 \to V(t) = V(0) - a_1 t \Rightarrow T(1) \le \frac{V(0) - \rho_R}{a_1}$$



But, what is a_1 ? No information!

Problem: $T(\varepsilon) \in O(f(G)), O(f(k_e)), O(f(k_s))$?

Theorem 2

Theorem

Suppose that all assumptions in Lemma 2 hold and the graph is undirected. Then, we have

$$T_1(1) \le \bar T_1 := \tau(c) + \frac{V(0) - \rho_R}{Q\beta\lambda_{min2}(L_G)} \left(\frac{1}{k_s} + \frac{2n}{k_e} \right)$$

$$Q := \frac{1}{n^2} \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}} \phi(e^{-\hat\xi\theta_{wo_i}} e^{\hat\xi\theta_{wo_j}})$$

In addition, if $k \le \beta/L^2,\ \beta > 0$, then

$$T(\varepsilon_R) \le \tau(c) + \bar T_1 + \max\{\bar T_2, \bar T_3\}$$

$$\bar T_2 := \frac{(1+\beta)(1-\varepsilon_R)}{k_e\beta},\quad \bar T_3 := \frac{(1-\varepsilon_R)\rho_R}{Q\beta\lambda_{min2}(L_G)} \left(\frac{1}{k_s} + \frac{2n}{\epsilon k_e} \right)$$

Corollary

Given n, c, ϵ, k ($\to \varepsilon_R$ in Theorem 1 is decides), then we have

$$T(\varepsilon_R) \in O\left(\frac{1}{k_s \lambda_{min2}(L_G)} \right)$$

Proof of Theorem 2

In the following, we consider only $\dot{V} \le -a_1$ If $(e^{\hat{\xi}\bar{\theta}_{ei}})_{i\in\mathcal{V}} \in \mathcal{S}_1$

Any lower bound of a_1 gives an upper bound of $T(1)$

$$a_1 := \beta \sum_{i\in\mathcal{V}} \left(k_e \phi(e^{-\hat{\xi}\theta_{wei}} e^{\hat{\xi}\bar{\theta}_{wei}}) + k_s \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat{\xi}\bar{\theta}_{wei}} e^{\hat{\xi}\bar{\theta}_{wej}}) \right)$$

$$\ge \min_{e^{\hat{\xi}\bar{\theta}_{wei}} \in SO(3),\ i\in\mathcal{V}} \beta \sum_{i\in\mathcal{V}} \left(k_e \phi(e^{-\hat{\xi}\theta_{wei}} e^{\hat{\xi}\bar{\theta}_{wei}}) + k_s \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat{\xi}\bar{\theta}_{wei}} e^{\hat{\xi}\bar{\theta}_{wej}}) \right)$$

$$= \min_{e^{\hat{\xi}\bar{\theta}_{wei}} \in SO(3),\ i\in\mathcal{V}} \frac{\beta}{2} \sum_{i\in\mathcal{V}} \left(k_e \| e^{\hat{\xi}\theta_{wei}} - e^{\hat{\xi}\bar{\theta}_{wei}} \|_F^2 + k_s \sum_{j\in\mathcal{N}_i} \| e^{\hat{\xi}\bar{\theta}_{wei}} - e^{\hat{\xi}\bar{\theta}_{wej}} \|_F^2 \right)$$

$$\ge \min_{R_i\in\mathbb{R}^{3\times3},\ i\in\mathcal{V}} \frac{\beta}{2} \sum_{i\in\mathcal{V}} \left(k_e \| R_i - e^{\hat{\xi}\theta_{wei}} \|_F^2 + k_s \sum_{j\in\mathcal{N}_i} \| R_i - R_j \|_F^2 \right)$$

Remove constraints $R_i^T R_i = I,\ \det(R_i) = 1$

Proof of Theorem 2

$$R_i = \begin{bmatrix} r_i^{11} & r_i^{12} & r_i^{13} \\ r_i^{21} & r_i^{22} & r_i^{23} \\ r_i^{31} & r_i^{32} & r_i^{33} \end{bmatrix},\quad e^{\hat{\xi}\theta_{wei}} = \begin{bmatrix} q_i^{11} & q_i^{12} & q_i^{13} \\ q_i^{21} & q_i^{22} & q_i^{23} \\ q_i^{31} & q_i^{32} & q_i^{33} \end{bmatrix}$$

