# Visual Feedback Pose Synchronization with Panoramic Vision

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**28th, February, 2011**



## Introduction



Visual Feedback Control [6-10]

Combination of Control Techniques with Vision

Advantages: suit to recognize unknown surroundings, especially in dynamical environments

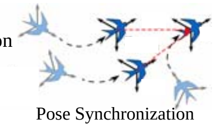
Visual Feedback Cooperative Control

In this work,

Visual Feedback Cooperative Control

Cooperative Control [1-5]

A distributed control strategy using local information so that the robotic network achieves specified tasks or behaviors



Pose Synchronization

Pose Synchronization [5]

To lead all rigid bodies' poses to a common (desired) value by utilizing distributed control strategies

Motivated to analyze and imitates cooperative behaviors where **vision plays a crucial role in cooperative behaviors in nature**

Flocking of Birds



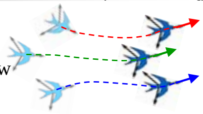
## Introduction



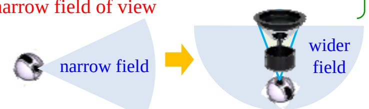
Visual Feedback Cooperative Control [11-15]

- Visual Feedback Attitude Synchronization [14]
  Propose visual feedback attitude synchronization law with theoretical guarantees
  - not consider position coordination
  - pinhole camera model for visual measurement

  (Visibility maintenance problems according to narrow field of view)

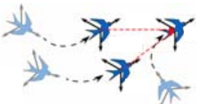


Attitude Synchronization




Pinhole Camera Model Panoramic Vision Model

Image of Panoramic Vision



Pose Synchronization

**Objective of Our Work**

To present a visual feedback pose synchronization control law with panoramic vision

[14] T. Ibuki, T. Hatanaka, M. Fujita and M. Spong, Proc. of the 49th IEEE Conference on Decision and Control, pp. 2486-2491, 2010.



## Outline



- Introduction
- Visual Robotic Network with Panoramic Vision
- Visual Feedback Pose Synchronization
- Visual Feedback Pose Synchronization Law
- Main Results
- Conclusion



## Visual Robotic Network: Rigid Body Motion

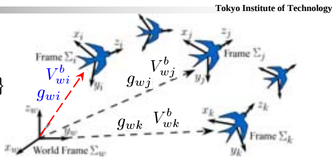


Kinematics of Rigid Bodies

Pose $(p_{wi}, e^{\hat{\xi}\theta_{wi}}) \in SE(3),\ i \in \mathcal{V}$

Rigid Body Set $\mathcal{V} := \{1, \cdots, n\}$

Exponential Coordinate for Rotation
- $\xi_{wi} \in \mathcal{R}^3$: rotation axis
- $\theta_{wi} \in \mathcal{R}$ : rotation angle




Homogeneous Representation

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{4\times4}$$

$$\wedge : \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$\vee$ : Inverse Operator of $\wedge$

Body Velocity

$$V^b_{wi} = \begin{bmatrix} v^b_{wi} \\ \omega^b_{wi} \end{bmatrix} \in \mathcal{R}^6$$

$$\hat{V}^b_{wi} := g^{-1}_{wi}\dot{g}_{wi} = \begin{bmatrix} \hat{\omega}^b_{wi} & v^b_{wi} \\ 0 & 0 \end{bmatrix} \in \mathcal{R}^{4\times4}$$

$v^b_{wi} \in \mathcal{R}^3$ : linear velocity

$\omega^b_{wi} \in \mathcal{R}^3$ : angular velocity

Rigid Body Motion

$$\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi} \quad (1)$$

**Rigid Body Motion**

$V^b_{wi} \rightarrow$ $\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi}$ $\rightarrow g_{wi} = (p_{wi}, e^{\hat{\xi}\theta_{wi}})$

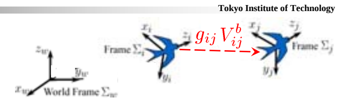


## Visual Measurement



Relative Pose

$$g_{ij} := g^{-1}_{wi}g_{wj} = (p_{ij}, e^{\hat{\xi}\theta_{ij}}) \in SE(3)$$

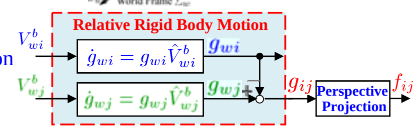



Body Velocity

$$\hat{V}^b_{ij} := g^{-1}_{ij}\dot{g}_{ij}$$

Relative Rigid Body Motion

$$V^b_{ij} = -\mathrm{Ad}_{(g^{-1}_{ij})}V^b_{wi} + V^b_{wj} \quad (2)$$




Visual Measurement (Relative Sensing)

$$f_i = (f_{ij})_{j\in\mathcal{N}_i} \quad (5) \qquad f_{ij} := [f^T_{ij_1} \ \cdots \ f^T_{ij_m}]^T \in \mathcal{R}^{2m}$$

