Passivity-based Cooperative Estimation for Networked Visual Motion Observers

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Cooperative Estimation

Cooperative Estimation

Objective

To present a distributed estimation algorithm by using not only sensed data but also some information from the other sensors

Motivation: Visual Sensor Networks

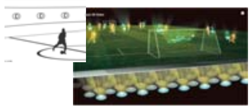
A network consisting of spatially distributed smart cameras

Smart Camera

Applications

Surveillance

Environmental Monitoring



http://www.itmedia.co.jp/news/articles/1005/27/news088.html

Entertainment

References

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking:Stability and L2-gain Performance Analysis," IEEE TCST, Vol.15, No.1, pp.40-52, 2007.
[2] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in *S E(3)," IEEE TCST, Vol. 17, No. 5, pp.*1119–1134, 2009.
[3] A. Nedic and A. Ozdaglar, "Distributed Subgradient Methods for Multi-agent Optimization," IEEE Trans. on Automatic Control, Vol. 54, No. 1, pp. 48-61, 2009.
[4] F. Bullo, J. Cortes and S. Martinez, "Distributed Control of Robotic Networks," Princeton Series in Applied Mathematics, 2009.
[5] M. Moakher, "Means and averaging in the group of rotations," SIAM Journal on Matrix Analysis and Applications, Vol. 24, No. 1, pp. 1–16, 2002.
[6] P. A. Absil, R. Mahony and R. Sepulchre, "Optimization Algorithms on Matrix Manifolds," Princeton Press, 2008.

Estimation Algorithm: [3] → [1] + [2]
Main Theorem (What to Prove): [3]
Approach to Proof: [2]
Problem Formulation: [4]
Basis on Average and Optimization on Manifolds: [5],[6]
Techniques in Proof: [5],[6] + Linear Algebra

Outline

- Introduction
- Definition of Visual Sensor Networks
- Passivity-based Visual Motion Observer (VMO) [1]
- Cooperative Estimation Algorithm
- Averaging Performance Analysis Corrected!
- On Convergence Speed New!
- Tracking Performance Analysis New!
- Conclusion

[1] M. Fujita, H. Kawai and M. W. Spong, "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE TCST*, Vol. 15, No. 1, pp. 40-52, 2007.

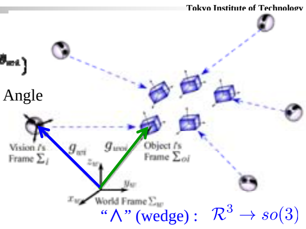
Rigid Body Motion

Pose of Vision Camera i relative to Σ_w

$p_{wi} \in \mathcal{R}^3$ $e^{\hat{\xi}\theta_{wi}} \in SO(3)$ ⇨ $g_{wi} = (p_{wi}, e^{\hat{\xi}\theta_{wi}})$

$\xi_{wi} \in \mathcal{R}^3$: Rotation Axis $\theta_{wi} \in \mathcal{R}$: Rotation Angle

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{4\times 4}$$



Body Velocity

$$\hat{V}^b_{wi} = g_{wi}^{-1}\dot{g}_{wi} = \begin{bmatrix} \hat{\omega}^b_{wi} & v^b_{wi} \\ 0 & 0 \end{bmatrix}$$

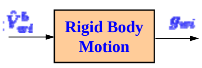
$v^b_{wi} \in \mathcal{R}^3$: Linear velocity $\omega^b_{wi} \in \mathcal{R}^3$: Angular velocity

"∧" (wedge) : $\mathcal{R}^3 \to so(3)$

$$\hat{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}^{\wedge} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Rigid Body Motion

$$\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi}$$



"∨" (vee) : $so(3) \to \mathcal{R}^3$ (Inverse Operator to Wedge)