Consider only (1,1)- component of the optimization problem

$$\min_{r_i^{11}\in\mathbb{R},\ i\in\mathcal{V}} \frac{\beta}{2} \sum_{i\in\mathcal{V}} \left(k_e \| r_i^{11} - q_i^{11} \|^2 + k_s \sum_{j\in\mathcal{N}_i} \| r_i^{11} - r_j^{11} \|^2 \right)$$

(Additional) Assumption 3: The graph *G* is undirected

$$\min_{r_i^{11}\in\mathbb{R},\ i\in\mathcal{V}} \frac{\beta}{2} \left(k_e \| r^{11} - q^{11} \|^2 + k_s (r^{11})^T L_G r^{11} \right) \qquad r^{11} = [r_1^{11}\ \cdots\ r_n^{11}]^T,\ q^{11} = [q_1^{11}\ \cdots\ q_n^{11}]^T$$

Graph Laplacian of the graph *G*

Optimal Solution:

$$k_e(r^{11} - q^{11}) + k_s L_G r^{11} = 0 \rightarrow k(r^{11} - q^{11}) + L_G r^{11} = 0$$

$$r^{11} = k(kI + L_G)^{-1} q^{11} = A q^{11},\ A = (I + L_G/k)^{-1}$$

Proof of Theorem 2

$$r^{11} = A q^{11},\ A = (I + L_G/k)^{-1} \longrightarrow \left(k_e \| r^{11} - q^{11} \|^2 + k_s (r^{11})^T L_G r^{11} \right)$$

$$\frac{\beta}{2} \left(k_e (q^{11})^T (I - A)^T (I - A) q^{11} + k_s (q^{11})^T A^T L_G A q^{11} \right)$$

$$= \frac{\beta}{2} (q^{11})^T \left(k_e (I - A)^T (I - A) + k_s A^T L_G A \right) q^{11}$$

$$= \frac{\beta}{2} (q^{11})^T \left(k_e (I - 2A) + A^T (k_e I + k_s L_G) A \right) q^{11}$$

$$= \frac{\beta}{2} (q^{11})^T \left(k_e (I - 2A) + k_e A^T A^{-1} A \right) q^{11}$$

$$= \frac{\beta}{2} (q^{11})^T \left(k_e (I - 2A) + k_e A \right) q^{11} = \frac{k_e\beta}{2} (q^{11})^T (I - A) q^{11}$$

$$= \frac{k_e\beta}{2} (q^{11})^T A (L_G/k) q^{11} = \frac{k_s\beta}{2} (q^{11})^T A (L_G) q^{11}$$

Proof of Theorem 2

$A L_G = L_G A$? (commutative)

$$(I + L_G/k)^{-1} L_G - L_G (I + L_G/k)^{-1}$$
$$= (I + L_G/k)^{-1} \{ L_G (I + L_G/k) - (I + L_G/k) L_G \} (I + L_G/k)^{-1} = 0$$

if A and B is symmetric and $AB = BA$, then $\lambda(AB) = \lambda(A)\lambda(B)$

$$\frac{k_s\beta}{2} (q^{11})^T A (L_G) q^{11} \ge \frac{k_s\beta}{2} \lambda_{min}(A) (q^{11})^T (L_G) q^{11}$$
$$= \frac{k_s\beta}{2} \frac{1}{1 + \lambda_{max}(L_G/k)} (q^{11})^T (L_G) q^{11}$$

The graph Laplacian L_G has $\mathbf{1} = [1\ \cdots\ 1]^T$ an eigenvector corresponding to the zero eigenvalue. Let the orthogonal component of q^{11} to the eigenspace is given by $\tilde{q}^{11} = M q^{11},\ M = (I - \mathbf{1}\mathbf{1}^T/n)$