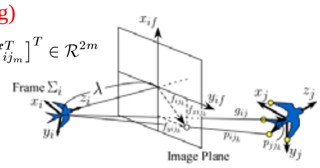
Vision Model (Perspective Projection)

$$f_{ij_k} = \frac{\lambda_i}{z_{ij_k}}\begin{bmatrix} x_{ij_k} \\ y_{ij_k} \end{bmatrix} \in \mathcal{R}^2$$

$\lambda_i \in \mathcal{R}$ : focal length

$k \in \{1, \cdots, m\}$

$p_{ij_k} = \begin{bmatrix} x_{ij_k} \\ y_{ij_k} \\ z_{ij_k} \end{bmatrix}$ : position of $k$ th feature point of body $j$ relative to body $i$




Panoramic Vision Model

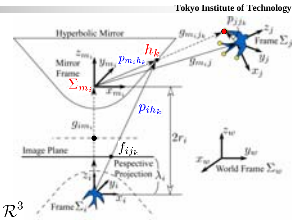
[14] T. Ibuki, T. Hatanaka, M. Fujita and M. Spong, Proc. of the 49th IEEE CDC, 2010.

## Visual Measurement extracted by Panoramic Vision



Panoramic Vision Model

Perspective Projection
+
Hyperbolic Mirror




$\Sigma_{m_i}$ : body $i$ 's mirror coordinate
(origin is located on the focal point)

$h_k$ : point on the hyperbolic mirror in 3D

pose of $h_k$ : $p_{m_ih_k} = [x_{m_ih_k}\ y_{m_ih_k}\ z_{m_ih_k}]^T \in \mathcal{R}^3$

$p_{ih_k} = [x_{ih_k}\ y_{ih_k}\ z_{ih_k}]^T \in \mathcal{R}^3$

Hyperbolic Equation

$$\frac{(z_{m_ih_k}+r_i)^2}{a_i^2} - \frac{x_{m_ih_k}^2 + y_{m_ih_k}^2}{b_i^2} = 1 \quad (4)$$

$a_i, b_i, r_i := \sqrt{a_i^2 + b_i^2}$

: hyperbolic mirror parameters

Pose of $\Sigma_{m_i}$ relative to $\Sigma_i$

$p_{im_i} = [0\ 0\ 2r_i]^T$

$e^{\hat{\xi}\theta_{im_i}} = I_3$

Perspective Projection

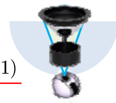
$$f_{ij_k} = \frac{\lambda_i}{z_{ih_k}} \begin{bmatrix} x_{ih_k} \\ y_{ih_k} \end{bmatrix} = \frac{\lambda_i}{2r_i + z_{m_ih_k}} \begin{bmatrix} x_{m_ih_k} \\ y_{m_ih_k} \end{bmatrix}$$

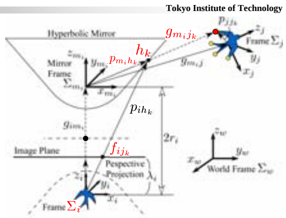


## Visual Measurement extracted by Panoramic Vision



Panoramic Vision Model






from the right figure,

$p_{m_ih_k} = cp_{m_ij_k} \quad (0 < c < 1)$

substituting $cp_{m_ij_k}$ into (4),

$$c(p_{m_ij_k}) = \frac{b_i^2(r_i z_{m_ij_k} + a_i\|p_{m_ij_k}\|_2)}{a_i^2 x_{m_ij_k}^2 + a_i^2 y_{m_ij_k}^2 - b_i^2 z_{m_ij_k}^2}$$

$$f_{ij_k} = \frac{\lambda_i c(p_{m_ij_k})}{2r_i + c(p_{m_ij_k}) z_{m_ij_k}} \begin{bmatrix} x_{m_ij_k} \\ y_{m_ij_k} \end{bmatrix}$$

※ Function of $p_{m_ij_k}$ (from $\Sigma_{m_i}$ to $\Sigma_j$)

Visual Measurement

$f_i(g_{m_ij}) = (f_{ij})_{j \in \mathcal{N}_i} \quad (5)$

$f_{ij} := \begin{bmatrix} f_{ij_1} \\ \vdots \\ f_{ij_m} \end{bmatrix}$

$g_{ij}$ → Panoramic Vision → $f_i$

Hyperbolic Equation

$$\frac{(z_{m_ih_k}+r_i)^2}{a_i^2} - \frac{x_{m_ih_k}^2 + y_{m_ih_k}^2}{b_i^2} = 1 \quad (4)$$

Perspective Projection

$$f_{ij_k} = \frac{\lambda_i}{z_{ih_k}} \begin{bmatrix} x_{ih_k} \\ y_{ih_k} \end{bmatrix} = \frac{\lambda_i}{2r_i + z_{m_ih_k}} \begin{bmatrix} x_{m_ih_k} \\ y_{m_ih_k} \end{bmatrix}$$



## Visibility Structure



Visibility Structure among Rigid Bodies

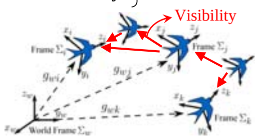
Visibility Set

$\mathcal{V} := \{1, \cdots, n\}$

$\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ $\big((j,i) \in \mathcal{E}$ : body $j$ is visible from body $i\big)$

