Passivity-based Cooperative Estimation for Networked Visual Motion Observers

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Motivation: Visual Sensor Networks
A network consisting of spatially distributed smart cameras

Applications
- Surveillance
- Environmental Monitoring
- Entertainment

References

Main Theorem (What to Prove): [3]
Approach to Proof: [2]
Problem Formulation: [4]
Basis on Average and Optimization on Manifolds: [5],[6]
Techniques in Proof: [5],[6] + Linear Algebra

Rigid Body Motion
Pose of Vision Camera relative to $\sum_i$:
- Rotation Axis $\gamma^i_0 \in \mathbb{R}^3$
- Rotation Angle $\gamma^i_0 \in \mathbb{R}$

Body Velocity $\dot{\gamma}^i_v = \gamma^i_0 \gamma^i_0^{-1} \dot{\theta}^i_0$
Angular velocity $\dot{\theta}^i_0 \in \mathbb{R}^3$
Linear velocity $\dot{\gamma}^i_v \in \mathbb{R}^3$

Rigid Body Motion
- Pose of Object $\gamma_{\text{Obj}} = \gamma_{\text{Obj}}^0 \gamma_{\text{Obj}}^1$
- Body Velocity $\dot{\gamma}_{\text{Obj}} = \gamma_{\text{Obj}}^0 \dot{\gamma}_{\text{Obj}}^1$

Relative Rigid Body Motion
Pose of Target Object relative to Vision Camera Frame
$\gamma_{\text{Obj}} = \gamma_{\text{Obj}}^0 \gamma_{\text{Obj}}^1$

Relative Rigid Body Motion $\gamma_{\text{Obj}} = \gamma_{\text{Obj}}^0 \gamma_{\text{Obj}}^1$

Visual measurement should be a function of relative pose $\dot{\gamma}_{\text{Obj}}$
Objective

Objective: Present an algorithm for visual sensor networks, so that the estimates $\hat{x}_i$ achieve the following requirements simultaneously:

- (Averaging) $\hat{x}_i$ gives an approximate average of $\{x_j\}_{j \in \mathcal{V}}$.
- (Tracking) $\hat{x}_i$ estimate tracks to the object motion $x_i$.

Position Ave.: $p^* = \frac{1}{n} \sum_{i \in \mathcal{V}} p_{\hat{x}_i} = \arg \min_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} (p_j - \hat{x}_i)^2$

Orientation Ave.: $\hat{q}^* = \arg \min_{q \in SO(3)} \sum_{i \in \mathcal{V}} \|q - q_{\hat{x}_i}\|^2$

This mechanism is called **Visual Motion Observer**.

Outline

- Introduction
- Definition of Visual Sensor Networks
- Passivity-based Visual Motion Observer (VMO) [1]
- Cooperative Estimation Algorithm
- Averaging Performance Analysis Corrected!
- On Convergence Speed New!
- Tracking Performance Analysis New!
- Conclusion


Visual Sensor Networks

Communication Graph: $\mathcal{G} = (\mathcal{V}, \mathcal{E})$.

Neighborhood: $N_i = \{j \in \mathcal{V} | (i,j) \in \mathcal{E}\}$

Number of Cameras and Targets: $\mathcal{V} = \{1, \ldots, n\}$

Relative Rigid Body Motion:

$\dot{\hat{x}}_i = -k_e e_{xi} + k_e \hat{e}_i \hat{V}_{wo}$

Visual Measurement:

$\hat{x}_i := [\hat{x}_{i,1}^{mf}, \ldots, \hat{x}_{i,mf}]^T, m \geq 4$

Position Ave.:

$\hat{x}^* = \arg \min_{x \in \mathcal{V}} \sum_{i \in \mathcal{V}} \|x - x_i\|^2$

On Convergence Speed:

$\dot{e}_{xi} = -k_e e_{xi} - k_e \hat{e}_i \hat{V}_{wo}$

Perspective Projection:

$\hat{f}_i := [\hat{f}_{i,1}^{mf}, \ldots, \hat{f}_{i,mf}]^T, m \geq 4$

Orientation Ave. (Euclidean Mean [5]):

$\hat{q}^* = \arg \min_{q \in SO(3)} \sum_{i \in \mathcal{V}} \|q - q_{\hat{x}_i}\|^2$

Cooperative Estimation Algorithm

- Estimation Error System
- Negative Feedback
Objective

\[ \rho_R = \sum \frac{e^{j\theta_{\text{mean}}}}{\text{mean}} \]

Sum of distances from the final object's orientation and the mean estimates

Fact: In the absence of communication, the VMO correctly estimates the target's orientation, namely the right hand side is estimation accuracy of the mean as a group in the absence of communication

\[ \Omega_R(\epsilon) := \left\{ \left( e^{j\theta_{\text{mean}}} \right)_{i \in V} \mid \sum_{i \in V} \frac{e^{j\theta_i}}{\text{mean estimates}} \leq \epsilon \right\} \]