$M = (I - \mathbf{1}\mathbf{1}^T/n)$: graph Laplacian for the complete graph

Proof of Theorem 2

$$\frac{k_s\beta}{2} \frac{1}{1 + \lambda_{max}(L_G/k)} (q^{11})^T (L_G) q^{11} \ge \frac{k_s\beta}{2} \frac{1}{1 + \lambda_{max}(L_G/k)} \lambda_{min2}(L_G) (\tilde{q}^{11})^T \tilde{q}^{11}$$

$$= \frac{k_s k_e \beta}{2(k_e + k_s \lambda_{max}(L_G))} \lambda_{min2}(L_G) ((q^{11})^T M q^{11}$$

$$= \frac{k_s k_e \beta}{2(k_e + k_s \lambda_{max}(L_G))} \lambda_{min2}(L_G) (\sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}} \| q_i^{11} - q_j^{11} \|^2 \frac{1}{n^2})$$

$$\min_{r_i^{11}\in\mathbb{R},\ i\in\mathcal{V}} \frac{\beta}{2} \sum_{i\in\mathcal{V}} \left(k_e \| r_i^{11} - q_i^{11} \|^2 + k_s \sum_{j\in\mathcal{N}_i} \| r_i^{11} - r_j^{11} \|^2 \right)$$

summation for (1,1) – (3,3)

$$\min_{R_i\in\mathbb{R}^{3\times3},\ i\in\mathcal{V}} \frac{\beta}{2} \sum_{i\in\mathcal{V}} \left(k_e \| R_i - e^{\hat{\xi}\theta_{wei}} \|_F^2 + k_s \sum_{j\in\mathcal{N}_i} \| R_i - R_j \|_F^2 \right)$$

$$a_1 \le \frac{k_s k_e \beta}{2(k_e + k_s \lambda_{max}(L_G))} \lambda_{min2}(L_G) (\sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}} \| e^{\hat{\xi}\theta_{wei}} - e^{\hat{\xi}\theta_{wej}} \|_F^2$$

$$= \frac{k_s k_e \beta Q}{k_e + k_s \lambda_{max}(L_G)} \lambda_{min2}(L_G),\ Q := \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_{wei}} e^{\hat{\xi}\theta_{wej}}) \frac{1}{n^2}$$

Outline

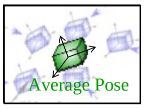
- Introduction
- Definition of Visual Sensor Networks
- Passivity-based Visual Motion Observer (VMO) [1]
- Cooperative Estimation Algorithm
- Averaging Performance Analysis Corrected!
- On Convergence Speed New!
- Tracking Performance Analysis New!
- Conclusion

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE TCST*, Vol. 15, No. 1, pp. 40-52, 2007.

Motion of Average

Euclidean Mean:

$$e^{\hat{\xi}\theta^*} := \arg\min_{e^{\hat{\xi}\theta}\in SO(3)} \sum_{j\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta}e^{\hat{\xi}\theta_{wo_j}})$$



If the target orientations $e^{\hat{\xi}\theta_{wo_j}}$ move, the mean $e^{\hat{\xi}\theta^*}$ also moves

Pose of Object: $g_{wo_i} = (p_{wo_i}, e^{\hat{\xi}\theta_{wo_i}})$

Body Velocity: $\hat{V}^b_{wo_i} = g^{-1}_{wo_i}\dot{g}_{wo_i}$

Rigid Body Motion

$$\dot{g}_{wo_i} = g_{wo_i}\hat{V}^b_{wo_i}$$

Assumption 4 (Target Object Motion)

$\hat{V}^b_{wo_i}$ is continuous in $t \geq 0$ and satisfies

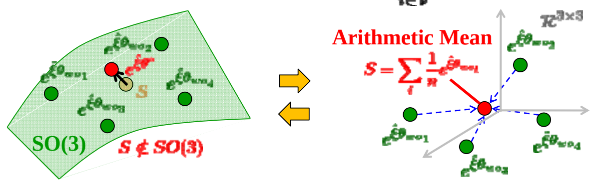
$$\|v^b_{wo_i}\|_2^2 \leq \bar{w}_p,\ \|\omega^b_{wo_i}\|_2^2 \leq \bar{w}_R\ \forall i \in \mathcal{V}$$