Visible Body Set

$\mathcal{N}_i := \{j \in \mathcal{V} \mid (j,i) \in \mathcal{E}\} \quad (3)$

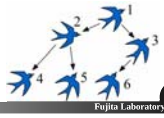



Assumption 1 (Leader-follower Type Visibility Structure)

- there exists a leader which has no visible body ($\mathcal{N}_1 = \emptyset$)
- the other bodies have a fixed visible body ($|\mathcal{N}_i| = 1$, and $\mathcal{N}_i$ is fixed $^\forall i \in \mathcal{V} \setminus \{1\}$)
- there exists a visibility path from each body to the leader ($^\forall i \in \mathcal{V} \setminus \{1\}$, $^\exists v_1, \cdots, v_r \in \mathcal{V}$ $s.t.$ $v_1 = 1$, $v_r = i$ $(v_k, v_{k+1}) \in \mathcal{E}$ $^\forall k \in \{1, \cdots, r-1\}$)

$G := (\mathcal{V}, \mathcal{E})$ : Graph

- Graph: Directed Spanning Tree






## Visual Robotic Network



Visual Robotic Network $\mathbf{\Sigma}$

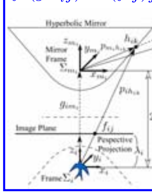
$n$ Rigid Bodies

$\dot{g}_{wi} = g_{wi}\hat{V}_{wi}^b, i \in \mathcal{V} \quad (1)$

$V_{wi}^b$ → Rigid Body Motion → $g_{wi} = (p_{wi}, e^{\hat{\xi}\theta_{wi}})$

Visual Measurement

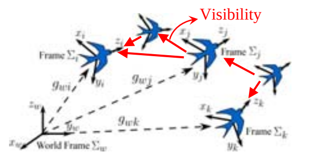
$f_i(g_{m_ij}) = (f_{ij})_{j \in \mathcal{N}_i}, i \in \mathcal{V} \quad (5)$




$g_{ij}$ → Panoramic Vision → $f_i$

Visibility Structure

$\mathcal{N}_i = \{j \in \mathcal{V} \mid (j,i) \in \mathcal{E}\}, i \in \mathcal{V} \quad (3)$



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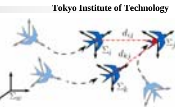
## Definition of Visual Feedback Pose Synchronization



Position Synchronization

$\lim_{t \to \infty} \|p_{wi} - p_{wj}\|_2 = 0 \ ^\forall i, j \in \mathcal{V}$



In application,

- Collision Occurrence
- Visibility Maintenance Problem


because of mirrors, weak to sense right below area and its neighbor


Introduction of biases $d_{ij} \in \mathcal{R}^3$ to overcome the above problems

Suppose that each rigid body has a bias relative to its neighbor $d_{ij}, j \in \mathcal{N}_i$

$(d_{ji} = -d_{ij}, d_{ik} = d_{ij} + d_{jk})$

Position Synchronization with Biases

$\lim_{t \to \infty} \|p_{wj} - p_{wi} - d_{ij}\|_2 = 0 \ ^\forall i, j \in \mathcal{V}$

## Definition of Visual Feedback Pose Synchronization



**Definition: Visual Feedback Pose Synchronization**

A visual robotic network $\Sigma$ is said to achieve visual feedback pose synchronization, if each velocity input consists of only visual measurement (5) $(V_{wi}^b(f_i))$ and

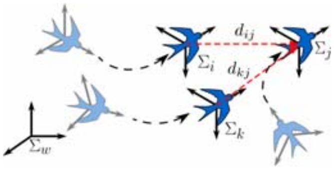
$$\lim_{t\to\infty} \Pi(\tilde{g}_{ij}) = 0 \ {}^\forall i,j \in \mathcal{V} \quad (6) \qquad \tilde{g}_{ij} := \begin{bmatrix} e^{\hat{\xi}\theta_{ij}} & p_{ij}-d_{ij} \\ 0 & 1 \end{bmatrix}$$

**Energy Function**

$$\Pi(g_{wi}) := \frac{1}{2}\|p_{wi}\|^2 + \phi(e^{\hat{\xi}\theta_{wi}}) \geq 0$$

$$\left(\phi(e^{\hat{\xi}\theta_{wi}}) := \frac{1}{2}\mathrm{tr}(I_3 - e^{\hat{\xi}\theta_{wi}}) \geq 0\right)$$

$$\Pi(g_{wi}) = 0 \Leftrightarrow g_{wi} = I_4$$




**Pose Synchronization**
- All relative positions converge to desired ones $d_{ij}$
- All orientations asymptotically converge to a common one

[5] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in SE(3)," *IEEE Trans. on Control System Technology*, Vol. 17, No. 5, pp. 1119-1134, 2009.