\( \epsilon \): A degree of improvement of mean estimation accuracy

The estimates \( e^{j\theta_{\text{mean}}} \) are said to achieve \( \epsilon \) - level averaging accuracy if there exists a finite \( T \) such that \( e^{j\theta_{\text{mean}}} \in \Omega_R(\epsilon) \) \( \forall t \geq T \)

Inspired by Bullo et al. [4]

Cooperative Estimation Algorithm

Definition

Relative Rigid Body Motion

relative estimates\[1\] + Synchronization\[2\] (Passivity) (Passivity)

\[ \frac{\partial}{\partial t} \text{error} \rightarrow 0 \]

Inspired by Bullo et al. [4]

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Assumptions

- Assumption 1 (Communication Graph)
- Assumption 2 (Target Object Pose)

\[ \Theta_{\text{mean}} \]

\[ \text{Set Membership Prior Information} \]

In this section... Target object is static: \( \Theta_{\text{mean}} = 0 \) \( \forall i \in V \)

Proposition: [2] + [5]

Suppose that \( e^{-j\theta_{\text{mean}}} > 0 \) holds at the initial time. Then, we have

\[ \phi(e^{-j\theta_{\text{mean}}}) \leq \Theta_{\text{mean}} := \max_{i \in V} \phi(e^{-j\theta_{\text{mean}}}) \]

Lemma 1 (Positively Invariance)

Under Assumptions 1 and 2, if \( e^{-j\theta_{\text{mean}}} > 0 \) holds at the initial time, then for any positive scalar \( \epsilon \), there exists a finite time \( \tau(c) \) s.t.

\[ \phi(e^{-j\theta_{\text{mean}}}) \leq \phi(e^{-j\theta_{\text{mean}}}) + \epsilon > \phi(e^{-j\theta_{\text{mean}}}) \]

Under Assumption 2, the region

\[ \mathcal{S} = \left\{ (e^{j\theta_{\text{mean}}} e^{j\theta_{i}})_{i \in V} > 0 \right\} \]

is positively invariant, i.e. if \( (e^{j\theta_{\text{mean}}} e^{j\theta_{i}})_{i \in V} > 0 \) holds at the initial time, then it also holds for all subsequent time
Lemma 2 (Mild Statement on Averaging Performance)

Under Assumptions 1 and 2, if \((\epsilon(\theta_{n_i}^0))_{i \in V} \in \mathcal{S}\) holds at the initial time, then the estimates achieve 1-level averaging accuracy

\[
\sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i}) \leq \sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i}) \forall t \geq T
\]

The left hand side is estimation accuracy of the mean as a group in the presence of communication.

Lemma 1 means that the estimation accuracy of the mean improves by using communication and cooperation BUT does not say how accurate estimates of the mean is provided.

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Proof of Lemma 2

Potential Function: \(V = \sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i})\)

Behavior of Estimates

\[
V = -\frac{1}{2} \sum_{i \in V} 2 \epsilon(\theta_{n_i})^2 + \sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i})
\]

\[
\Phi_1 := \sum_{j \in N_i} \phi(e^{-\epsilon(\theta_{n_j}^0)}; \hat{\theta}_{n_j})
\]

\[
\Phi_2 := \sum_{j \in N_i} \phi(e^{-\epsilon(\theta_{n_j}^0)}; \hat{\theta}_{n_j})
\]

\[
\phi(\epsilon(\theta_{n_j}^0); \hat{\theta}_{n_j}) \leq \phi(\epsilon(\theta_{n_j}^0); \hat{\theta}_{n_j}) - \alpha_i \phi(\epsilon(\theta_{n_j}^0); \hat{\theta}_{n_j})
\]

Suppose that \(\sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i}) \leq 0\)

\[
\sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i}) \leq \sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i})
\]

under Assumption 2, this term is strictly negative

From theorem on Ultimate Boundedness in the book by Khalil, the trajectories of estimates ultimately converge to the set satisfying

\[
\sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i}) > 0
\]

Notice : We do not use the fact that \(e^{\epsilon(\theta)}\) is an average!

\[
\sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i}) \leq \sum_{i \in V} \phi(e^{-\epsilon(\theta_{n_i}^0)}; \hat{\theta}_{n_i})
\]

for a common \(j\)

We next evaluate closeness between estimates \(\hat{\theta}_{n_i}^0\) and \(\hat{\theta}_{n_j}^0\)
Division of A Set $S_1$

$S_1$ \[ S \]

Estimates are not close

$S_1(k, s) = \left\{ \begin{array}{l}
\left( e^{\Theta_{in}}, \beta \sum_{n=1}^{N} \left( e^{\Theta_{in}} e^{\theta_{in}} \right) \right) \geq k_{in} \\
\left( e^{\Theta_{in}} e^{\theta_{in}} \right) \end{array} \right\}
$

$S_2(k, s) = S_1 \setminus \left( \Omega(k, \Omega_{out}) \right)$

Estimates are close enough

$\beta = 1 - \sqrt{2k_c}$

Lemma 3

Suppose that all assumptions in Lemma 2 hold. Then, if $\beta > 0$, then there exists a positive scalar $a_0$ such that $\psi \leq a_0$, $a_0 + \beta > 0$ holds at least after the time $\tau(e)$ as long as $V(\psi(\psi(e_{in}), e_{in})) \in S_2(k, s)$.