For all time $t \geq 0$, there exist $i, j \in \mathcal{V}$ such that $e^{\hat{\xi}\theta_{wo_i}} \neq e^{\hat{\xi}\theta_{wo_j}}$

$e^{-\hat{\xi}\theta_i^*}e^{\hat{\xi}\theta_{io_i}} > 0\ \forall i \in \mathcal{V}$ and $t \geq 0$

Nature of Euclidean Mean

$$e^{\hat{\xi}\theta^*} = \mathrm{Proj}(S(t)),\ S(t) = \frac{1}{n}\sum_{i\in\mathcal{V}} e^{\hat{\xi}\theta_{wo_i}}$$



$M = U_M\Sigma_M V_M^T$ (Singular Value Decomposition)

$\Rightarrow \mathrm{Proj}(M) = U_M V_M^T \in SO(3)$ [5]

or from polar decomposition

$$e^{\hat{\xi}\theta^*}(t) = S(t)P_S(t)^{-1},\ P_S^2(t) = S^T(t)S(t)$$

Body Velocity of Average

$$e^{\hat{\xi}\theta^*}(t) = S(t)P_S(t)^{-1},\ P_S^2(t) = S^T(t)S(t)$$

$$S(t) = \frac{1}{n}\sum_{i\in\mathcal{V}} e^{\hat{\xi}\theta_{wo_i}} + \dot{g}_{wo_i} = g_{wo_i}\hat{V}^b_{wo_i} + \textbf{Assumption 4}$$

$S(t) > 0$ and hence invertible, and $S(t)$ is continuous and differentiable

$\Rightarrow$ $P_S(t)$ is continuous and differentiable

Assumption 4 $\Rightarrow$ $P_S(t)$ is continuously differentiable

$\Rightarrow$ **$e^{\hat{\xi}\theta^*}(t)$ is well-defined and continuous**

Since $e^{\hat{\xi}\theta^*}(t) \in SO(3)$, $\dot{e}^{\hat{\xi}\theta^*}(t) \in T_{e^{\hat{\xi}\theta^*}(t)}SO(3)$ holds

Tangent Space: $T_{e^{\hat{\xi}\theta^*}(t)}SO(3) = \{e^{\hat{\xi}\theta^*}(t)X \mid X \in so(3)\}$

The motion of the average is represented as $\dot{e}^{\hat{\xi}\theta^*} = e^{\hat{\xi}\theta^*}\hat{\omega}^{b,*}$

Relation between Body Velocities

Lemma 6 (proved from [7])

Suppose that $(e^{\hat{\xi}\theta_{wo_i}})_{i\in\mathcal{V}}$ satisfies $\|e^{\hat{\xi}\theta^*} - S(t)\|_F \leq \gamma\ \forall t \geq 0$

Then, the following inequality holds.

$$\|\omega^{b,*}(t)\|^2 \leq \mu^2(\gamma)\|w_R(t)\|^2/n,\ \mu(\gamma) := \frac{\sqrt{2}}{\sqrt{2}-\gamma}$$

In addition, $\|v^{b,*}(t)\|^2 = \|w_p(t)\|^2/n$

$$w_p(t) = [(v^b_{wo_1})^T \ \cdots \ (v^b_{wo_n})^T]^T,\ w_R(t) = [(\omega^b_{wo_1})^T \ \cdots \ (\omega^b_{wo_n})^T]^T$$

$$\|e^{\hat{\xi}\theta^*} - S(t)\|_F^2 \leq \theta_m := \max_{i,j\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_{wo_i}}e^{\hat{\xi}\theta_{wo_j}})\ \forall i \in \mathcal{V}$$

$\Rightarrow$ γ can be estimated from the prior set-valued information on the targets' orientations