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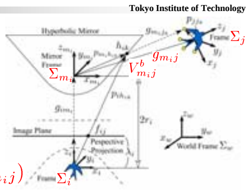
## Preliminaries



**Relative Rigid Body Motion from $\Sigma_{m_i}$ to $\Sigma_j$**

$$V_{m_ij}^b = -\mathrm{Ad}_{(g_{m_ij}^{-1})} V_{wm_i}^b + V_{wj}^b$$
$$= -\mathrm{Ad}_{(g_{m_ij}^{-1})} \mathrm{Ad}_{(g_{im_i}^{-1})} V_{wi}^b + V_{wj}^b \quad (7)$$

Body $i$'s Velocity Input




Measured Output: Visual Measurement $f_i(g_{m_ij})$

➡ use a vision-based nonlinear observer to estimate $g_{m_ij}$

Then, estimate of relative pose $g_{ij}$: $g_{im_i}\bar{g}_{m_ij}$

**Estimated Relative Pose**

$$\bar{g}_{m_ij} = (\bar{p}_{m_ij}, e^{\hat{\xi}\bar{\theta}_{m_ij}}) \in SE(3)$$

**Estimation Error**

$$g_{eij} := \bar{g}_{m_ij}^{-1} g_{m_ij} = (p_{eij}, e^{\hat{\xi}\theta_{eij}}) \in SE(3)$$

$$\bar{g}_{m_ij} \to g_{m_ij} \ (g_{eij} \to I_4)$$

**Control Error**

$$g_{cij} := g_{dij}^{-1} g_{im_i} \bar{g}_{m_ij} = (p_{cij}, e^{\hat{\xi}\theta_{cij}}) \in SE(3)$$

$$\left(g_{dij} := \begin{bmatrix} I_3 & d_{ij} \\ 0 & 1 \end{bmatrix} \to \tilde{g}_{ij} = g_{dij}^{-1} g_{ij}\right)$$

If $g_{cij} = I_4$, $g_{eij} = I_4$ ${}^\forall j \in \mathcal{N}_i, i \in \mathcal{V}$, then $\tilde{g}_{ij} = I_4$ ${}^\forall i,j \in \mathcal{V}$ (Pose Sync.)



## Visual Feedback Pose Synchronization Law



**Visual Feedback Pose Synchronization Law**

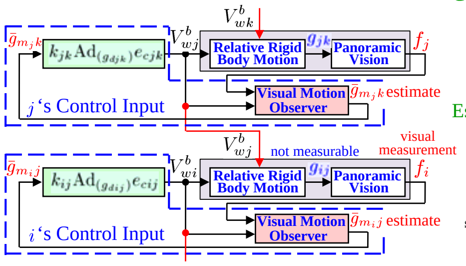
$$V_{wi}^b = k_{ij} \mathrm{Ad}_{(g_{dij})} e_{cij}$$ — Velocity Input

$$\bar{V}_{m_ij}^b = -\mathrm{Ad}_{(\bar{g}_{m_ij}^{-1})} \mathrm{Ad}_{(g_{im_i}^{-1})} V_{wi}^b + u_{ij} \quad (8)$$ (model of (7))

$$u_{ij} = k_{eij}\left(e_{eij} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{cij}})} e_{cij}\right)$$

$$j \in \mathcal{N}_i, \ i \in \mathcal{V}$$

Visual Motion Observer to estimate relative pose $g_{m_ij}$




**Control Error Vector**

$$e_{cij} := \begin{bmatrix} p_{cij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{cij}})^\vee \end{bmatrix}$$

$$\left(g_{cij} = g_{dij}^{-1} g_{im_i} \bar{g}_{m_ij}\right)$$

**Estimation Error Vector**

$$e_{eij} := \begin{bmatrix} p_{eij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{eij}})^\vee \end{bmatrix}$$

$$\left(g_{eij} = \bar{g}_{m_ij}^{-1} g_{m_ij}\right)$$

$$\mathrm{sk}(e^{\hat{\xi}\theta_{ij}}) := \frac{1}{2}(e^{\hat{\xi}\theta_{ij}} - e^{-\hat{\xi}\theta_{ij}})$$



## Interpretation of Velocity Input



**Velocity Input for Visual Feedback Pose Synchronization**

$$V_{wi}^b = k_{ij}\mathrm{Ad}_{(g_{dij})} e_{cij} \quad e_{cij} := \begin{bmatrix} p_{cij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{cij}})^\vee \end{bmatrix} \quad g_{cij} = g_{dij}^{-1} g_{im_i} \bar{g}_{m_ij} \quad \mathrm{Ad}_{(g_{dij})} = \begin{bmatrix} I_3 & \hat{d}_{ij} \\ 0 & I_3 \end{bmatrix}$$

If $\bar{g}_{m_ij} = g_{m_ij}$, then

$$g_{cij} = g_{dij}^{-1} g_{im_i} g_{m_ij} = g_{dij}^{-1} g_{ij} = \begin{bmatrix} e^{\hat{\xi}\theta_{ij}} & p_{ij}-d_{ij} \\ 0 & 1 \end{bmatrix}, e_{cij} = \begin{bmatrix} p_{ij}-d_{ij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix}$$

Therefore,

$$V_{wi}^b = k_{ij}\mathrm{Ad}_{(g_{dij})} e_{cij} = k_{ij}\mathrm{Ad}_{(g_{dij})} \begin{bmatrix} p_{cij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{cij}})^\vee \end{bmatrix} = \begin{bmatrix} p_{ij}-d_{ij} + \hat{d}_{ij}\mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix}$$

**[4] Velocity Input for Pose Synchronization**

$$V_{wi}^b = k_i \sum_{j\in\mathcal{N}_i} \begin{bmatrix} p_{ij}-d_{ij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix}$$

If attitudes are synchronized, then this term is 0 since $\mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee = 0$

not the same velocity input completely, but almost the same

[4] T. Hatanaka, Y. Igarashi, M. Fujita and M. W. Spong, "Passivity-based Pose Synchronization and Flocking in Three Dimensions," *IEEE Trans. on Automatic Control*, 2011 (conditionally accepted).