Pose of Object: $g_{wo_i} = (p_{wo_i}, e^{\hat{\xi}\theta_{wo_i}})$

Body Velocity: $\hat{V}^b_{wo_i} = g_{wo_i}^{-1}\dot{g}_{wo_i}$

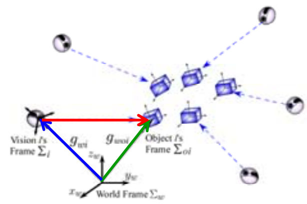
⇨ Rigid Body Motion

$$\dot{g}_{wo_i} = g_{wo_i}\hat{V}^b_{wo_i}$$

Relative Rigid Body Motion

Pose of Target Object relative to Vision Camera Frame

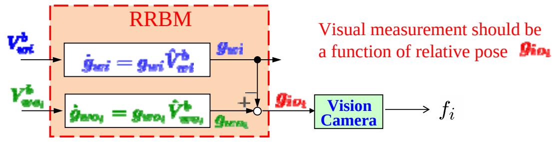
$$g_{io_i}(p_{io_i}, e^{\hat{\xi}\theta_{io_i}}) = g_{wi}^{-1}g_{wo_i}$$



Relative Rigid Body Motion

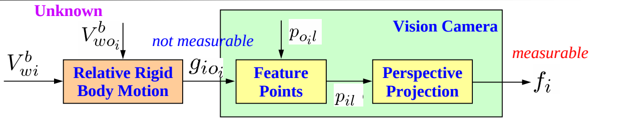
$$\dot{g}_{io_i} = -\hat{V}^b_{wi}g_{io_i} + g_{io_i}\hat{V}^b_{wo_i}$$

Body Velocity of Object
Body Velocity of Vision Camera



Visual measurement should be a function of relative pose g_{io_i}

Visual Measurement



Feature Points of Object

Position of feature points relative to object frame

$p_{o_i l} \quad l = 1, \cdots, m$

Position of feature points relative to camera frame

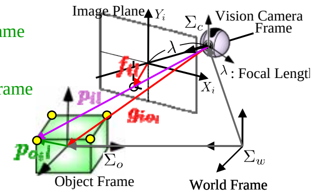
$p_{il} = g_{io_i} p_{o_i l} = [x_{il} \ y_{il} \ z_{il}]^T$

Perspective Projection:

$$f_{il} = \frac{\lambda_i}{z_{il}} \begin{bmatrix} x_{il} \\ y_{il} \end{bmatrix}$$

Visual Measurement

$f_i := [f_{i1}^T \ \cdots \ f_{im}^T]^T \quad m \geq 4$



$p_{il} = g_{io_i} p_{o_i l}$

known

$\Rightarrow f_i = f_i(g_{io_i})$

Visual Sensor Networks

Visual Sensor Networks

Communication

Communication Graph : $G = (\mathcal{V}, \mathcal{E})$

$(i, j) \in \mathcal{E} \Leftrightarrow i$ gets some info. of j

Neighbor Set: $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$

Number of Cameras and Targets

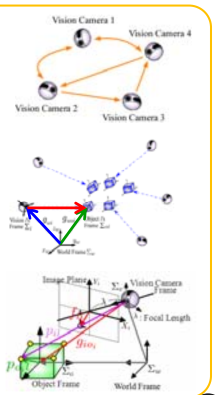
$\mathcal{V} := \{1, \cdots, n\}$

Relative Rigid Body Motion

$\dot{g}_{io_i} = -\hat{V}^b_{wi} g_{io_i} + g_{io_i} \hat{V}^b_{wo_i} \quad i \in \mathcal{V}$

Visual Measurement

$f_i := [f_{i1}^T \ \cdots \ f_{im}^T]^T \quad m \geq 4$



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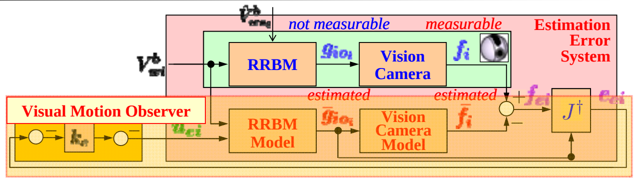
Visual Motion Observer[1]