[7] I. Soderkvist, "Perturbation of the Orthogonal Procrustes Problem," BIT Numerical Mathematics, Springer, Vol. 33, No. 4, pp. 687-694, 1993

Problem Reformulation

Averaging Performance $\Omega_R(\varepsilon) := \left\{(e^{\hat{\bar{\xi}}\bar{\theta}_{io_i}})_{i\in\mathcal{V}} \,\middle|\, \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_i^*}e^{\hat{\bar{\xi}}\bar{\theta}_{io_i}}) \leq \varepsilon\rho_R\right\}$

ε: A degree of improvement of mean estimation accuracy

$$\rho_R := \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_i^*}e^{\hat{\bar{\xi}}\bar{\theta}_{io_i}}) \Rightarrow \rho'_R := \sup_{t\geq0}\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_i^*}e^{\hat{\bar{\xi}}\bar{\theta}_{io_i}}) < \infty$$

Accordingly, $\Omega'_R(\varepsilon) := \left\{(e^{\hat{\bar{\xi}}\bar{\theta}_{io_i}})_{i\in\mathcal{V}} \,\middle|\, \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_i^*}e^{\hat{\bar{\xi}}\bar{\theta}_{io_i}}) \leq \varepsilon\rho'_R\right\}$

Definition

The estimates $(e^{\hat{\bar{\xi}}\bar{\theta}_{io_i}})_{i\in\mathcal{V}}$ are said to achieve ε **- level Tracking performance** if there exists a finite T such that

$$(e^{\hat{\bar{\xi}}\bar{\theta}_{io_i}})_{i\in\mathcal{V}} \in \Omega'_R(\varepsilon)\ \forall t \geq T,\ w_R \text{ satisfying Assumption 4}$$

Theorem 3

Theorem

Under Assumptions 1 and 3, Then, if $k_e > 2\mu^2$, then the position and orientation estimates achieve ε'_p **and** ε'_R - level tracking performances respectively with

$$\varepsilon'_p := 1 + \frac{1}{k_e - 1} + \frac{\bar{w}_R}{\rho'_R(k_e - 1)}$$

$$\varepsilon'_R := 1 + \frac{2\mu^2}{k_e - 2\mu^2} + \frac{\bar{w}_R}{\rho'_R(k_e - 2\mu^2)}$$

The tracking performance improves as the feedback gain of the visual motion observer gets strong

Proof of Theorem 3

$$V = \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_i^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})$$

time varying

$$\dot V = \sum_{i\in\mathcal{V}} (\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee)^T(\omega_{uei} - \omega^{b,*})$$

$$= \underbrace{\sum_{i\in\mathcal{V}} (\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee)^T\omega_{uei}}_{G_1 =} - \underbrace{\sum_{i\in\mathcal{V}} (\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee)^T\omega^{b,*}}_{G_2 =}$$

From the proof of Lemma 2

$$G_1 \le k_e \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}}) - a_1$$

$$< k_e \rho_R' - \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})$$

Proof of Theorem 3

$$G_2 = -\sum_{i\in\mathcal{V}} (\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee)^T\omega^{b,*}$$

$$= -\sum_{i\in\mathcal{V}} \Big((1/\mu^2)\|\mu^2\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee + \omega^{b,*}\|^2 - \mu^2\|\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee\|^2 - (1/\mu^2)\|\omega^{b,*}\|^2\Big)$$

$$\le \sum_{i\in\mathcal{V}} \Big(\mu^2\|\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee\|^2\Big) + \underline{(n/\mu^2)\|\omega^{b,*}\|^2}$$

Lemma 6
$\|\omega^{b,*}(t)\|^2 \le \mu^2(\gamma)\|w_R(t)\|^2/n$

$$\le \sum_{i\in\mathcal{V}} \Big(\mu^2\|\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee\|^2\Big) + \|w_R\|^2 \le \sum_{i\in\mathcal{V}} \Big(\mu^2\|\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee\|^2\Big) + \bar{w}_R^2$$