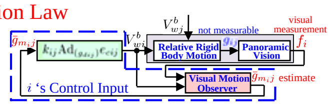


## Visual Feedback Pose Synchronization Law



**Visual Feedback Pose Synchronization Law**

$$V_{wi}^b = k_{ij}\mathrm{Ad}_{(g_{dij})} e_{cij}$$
$$\bar{V}_{m_ij}^b = -\mathrm{Ad}_{(\bar{g}_{m_ij}^{-1})} \mathrm{Ad}_{(g_{im_i}^{-1})} V_{wi}^b + u_{ij}$$
$$u_{ij} = k_{eij}\left(e_{eij} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{cij}})} e_{cij}\right) \quad j \in \mathcal{N}_i, \ i \in \mathcal{V}$$




**Lemma 1**

Visual Feedback Pose Synchronization Law consists of only visual measurement $f_i$ under the appropriate assumptions.

**Sketch of Proof (Refer to Appendix 1)**

- $e_{cij}$ can be calculated by $g_{cij}$ and $g_{cij} = g_{dij}^{-1} g_{im_i} \bar{g}_{m_ij}$ (known, estimated)
- $e_{eij}$ can be calculated as follows

$$p_{m_ij_k} - \bar{p}_{m_ij_k} = e^{\hat{\xi}\bar{\theta}_{m_ij}}[I_3 - \hat{p}_{jj_k}]e_{eij}$$ under the assumption that $|\theta_{eij}| << 1$

$$f_{ij_k} - \bar{f}_{ij_k} = \left[\frac{\partial f_{ij_k}}{\partial x_{m_ij_k}}\Big|_{p_{m_ij_k}=\bar{p}_{m_ij_k}} \ \frac{\partial f_{ij_k}}{\partial y_{m_ij_k}}\Big|_{p_{m_ij_k}=\bar{p}_{m_ij_k}} \ \frac{\partial f_{ij_k}}{\partial z_{m_ij_k}}\Big|_{p_{m_ij_k}=\bar{p}_{m_ij_k}}\right](p_{m_ij_k} - \bar{p}_{m_ij_k})$$

First-order Taylor expansion approximation

➡ $f_i - \bar{f}_i = J_i(\bar{g}_{m_ij}) e_{eij} \to e_{eij} = J_i^\dagger(\bar{g}_{m_ij})(f_i - \bar{f}_i)$

## Outline







## Visual Feedback Pose Synchronization

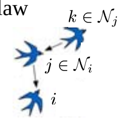


**Theorem 1: Visual Feedback Pose Synchronization**

Suppose the leader does not move $(V_{w1}^b = 0)$. Then, under Assumption 1, a visual robotic network $\Sigma$ with control law (8) achieves visual feedback pose synchronization if

$$\begin{cases} \begin{bmatrix} (k_{i1}+k_{ei1})I_6 - k_{i1}D_{i1} & -k_{ei1}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ci1}})} \\ -k_{ei1}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ci1}})} & k_{ei1}I_6 \end{bmatrix} > 0, \ i \in \mathcal{V}_p \\ k_{jk} < \frac{2k_{ij}k_{eij}}{k_{ij}+k_{eij}}, \ k \in \mathcal{N}_j, \ j \in \mathcal{N}_i, \ i \in \mathcal{V}_q \\ \begin{bmatrix} (k_{ij}+k_{eij})I_6 - k_{ij}D_{ij} & -k_{eij}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{cij}})} \\ -k_{eij}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{cij}})} & (k_{eij} - \frac{1}{2}k_{jk})I_6 \end{bmatrix} > 0,, \ k \in \mathcal{N}_j, \ j \in \mathcal{N}_i, \ i \in \mathcal{V}_r \end{cases} \quad (12)$$

$$D_{ij} := \frac{1}{2}\mathrm{Ad}_{(g_{dij})}^T \mathrm{Ad}_{(g_{dij})}$$



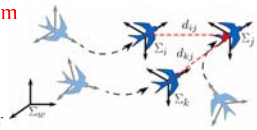

**Sketch of Proof**

Introduce control and estimation error system for all $(j,i) \in \mathcal{E}$

Lyapunov Function Candidate:

$$U := \sum_{i=2}^{n} \sum_{j \in \mathcal{N}_i} q_i \left(\Pi(g_{cij}) + \Pi(g_{eij})\right) \geq 0$$