Estimation Error System

$\dot{g}_{ei} = -u_{ei} g_{ei} + g_{ei} \hat{V}_{wo_i}$

Negative Feedback

$u_{ei} = k_e e_{ei}, \ k_e > 0$

$(-k_e(-e_{ei}))$

This mechanism is called ***Visual Motion Observer***

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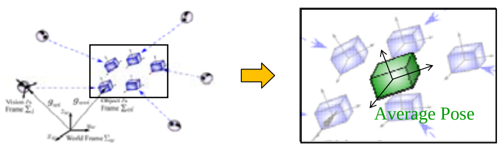
Objective



Objective: Present an algorithm for visual sensor networks, so that the estimates $\bar{g}_{io_i}$ achieve the following requirements simultaneously

- **(Averaging) : estimate gives an approximate average of $\{g_{o_j}\}_{j \in \mathcal{V}}$**
- **(Tracking) : estimate tracks to the object motion g_{io_i}**

Position Ave. : $p^* = \frac{1}{n} \sum_{j \in \mathcal{V}} p_{wo_j} = \arg\min_p \sum_{j \in \mathcal{V}} \|p - p_{wo_j}\|^2$

Orientation Ave.: $e^{\hat{\xi}\theta^*} := \arg \min_{e^{\hat{\xi}\theta} \in SO(3)} \sum_{j \in \mathcal{V}} \phi(e^{-\hat{\xi}\theta} e^{\hat{\xi}\theta_{wo_j}})$

(Euclidean Mean [5])

most closest element to all the objects' orientations in terms of metric ϕ

$\phi(R_1^T R_2) = \frac{1}{2} \|R_1 - R_2\|_F^2$

$\phi(e^{\hat{\xi}\theta}) = \mathrm{tr}(I_3 - e^{\hat{\xi}\theta})$

Objective

$$\rho_R = \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}})$$

mean — object's orientation

Sum of distances from the final estimates and the mean

Fact: In the absence of communication, the VMO correctly estimates the target's orientation $e^{\hat{\xi}\theta_{io_i}}$, namely the right hand side is estimation accuracy of the mean as a group **in the absence of communication**

$$\Omega_R(\varepsilon) := \left\{ (e^{\hat{\xi}\bar{\theta}_{io_i}})_{i\in\mathcal{V}} \,\middle|\, \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}}) \le \varepsilon\rho_R \right\}$$

mean — estimates

ε : A degree of improvement of mean estimation accuracy

Definition

The estimates $(e^{\hat{\xi}\bar{\theta}_{io_i}})_{i\in\mathcal{V}}$ are said to achieve **ε - level averaging accuracy** if there exists a finite T such that

$$(e^{\hat{\xi}\bar{\theta}_{io_i}})_{i\in\mathcal{V}} \in \Omega_R(\varepsilon)\ \forall t \ge T$$

Inspired by Bullo et al. [4]

Cooperative Estimation Algorithm

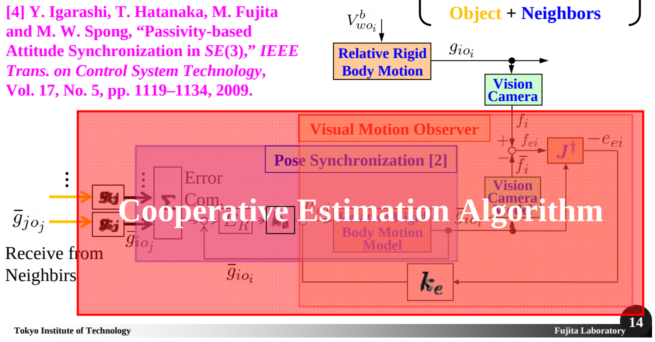
$$\dot{\bar{g}}_{io_i} = \bar{g}_{io_i}\hat{u}_{ei},\quad u_{ei} = k_e e_{ei} + k_s \sum_{j\in\mathcal{N}_i} E_R(\bar{g}_{io_i}^{-1}\bar{g}_{io_j})$$

VMO[1] — Synchronization[2]

VMO[1] + Synchronization[2]
(Passivity) (Passivity)

Gradient Decent[6] + Sync.
Error Feedback from
Object + Neighbors

[4] Y. Igarashi, T. Hatanaka, M. Fujita and M. W. Spong, "Passivity-based Attitude Synchronization in *SE*(3)," *IEEE Trans. on Control System Technology*, Vol. 17, No. 5, pp. 1119–1134, 2009.