Since $\sum_{i\in\mathcal{V}} \Big(\|\mathrm{sk}(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})^\vee\|^2\Big) \le 2\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})$ from $\sin^2\theta \le 2 - 2\cos\theta$

$$\le \bar{w}_R^2 + 2\mu^2 \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})$$

Proof of Theorem 3

$$\dot V = G_1 + G_2$$

$$G_1 < k_e\rho_R' - \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}}) \qquad G_2 \le \bar{w}_R^2 + 2\mu^2 \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})$$

⇨ $$\dot V < \bar{w}_R^2 + k_e\rho_R' - (k_e - 2\mu^2)\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}})$$

The estimates ultimately satisfy

$$\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\bar{\xi}}\bar{\theta}_{wo_i}}) \le \frac{1}{k_e - 2\mu^2}(\bar{w}_R^2 + k_e\rho_R')$$

$$= \frac{\rho_R'}{k_e - 2\mu^2}(\bar{w}_R^2/\rho_R' + k_e) = \rho_R'\left(\frac{k_e}{k_e - 2\mu^2} + \frac{\bar{w}_R^2}{\rho_R'(k_e - 2\mu^2)}\right)$$

$$= \rho_R'\left(1 + \frac{2\mu^2}{k_e - 2\mu^2} + \frac{\bar{w}_R^2}{\rho_R'(k_e - 2\mu^2)}\right)$$

⇨ ε_R' -level tracking performance $\quad \varepsilon_R' := 1 + \dfrac{2\mu^2}{k_e - 2\mu^2} + \dfrac{\bar{w}_R^2}{\rho_R'(k_e - 2\mu^2)}$

Summary

Definition
The estimates $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}}$ are said to achieve **ε - level averaging accuracy** if there exists a finite T such that
$$(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \Omega_R(\varepsilon)\ \forall t \ge T$$

Suppose that all assumptions in Lemma 2 hold. Then, for any $\epsilon \in (0,1)$ position estimates $(\bar{p}_{io_i})_{i\in\mathcal{V}}$ achieves **ε_p - level averaging accuracy** with
$$\varepsilon_p = \begin{cases} 1 - (1-\epsilon)(1 - \sqrt{k}L)^2 & \text{if } k \le 1/L^2 \\ 1 & \text{otherwise} \end{cases}$$
In addition, the orientation estimates $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}}$ achieves **ε_R - level averaging accuracy** with
$$\varepsilon_R = \begin{cases} 1 - (1-\epsilon)(\sqrt{\beta} - \sqrt{k}L)^2 & \text{if } k \le \beta/L^2,\ \beta > 0 \\ 1 & \text{otherwise} \end{cases}$$

Definition
The estimates $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}}$ are said to achieve **ε - level Tracking performance** if there exists a finite T such that
$$(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \Omega_R'(\varepsilon)\ \forall t \ge T,\ w_R \text{ satisfying Assumption 4}$$

Theorem
Under Assumptions 1 and 3, Then, if $k_e > 2\mu^2$, then the position and orientation estimates achieve ε_p' and ε_R' - level tracking performances respectively with
$$\varepsilon_p' := 1 + \frac{1}{k_e - 1} + \frac{\bar{w}_p^2}{\rho_R'(k_e - 1)}$$
$$\varepsilon_R' := 1 + \frac{2\mu^2}{k_e - 2\mu^2} + \frac{\bar{w}_R^2}{\rho_R'(k_e - 2\mu^2)}$$

Definition
Given target orientations $(e^{\hat{\xi}\theta_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}$ and initial estimates $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}$, when the present estimation algorithm is applied, **the time complexity of ε – level averaging accuracy task** is defined by
$$T(\varepsilon) := \inf T \text{ s.t. } (e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \Omega_R(\varepsilon)\ \forall t \ge T$$