Control Error — Estimation Error



➡ Origin of the error system $x_e = 0$ is asymptotically stable

$x_e = 0$ means Visual Feedback Pose Synchronization

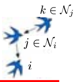


## Analysis of Gain Condition (12)



If $d_{ij} = 0$ (i.e. $D_{ij} = (1/2)I_6$) $^\forall i, j \in \mathcal{V}$, gain condition (12) becomes

$$\begin{cases} k_{jk} < \frac{2k_{ij}k_{eij}}{k_{ij}+k_{eij}}, \ k \in \mathcal{N}_j, \ j \in \mathcal{N}_i, \ i \in \mathcal{V}_q \\ k_{jk} < \frac{2k_{ij}k_{eij}}{k_{ij}+2k_{eij}}, \ k \in \mathcal{N}_j, \ j \in \mathcal{N}_i, \ i \in \mathcal{V}_r \end{cases}$$




These inequalities implies that if the backward rigid bodies move fast, then visual feedback attitude synchronization is achieved [14]

$$\begin{bmatrix} (k+k_e)I_6 - kD & -k_e\mathrm{Ad}_{(e^{\hat{\xi}\theta_c})} \\ -k_e\mathrm{Ad}_{(e^{-\hat{\xi}\theta_c})} & k_eI_6 \end{bmatrix} > 0$$

$(k+k_e)I_6 - kD > 0$ : necessary condition

**Schur Complements**

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0 \Leftrightarrow \begin{cases} Q(x) > 0 \\ R(x) - S(x)Q(x)^{-1}S(x)^T > 0 \end{cases}$$

$$\begin{bmatrix} \left(\frac{1}{2}k + k_e\right)I_3 & -\frac{1}{2}k\hat{d} \\ -\frac{1}{2}k\hat{d}^T & \left(\frac{1}{2}k + k_e\right)I_3 - \frac{1}{2}k\hat{d}^T\hat{d} \end{bmatrix} > 0$$

➡ $$\left(\frac{k+2k_e}{2}\right)I_3 - \frac{k(k+k_e)}{k+2k_e}\hat{d}^T\hat{d} \geq \left(\frac{k+2k_e}{2}\right)I_3 - \frac{k(k+k_e)}{k+2k_e}\|d\|_2^2 I_3 > 0$$

$(\|d\|_2^2 I_3 \geq \hat{d}^T\hat{d})$

➡ $$\frac{k+2k_e}{2} - \frac{k(k+k_e)}{k+2k_e}\|d\|_2^2 > 0$$

undesired!

Gain condition (12) depends on the distance of the desired relative position

Perhaps, this is because the energy function includes $d_{ij}$ explicitly

[14] T. Ibuki, T. Hatanaka, M. Fujita and M. Spong, Proc. of the 49th IEEE CDC, 2010.



## Pose Synchronization with Desired Velocity

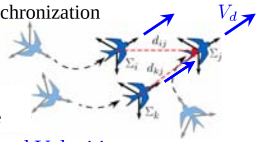


From the practical point of view, it is required for bodies to move in the desired direction while achieving pose synchronization

**Assumption 2**

All bodies have a common desired velocity in its own frame (i.e. $\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{wi}})}V_d$ )

This can be implemented by a beacon in workspace




**Visual Feedback Pose Synch. Law with Desired Velocities**

$$\begin{cases} V_{wi}^b = k_{ij}\mathrm{Ad}_{(g_{dij})}e_{cij} + \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{wi}})}V_d \\ \bar{V}_{m_ij}^b = -\mathrm{Ad}_{(\bar{g}_{m_ij}^{-1})}\mathrm{Ad}_{(g_{im_i}^{-1})}(V_{wi}^b - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{wi}})}V_d) + u_{ij} \\ u_{ij} = k_{eij}\left(e_{eij} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{cij}})}e_{cij}\right) \end{cases} \quad j \in \mathcal{N}_i, \ i \in \mathcal{V}$$

**Corollary 1: Visual Feedback Pose Synch. with Desired Velocities**

Suppose the leader's velocity is $\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{w1}})}V_d$ . Then, under Assumption 1, a visual robotic network $\Sigma$ with control law (8) achieves visual feedback pose synchronization if gain condition (12) is satisfied.

Sketch of Proof: Note that relative rigid body motion (2) does not change

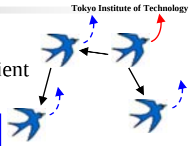


## Tracking Performance Analysis



Theorem 1: $V_{w1}^b = 0$

➡ For a moving leader $(V_{w1}^b \neq 0)$, is there sufficient tracking performance ?