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Assumptions

Assumption 1 (Communication Graph)
The communication graph $G=(\mathcal{V},\mathcal{E})$ is fixed, balanced and strongly connected.

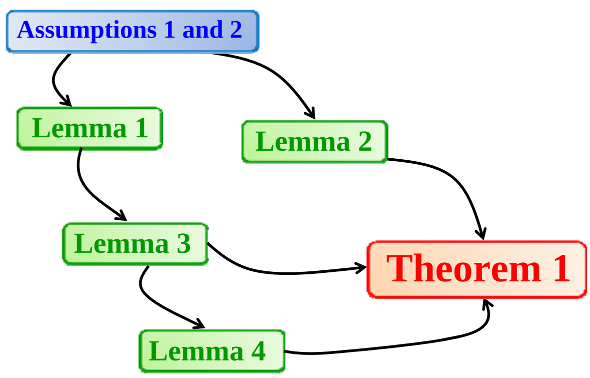
Assumption 2 (Target Object Pose)
There exist $i, j\in\mathcal{V}$ such that $e^{\hat{\xi}\theta_{wo_i}} \neq e^{\hat{\xi}\theta_{wo_j}}$
For any $i\in\mathcal{V}$, $e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}} > 0$ holds true, that is, the relative angle between each target's orientation and mean is smaller than (Set Membership Prior Information)

In this section… Target object is static: $V^b_{wo_i} = 0\ \forall i\in\mathcal{V}$

Proposition: [2] + [5]
Suppose that $e^{-\hat{\xi}\theta_{wo_i}} e^{\hat{\xi}\theta_{wo_j}} > 0\ \forall i, j\in\mathcal{V}$. Then, we have

$$\phi(e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}}) \le \theta_m := \max_{i,j\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_{wo_i}} e^{\hat{\xi}\theta_{wo_j}})\ \forall i\in\mathcal{V}$$

Procedure of Proof



Lemma 1 (Positively Invariance)

Under Assumptions 1 and 2, if $e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}} > 0$ holds at the initial time,
(mean, estimate)
then for any positive scalar c there exists a finite time $\tau(c)$ s.t.

$$\phi(e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}}) \le \phi(e^{-\hat{\xi}\bar{\theta}^*} e^{\hat{\xi}\theta_{wo_h}}) + c\ \forall t \ge \tau(c),\ i\in\mathcal{V}$$

mean estimate — mean — orientation farthest from the mean

$$h := \arg\max_j \phi(e^{-\hat{\xi}\bar{\theta}^*} e^{\hat{\xi}\theta_{wo_j}}) \qquad \delta := \phi(e^{-\hat{\xi}\bar{\theta}^*} e^{\hat{\xi}\theta_{wo_h}}),\ \delta_c = \delta + c$$

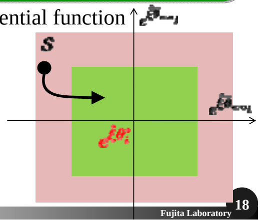
Proof: Use $V' = \max_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}})$ as a potential function

Inspired by Igarashi et al [2] (Proposition 1)

Under Assumption 2, the region

$$S = \left\{ (e^{\hat{\xi}\bar{\theta}_{io_i}})_{i\in\mathcal{V}} \middle| e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}} > 0\ \forall i\in\mathcal{V} \right\}$$

is positively invariant, i.e. if $e^{-\hat{\xi}\bar{\theta}_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}} > 0$ holds at the initial time then it also holds for all subsequent time



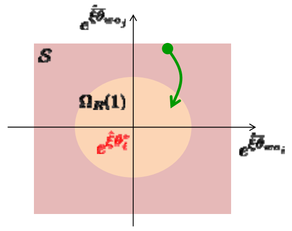
Lemma 2 (Mild Statement on Averaging Performance)