Proof of Lemma 3

Lemma 5: $\psi = \lambda_{min}(\sigma_{max}(e^{\psi_{in}} e^{\epsilon_{in}})) \geq \beta : = 1 - \sqrt{2k_c}$

proved by perturbation theorem

Lemma 4

Suppose that $a_2 = \sum_{n=1}^{N} \left( e^{\Theta_{in}} e^{\theta_{in}} \right)$ strictly negative under Assumption 2

Proof of Lemma 4

$\forall t > \tau(e)$, we get

$\dot{a}_2 = \sum_{n=1}^{N} \left( e^{\Theta_{in}} e^{\theta_{in}} \right) \leq c \sum_{n=1}^{N} \left( e^{\Theta_{in}} e^{\theta_{in}} \right)$

Then, $\psi \leq k_c \sum_{n=1}^{N} \left( e^{\Theta_{in}} e^{\theta_{in}} \right)$

Theorem 1 (Averaging Performance Analysis)

Suppose that all assumptions in Lemma 2 hold. Then, for any $e \in (0, 1)$ position estimates $(\psi_{in})_{e_{in}}$ achieve $e_p$-level averaging accuracy with

$e_p = \begin{cases} 
1 - (1 - \epsilon)(1 - \sqrt{k_c})^2 & \text{if } k \leq 1/L^2 \\
1 & \text{otherwise}
\end{cases}$

In addition, the orientation estimates $(\psi_{in})_{e_{in}}$ achieve $e_R$-level averaging accuracy with

$e_R = \begin{cases} 
1 - (1 - \epsilon)(1 - \sqrt{k_c})^2 & \text{if } k \leq 1/L^2, \beta > 0 \\
1 & \text{otherwise}
\end{cases}$

$k = k_c / k_s \rightarrow 0(k_c >> k_s) \Rightarrow \psi_p, \psi_R \rightarrow 0, e \rightarrow \infty$

same conclusion as multi-agent optimization[3]

Noticing that $\beta < 1$, we see that an offset appears only on orientations

Difference between SO(3) and Vector Space

In general, all the local issues on a manifold can be approximated by those on a vector space

$\beta := 1 - \sqrt{2k_c}$

$\delta \rightarrow 0 \Rightarrow \beta \rightarrow 1$

$s := \phi(e^{-2\theta_{in}} e^{\Theta_{in}}), \delta_c = \delta + c$

Targets’ orientations are close to each other

Note that the discussion does NOT imply that the theory is valid on a just very limited region on the manifold.

The estimates can be far from each other.

Convergence to a neighbor of the mean is semi-global. Only how close to the mean the estimates get after entering the neighbor is an issue on a local region.
**Proof of Theorem 1**

It is sufficient to prove that $\dot{V} < 0$ in the region $S_0(k, \varepsilon)$ at least after the time $\tau(\varepsilon)$.

$$\dot{V} \leq \sum_{i \in V} \left[ k_i \left( e^{\beta \cdot \varepsilon} \cdot \varepsilon \right) - e^{\beta \cdot \varepsilon} \cdot \varepsilon \right] = -a \varepsilon^2$$

**Lemma 4**

If $e^{\beta \cdot \varepsilon^2}$ is an average, then

$$\dot{V} \leq k_1 \left( \sum_{i \in V} e^{\beta \cdot \varepsilon^2} \cdot \varepsilon \right) - e^{\beta \cdot \varepsilon^2} \cdot \varepsilon = -a \varepsilon^2$$

The function $V$ is strictly decreasing except for the case $(e^{\beta \cdot \varepsilon^2})_{t \in V} \in \Omega(0, \varepsilon)$.

**Summary**

The average pose is defined as:

$$\bar{\theta} = \frac{1}{|V|} \sum_{i \in V} \theta_i$$

where $V$ is the set of all agents.

The estimation $(\hat{x})_{t \in V}$ is said to satisfy the $\varepsilon$-level averaging accuracy if there exists a finite $T$ such that

$$|\hat{x}_i(t) - \bar{\theta}| < \varepsilon$$

for all $t \geq T$. In addition, the orientation estimates $(\hat{\theta})_{t \in V}$ satisfy the $\varepsilon$-level averaging accuracy with

$$|\hat{\theta}_i(t) - \bar{\theta}| < \varepsilon$$

for all $t \geq T$. 

There is no escape other than $\Omega(0, \varepsilon)$ in a finite time and remain in the set for all subsequent time.

$$\varepsilon^2$$ - level averaging accuracy