Evaluate attitude errors regarding the leader's velocity as an external disturbance

$x_e$ : All rigid bodies' control and estimation errors

**Quantitative Evaluation: $\mathcal{L}_2$-gain Performance Analysis**

For any energy bounded input, the output will be bounded

$$\|x_e\|_{\mathcal{L}_2} \leq \gamma\|V_{w1}^b\|_{\mathcal{L}_2} + \delta \qquad \gamma, \ \delta \geq 0$$

**Qualitative Evaluation: Input-to-state Stability**

For any bounded input, the state will be bounded

$$\|x_e(t)\|_2 \leq \beta(\|x_0\|_2, t) + \alpha(\|V_{w1}^b\|_{\mathcal{L}_\infty}), \ ^\forall t \geq 0 \quad \alpha(\cdot) \in \mathcal{K}, \ \beta(\cdot,\cdot) \in \mathcal{KL}$$

H. K. Khalil, *Nonlinear Systems*, Third Edition, Prentice Hall, 2002.



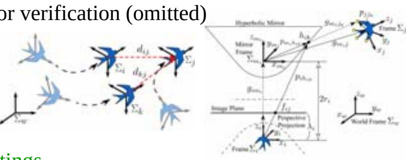
## Outline

# Summary



In this talk, we have

- presented, Visual Robotic Network with Panoramic Vision
- defined visual feedback pose synchronization
- proposed visual feedback pose synchronization law with theoretical guarantees
- analyzed tracking performance of the network (omitted)
- conducted simulations for verification (omitted)



Future Works

- To conduct experiments
- To consider problem settings
- To deal with a wide class of visibility structure
  - eliminate the leader
  - bidirectional visibility



# Appendix 1: Proof of Lemma 1



$$\begin{aligned} p_{m_ij_k} - \bar{p}_{m_ij} &= (p_{m_ij} - \bar{p}_{m_ij_k}) + (e^{\hat{\xi}\theta_{m_ij}} - e^{\hat{\bar{\xi}}\bar{\theta}_{m_ij}})p_{jj_k} \\ &= e^{\hat{\bar{\xi}}\bar{\theta}_{m_ij}} p_{eij} + e^{\hat{\bar{\xi}}\bar{\theta}_{m_ij}}(e^{-\hat{\bar{\xi}}\bar{\theta}_{m_ij}} e^{\hat{\xi}\theta_{m_ij}} - I_3)p_{jj_k} \\ &\approx e^{\hat{\bar{\xi}}\bar{\theta}_{m_ij}} p_{eij} + e^{\hat{\bar{\xi}}\bar{\theta}_{m_ij}} \mathrm{sk}(e^{\hat{\xi}\theta_{eij}})p_{jj_k} \quad (|\theta_{eij}| << 1 \rightarrow e^{\hat{\xi}\theta_{eij}} \approx I_3 + \mathrm{sk}(e^{\hat{\xi}\theta_{eij}})) \\ &= e^{\hat{\bar{\xi}}\bar{\theta}_{m_ij}}(p_{eij} - \hat{p}_{jj_k}\mathrm{sk}(e^{\hat{\xi}\theta_{eij}})^{\vee}) \\ &= e^{\hat{\bar{\xi}}\bar{\theta}_{m_ij}}[I_3 - \hat{p}_{jj_k}]e_{eij} \end{aligned}$$

$$f_{ij_k} - \bar{f}_{ij_k} = \begin{bmatrix} \frac{\partial f_{ij_k}}{\partial x_{m_ij_k}}|_{p_{m_ij_k}=\bar{p}_{m_ij_k}} & \frac{\partial f_{ij_k}}{\partial y_{m_ij_k}}|_{p_{m_ij_k}=\bar{p}_{m_ij_k}} & \frac{\partial f_{ij_k}}{\partial z_{m_ij_k}}|_{p_{m_ij_k}=\bar{p}_{m_ij_k}} \end{bmatrix}(p_{m_ij_k} - \bar{p}_{m_ij_k})$$

$$\begin{cases} \frac{\partial f_i}{\partial x_{m_ij_k}} = \frac{2r_i\lambda_i c_x(p_{m_ij_k})}{(2r_i + c_x(p_{m_ij_k})z_{m_ij_k})^2}\begin{bmatrix} x_{m_ij_k} \\ y_{m_ij_k} \end{bmatrix} = \frac{\lambda_i c_x(p_{m_ij_k})}{2r_i + c_x(p_{m_ij_k})z_{m_ij_k}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \frac{\partial f_i}{\partial y_{m_ij_k}} = \frac{2r_i\lambda_i c_y(p_{m_ij_k})}{(2r_i + c_y(p_{m_ij_k})z_{m_ij_k})^2}\begin{bmatrix} x_{m_ij_k} \\ y_{m_ij_k} \end{bmatrix} = \frac{\lambda_i c_y(p_{m_ij_k})}{2r_i + c_y(p_{m_ij_k})z_{m_ij_k}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \frac{\partial f_i}{\partial z_{m_ij_k}} = \frac{2r_i\lambda_i c_z(p_{m_ij_k})}{(2r_i + c_z(p_{m_ij_k})z_{m_ij_k})^2}\begin{bmatrix} x_{m_ij_k} \\ y_{m_ij_k} \end{bmatrix} = \frac{\lambda_i c_z(p_{m_ij_k})}{2r_i + c_z(p_{m_ij_k})z_{m_ij_k}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{cases}$$

$$\left( c_x(p_{m_ij_k}) = \frac{\partial c(p_{m_ij_k})}{\partial x_{m_ij_k}} = \frac{a_ib_i^2r_iy_{m_ij_k}z_{m_ij_k}\|p_{m_ij_k}\|_2 + a_ib_i^2x_{m_ij_k}(r_i^2z_{m_ij_k}^2 + a_i^2\|p_{m_ij_k}\|_2^2)}{(a_i^2x_{m_ij_k}^2 + a_i^2y_{m_ij_k}^2 - b_i^2z_{m_ij_k}^2)^2\|p_{m_ij_k}\|_2} \right)$$