Under Assumptions 1 and 2, if $(e^{\hat\xi\theta_{io_i}})_{i\in\mathcal{V}} \in \mathcal{S}$ holds at the initial time, then the estimates achieve **1-level averaging accuracy**

$$\sum_{i\in\mathcal{V}} \phi(\underbrace{e^{-\hat\xi\theta_i^*}}_{\text{mean}} \underbrace{e^{\hat\xi\bar\theta_{io_i}}}_{\text{estimate}}) \le \sum_{i\in\mathcal{V}} \phi(\underbrace{e^{-\hat\xi\theta_i^*}}_{\text{mean}} \underbrace{e^{\hat\xi\theta_{io_i}}}_{\text{object's orientation}}) \ \forall t \ge T$$

The left hand side is estimation accuracy of the mean as a group **in the presence of communication**

Lemma 1 means that the estimation accuracy of the mean improves by using communication and cooperation BUT does not say how accurate estimates of the mean is provided.



$$\mathcal{S}_1 = \mathcal{S} \cap \Omega_R(1)$$

Proof of Lemma 2

Potential Function: $V = \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta_i^*} e^{\hat\xi\bar\theta_{io_i}})$

Summation of Individual Energy Function: Inspired by Igarashi et al. [2] or Chopra & Spong (2006)

Behavior of Estimates

$$\dot{e}^{\hat\xi\bar\theta_{io_i}} = e^{\hat\xi\bar\theta_{io_i}} \left(k_e \mathrm{sk}(e^{-\hat\xi\bar\theta_{io_i}} e^{\hat\xi\theta_{io_i}}) + \mathrm{sk}(e^{-\hat\xi\bar\theta_{io_i}} e^{\hat\xi\bar\theta_{io_j}})\right)$$

$$\dot V = 2\sum_{i\in\mathcal{V}} (\mathrm{sk}(e^{-\hat\xi\theta_i^*} e^{\hat\xi\bar\theta_{io_i}})^\vee)^T \omega_{uei} = 2\sum_{i\in\mathcal{V}} (\mathrm{sk}(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}})^\vee)^T \omega_{uei}$$

$$= -\sum_{i\in\mathcal{V}} \mathrm{tr}\left(\mathrm{sk}(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) \left\{k_e \mathrm{sk}(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}}) + \sum_{j\in\mathcal{N}_i} \mathrm{sk}(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}})\right\}\right)$$

$$= -\frac{1}{2}\sum_{i\in\mathcal{V}} \mathrm{tr}\Big\{k_e \underbrace{\left(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}} - e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}} e^{-\hat\xi\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_i}}\right)}_{\Phi_1 :=}$$

$$+ k_s \sum_{j\in\mathcal{N}_i} \underbrace{\left(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_j}} - e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}} e^{-\hat\xi\bar\theta_{wo_j}} e^{\hat\xi\bar\theta_{wo_i}}\right)}_{\Phi_2 :=} \Big\}$$

Proof of Lemma 2

Lemma 4

$$\frac{1}{2}\mathrm{tr}(R_1^T R_2 - R_1^T R_3 R_2^T R_3) \ge \phi(R_1^T R_2) - \phi(R_1^T R_3) + \lambda_{min}(sym(R_1^T R_3))\phi(R_3^T R_2) \ \forall R_1, R_2, R_3 \in SO(3)$$