Theorem
Suppose that all assumptions in Lemma 2 hold and the graph is undirected. Then, we have
$$T_1(1) \le T_1 := \tau(\epsilon) + \frac{V(0) - \rho_R}{Q\beta\lambda_{min2}(L_G)}\left(\frac{1}{k_e} + \frac{2n}{k_e}\right)$$
$$Q := \frac{1}{n^2}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_{wo_i}} e^{\hat{\xi}\theta_{wo_j}})$$
In addition, if $k \le \beta/L^2,\ \beta > 0$, then
$$T(\varepsilon_R) \le \tau(\epsilon) + T_1 + \max\{T_2, T_3\}$$
$$T_2 := \frac{(1+\beta)(1-\varepsilon_R)}{k_e\beta},\quad T_3 := \frac{(1-\varepsilon_R)\rho_R}{Q\beta\lambda_{min2}(L_G)}\left(\frac{1}{k_e} + \frac{2n}{\epsilon k_e}\right)$$

We have derived

- **Averaging Performance**
- **Speed of Convergence**
- **Tracking Performance**

On Gain Selection

On Gain Selection $\quad u_{ei} = k_e e_{ei} + k_s E_R(g_{io_i}^{-1} g_{io_j})$

In order to achieve good tracking performance and high convergence speed, the feedback gain k_e should be large (in practice it is limited by the sensing accuracy, i.e. effect of noise).
However, a large k_e makes the averaging performance poor since $k = k_e/k_s$ gets small. To achieve a good averaging performance simultaneously, the mutual feedback gain k_s should be much larger

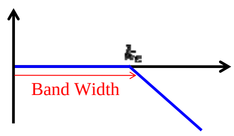
Good Experimental Study: What happens for quite large k_s ?
(In consensus, a strong feedback is fragile against delays)

In the demonstration, we had to choose quite small k_e, which results in a long waiting time. The boring problem could be overcome by the modification of the input

On Gain Selection

Simple Interpretation of VMO

$$\dot y = k_e(r - y) \rightarrow y(s) = \frac{k_e}{s + k_e} r(s)$$



Performance Limitations

- Unstable Zeros
- Unstable Pole
- Model Reliability
- (Feedback) Time Delay
- Actuator Quality
- Sensor Quality (Noise Effect)
- Computation Capability

Sampling Frequency: about 30[Hz]
Nyquist Frequency: about 15[Hz]
Available Frequency: 1.5 – 3 [Hz]
→ **about 9 – 18[rad/s]**

Estimation of Sensor Quality is rather difficult, though I do not intend to say it's impossible (Actually, Wasa kun has already estimated it for the overhead camera). BUT, …

$$k_e, k_s \lesssim 10 \sim 20$$

It might be better to run computation much faster (I'm not sure what happens without synchronization with sensing)

Plan of Submission for CDC 2011

- Introduction
- Definition of Visual Sensor Networks
- Passivity-based Visual Motion Observer (VMO) [1]
- Cooperative Estimation Algorithm
- Averaging Performance Analysis Corrected!
- On Convergence Speed New!
- Tracking Performance Analysis New!
- Conclusion

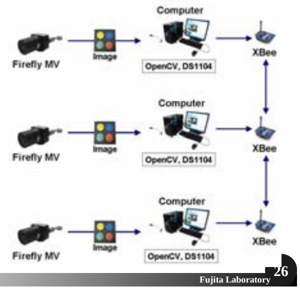
Additional Issues

Promised Future Work

- Switching Topology Analysis via **Brief Instability**
- Communication Delay (**Asynchronous Case**: given in CDC)
- Extension to Omni-directional Vision Cameras

I hope these issues are completed by students

New Experimental System under Construction with Helps of Nishi & Sunaga(Real Distributed System)



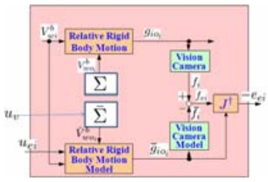
Additional Issues

Challenging Future Works

VMO + Baysian Decision-Making

Preparation: Integration of target object motion model with VMO

Problem: VMO in the framework of Statistics



Extension to Distributed Version is much more challenging

Game-Theoretic Task Assignment in Visual Sensor Networks with Helps of Nomura

Auditory Sensor, Stereo Vision, Spherical Camera, etc….