# Appendix 2: Proof of Theorem 1



Lyapunov Function Candidate: $U := \sum_{i=2}^{n}\sum_{j\in\mathcal{N}_i} q_i\left(\Pi(g_{cij}) + \Pi(g_{eij})\right) \geq 0$

Time Derivative of $U$

$$\begin{aligned} \dot{U} &= \sum_{i=2}^{n}\sum_{j\in\mathcal{N}_i} q_i\left(e_{cij}^T\mathrm{Ad}_{(e^{\hat{\xi}\theta_{cij}})}V_{cij}^b + e_{eij}^T\mathrm{Ad}_{(e^{\hat{\xi}\theta_{eij}})}V_{eij}^b\right) \\ &= \sum_{i=2}^{n}\sum_{j\in\mathcal{N}_i} q_i\left(-e_{ij}^T\begin{bmatrix} I_6 & -\mathrm{Ad}_{(e^{\hat{\xi}\theta_{cij}})} \\ 0 & I_6 \end{bmatrix}\begin{bmatrix} \mathrm{Ad}_{(g_{dij}^{-1})}V_{wi}^b \\ u_{ij} \end{bmatrix} + e_{eij}^T\mathrm{Ad}_{(e^{\hat{\xi}\theta_{eij}})}V_{wj}^b\right) \\ &= \sum_{i=2}^{n}\sum_{j\in\mathcal{N}_i} q_i\left(-e_{ij}^TQ_{ij}e_{ij} + e_{eij}^T\mathrm{Ad}_{(e^{\hat{\xi}\theta_{eij}})}V_{wj}^b\right) \quad \left(Q_{ij} := \begin{bmatrix} (k_{ij}+k_{eij})I_6 & -k_{eij}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{cij}})} \\ -k_{eij}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{cij}})} & k_{eij}I_6 \end{bmatrix}\right) \end{aligned}$$

$$\begin{aligned} e_{eij}^T\mathrm{Ad}_{(e^{\hat{\xi}\theta_{eij}})}V_{wj}^b &= k_{jk}e_{eij}^T\mathrm{Ad}_{(e^{\hat{\xi}\theta_{eij}})}\mathrm{Ad}_{(g_{djk})}e_{cjk} \\ &= \frac{1}{2}k_{jk}\left(\left\|\mathrm{Ad}_{(g_{djk})}e_{cjk}\right\|_2^2 + \|e_{eij}\|_2^2 - \left\|\mathrm{Ad}_{(g_{djk})}e_{cjk} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{eij}})}e_{eij}\right\|_2^2\right) \end{aligned}$$

$$k_{jk}\frac{1}{2}e_{cjk}^T\mathrm{Ad}_{(g_{djk})}^T\mathrm{Ad}_{(g_{djk})}e_{cjk}$$

$D_{jk}$



# Appendix 2: Proof of Theorem 1



Proof of Theorem 1

Define

$$\psi_i := \begin{cases} -e_{i1}^T\begin{bmatrix} (k_{i1}+k_{ei1})I_6 - k_{i1}D_{i1} & -k_{ei1}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{ci1}})} \\ -k_{ei1}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ci1}})} & k_{ei1}I_6 \end{bmatrix}e_{i1}, \ i \in \mathcal{V}_p \\ -e_{ij}^T\begin{bmatrix} (k_{ij}+k_{eij})I_6 & -k_{eij}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{cij}})} \\ -k_{eij}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{cij}})} & \left(k_{eij} - \frac{1}{2}k_{jk}\right)I_6 \end{bmatrix}e_{ij} \\ \quad -k_{jk}\left\|\mathrm{Ad}_{(g_{djk})}e_{cjk} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{eij}})}e_{eij}\right\|_2^2, \ k \in \mathcal{N}_j, \ j \in \mathcal{N}_i, \ i \in \mathcal{V}_q \\ -e_{ij}^T\begin{bmatrix} (k_{ij}+k_{eij})I_6 - k_{ij}D_{ij} & -k_{eij}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{cij}})} \\ -k_{eij}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{cij}})} & \left(k_{eij} - \frac{1}{2}k_{jk}\right)I_6 \end{bmatrix}e_{ij} \\ \quad -k_{jk}\left\|\mathrm{Ad}_{(g_{djk})}e_{cjk} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{eij}})}e_{eij}\right\|_2^2, \ k \in \mathcal{N}_j, \ j \in \mathcal{N}_i, \ i \in \mathcal{V}_r \end{cases}$$

Then, if gain condition (12) is satisfied, then $\psi_i \leq 0 \ {}^{\forall}i \in \mathcal{V}$

Therefore, $\dot{U} = \sum_{i=2}^{n} q_i\psi_i \leq 0$