Igarashi et al [2]

$$\Phi_1 := \Big(\underbrace{e^{-\hat\xi\theta^*}}_{R_1} \underbrace{e^{\hat\xi\theta_{wo_i}}}_{R_2} - \underbrace{e^{-\hat\xi\theta^*}}_{R_1} \underbrace{e^{\hat\xi\bar\theta_{wo_i}}}_{R_3} \underbrace{e^{-\hat\xi\theta_{wo_i}}}_{R_2} \underbrace{e^{\hat\xi\bar\theta_{wo_i}}}_{R_3}\Big)$$

$$-\frac{1}{2}\mathrm{tr}(\Phi_1) \le \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}}))$$

$$\sigma_i = \lambda_{min}(sym(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}))$$

$$\Phi_2 := \sum_{j\in\mathcal{N}_i} \Big(\underbrace{e^{-\hat\xi\theta^*}}_{R_1} \underbrace{e^{\hat\xi\bar\theta_{wo_j}}}_{R_2} - \underbrace{e^{-\hat\xi\theta^*}}_{R_1} \underbrace{e^{\hat\xi\bar\theta_{wo_i}}}_{R_3} \underbrace{e^{-\hat\xi\bar\theta_{wo_j}}}_{R_2} \underbrace{e^{\hat\xi\bar\theta_{wo_i}}}_{R_3}\Big)$$

$$-\frac{1}{2}\mathrm{tr}(\Phi_2) \le \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_j}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}}))$$

Proof of Lemma 2

$$-\frac{1}{2}\mathrm{tr}(\Phi_1) \le \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}})$$

$$-\frac{1}{2}\mathrm{tr}(\Phi_2) \le \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_j}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}})$$

$$\dot V = -\frac{1}{2}\sum_{i\in\mathcal{V}} (k_e \mathrm{tr}(\Phi_1) + k_s \mathrm{tr}(\Phi_2))$$

$$\le k_e \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}})$$

$$+ k_s \underbrace{\sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_j}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}})}_{=0} - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}})$$

$= 0$ under Assumption 1 (similarly to Igarashi et al [2])

$$= k_e \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}})$$

$$- k_s \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}})$$

Proof of Lemma 2

$$\dot V \le k_e \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}})$$

$$- \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}})$$

Suppose that $\sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) \le 0$

$$\dot V \le k_e \sum_{i\in\mathcal{V}} -\sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}}) - k_s \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}})$$

under Assumption 2, this term is strictly negative

From theorem on **Ultimate Boundedness** in the book by Khalil, the trajectories of estimates ultimately converge to the set satisfying

$$\sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) \ge 0$$

$$\Rightarrow \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) \le \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}})$$

How to Think?

Notice : We do not use the fact that $e^{\hat\xi\theta^*}$ is an average !

$$\sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) \le \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta} e^{\hat\xi\theta_{wo_i}}) \ \forall e^{\hat\xi\theta}$$

$$\dot V \le k_e \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) - \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\bar\theta_{wo_i}}) - \sigma_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}})$$

$$- \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} b_i \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\bar\theta_{wo_j}})$$

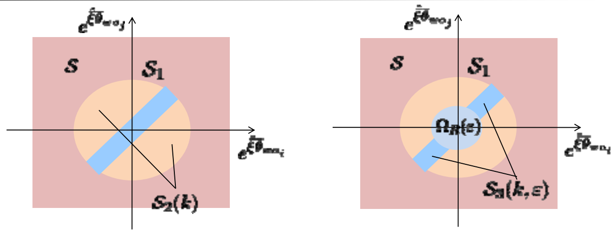
$$\sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\bar\theta_{wo_i}} e^{\hat\xi\theta_{wo_i}}) \ \Rightarrow \ \sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\bar\theta_{wo_j}} e^{\hat\xi\theta_{wo_i}})$$

If $e^{\hat\xi\bar\theta_{wo_i}} \approx e^{\hat\xi\bar\theta_{wo_j}}$ for a common j

$$\sum_{i\in\mathcal{V}} \phi(e^{-\hat\xi\theta^*} e^{\hat\xi\theta_{wo_i}}) \le \phi(e^{-\hat\xi\bar\theta_{wo_j}} e^{\hat\xi\theta_{wo_i}})$$

We next evaluate closeness between estimates $e^{\hat\xi\bar\theta_{wo_i}}$ and $e^{\hat\xi\bar\theta_{wo_j}}$

Division of A Set $\mathcal{S}_1$



$$\mathcal{S}_2(k) = \left\{ (e^{\hat{\xi}\bar{\theta}_{wo_i}}) \,\middle|\, \beta \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\bar{\theta}_{wo_j}}) \geq k\rho_R \right\}$$ Estimates are not close

$\mathcal{S}_3(k,\varepsilon) = \mathcal{S}_1 \setminus (\mathcal{S}_2(k) \cup \Omega_R(\varepsilon))$ Estimates are close enough

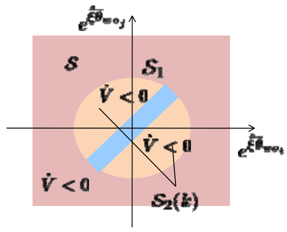
$$\beta = 1 - \sqrt{2\delta_c} \qquad k = k_e / k_s$$

Lemma 3

Suppose that all assumptions in Lemma 2 hold. Then, if $\beta > 0$, then there exists a positive scalar a_2 such that $\dot{V} \leq -a_2,\ a_2 > 0$ holds at least after the time $\tau(c)$ as long as $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}_2(k)$

In the previous version, we proved the main theorem under the misunderstanding that $\mathcal{S}_1 \setminus \mathcal{S}_2(k)$ is positively invariant but it was wrong! The trajectories can get out of the region! Nevertheless, we can prove the main theorem without such a statement as long as V decreases in the region

$$\mathcal{S}_3(k,\varepsilon) = \mathcal{S}_1 \setminus (\mathcal{S}_2(k) \cup \Omega_R(\varepsilon))$$



$V = \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_i^*} e^{\hat{\xi}\bar{\theta}_{wo_i}})$
strictly decreases

Proof of Lemma 3

Lemma 1: $\phi(e^{-\hat{\xi}\theta_i^*} e^{\hat{\xi}\bar{\theta}_{io_i}}) \leq \delta_c \quad \forall t \geq \tau(c)\ ,\ i \in \mathcal{V}$

Lemma 5: $\sigma_i = \lambda_{min}(sym(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}})) \geq \beta := 1 - \sqrt{2\delta_c}$

proved by **perturbation theorem**

$\forall t \geq \tau(c)$, we get

$$\dot{V} \leq \sum_{i\in\mathcal{V}} \left\{ k_e \left(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{io_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - \beta\phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\bar{\theta}_{io_i}}) \right) - k_s\beta \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\bar{\theta}_{wo_j}}) \right\}$$

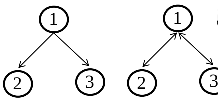
Suppose that $\sum_{i\in\mathcal{V}} \left\{ k_e\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{io_i}}) - k_s\beta \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\bar{\theta}_{wo_j}}) \right\} \leq 0 \ \ldots\ (e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}_2(k)$

Then, $\dot{V} \leq k_e \sum_{i\in\mathcal{V}} \left\{ -\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - \beta\phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\bar{\theta}_{io_i}}) \right\}$

$a_2 =$ strictly negative under Assumption 2

Lemma 4

$$L := \left(\min_{j\in\mathcal{V}} \sum_{i\in\mathcal{V}} l_{ij}^2 \right)^{1/2}$$

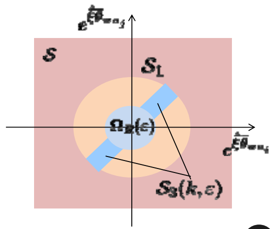


$l_{12} = 1, l_{13} = 1, l_{23} = 2$
$L = \sqrt{1+1} = \sqrt{2}$
(when j = 1)

l_{ij} : Size of the shortest path from note i to j along the graph G whose edges are replaced by undirected ones

Suppose that $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}_3(k,\varepsilon)$ and $\beta > 0$. Then there exists a $j \in \mathcal{V}$ such that the following inequality holds at least after time $\tau(c)$

$$\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\bar{\theta}_{wo_j}}) \leq \frac{L^2 k\rho_R}{\beta} \ \forall i \in \mathcal{V}$$



Theorem 1(Averaging Performance Analysis)

Suppose that all assumptions in Lemma 2 hold. Then, for any $\varepsilon \in (0,1)$ position estimates $(\bar{p}_{io_i})_{i\in\mathcal{V}}$ achieve **ε_p - level averaging accuracy** with

$$\varepsilon_p = \begin{cases} 1 - (1-\varepsilon)(1 - \sqrt{kL})^2 & \text{if } k \leq 1/L^2 \\ 1 & \text{otherwise} \end{cases}$$

In addition, the orientation estimates $(e^{\hat{\xi}\bar{\theta}_{io_i}})_{i\in\mathcal{V}}$ achieve **ε_R - level averaging accuracy** with

$$\varepsilon_R = \begin{cases} 1 - (1-\varepsilon)(\sqrt{\beta} - \sqrt{kL})^2 & \text{if } k \leq \beta/L^2,\ \beta > 0 \\ 1 & \text{otherwise} \end{cases}$$

$k = k_e/k_s \to 0(k_s >> k_e) \Rightarrow \varepsilon_p,\ \varepsilon_R$ get small

$$k \to 0 \Rightarrow \varepsilon_p \approx 0,\ \varepsilon_R \approx 1 - \beta$$

same conclusion as multi-agent optimization[3]

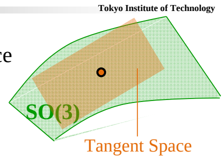
Noticing that $\beta < 1$, we see that an offset appears only on orientations

Difference between SO(3) and Vector Space

In general, all the local issues on a manifold can be approximated by those on a vector space

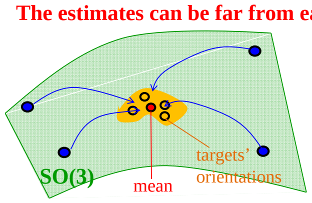
$$\beta := 1 - \sqrt{2\delta_c} \qquad \delta \to 0 \Rightarrow \beta \to 1$$

$$\delta := \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_h}}),\ \ \delta_c = \delta + c$$



Targets' orientations are close to each other

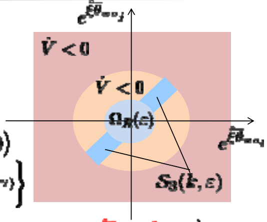
Note that the discussion does **NOT** imply that the theory is valid on a just very limited region on the manifold.
The estimates can be far from each other.



Convergence to a neighbor of the mean is **semi-global.** Only how close to the mean the estimates get after entering the neighbor is an issue on a local region

Proof of Theorem 1

It is sufficient to prove that $\dot{V} < 0$ in the region $\mathcal{S}_3(k,\varepsilon)$ at least after the time $\tau(\varepsilon)$



$$\dot{V} \le \sum_{i\in\mathcal{V}} \Big\{ k_e \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - \beta\phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\theta_{wo_i}})\big) - k_s\beta \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\bar{\theta}_{wo_j}}) \Big\}$$

$$\dot{V} \le \sum_{i\in\mathcal{V}} \Big\{ k_e \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - (1-\epsilon)\beta\phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\theta_{wo_i}})\big)$$

$$- k_e\epsilon\beta\phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\theta_{wo_i}}) - k_s\beta \sum_{j\in\mathcal{N}_i} \phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\bar{\theta}_{wo_j}}) \Big\}$$

strictly negative under Assumption 2

$$\dot{V} \le \sum_{i\in\mathcal{V}} \Big\{ k_e \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - (1-\epsilon)\beta\phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\theta_{wo_i}})\big) \Big\} - a_3$$

Proof of Theorem 1

$$\dot{V} \le \sum_{i\in\mathcal{V}} \Big\{ k_e \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - (1-\epsilon)\beta\phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\theta_{wo_i}})\big) \Big\} - a_3$$

$$\phi(e^{-\hat{\xi}\bar{\theta}_{wo_i}} e^{\hat{\xi}\theta_{wo_i}}) \ge a\phi(e^{-\hat{\xi}\theta_{wo_i}}, e^{\hat{\xi}\bar{\theta}_{wo_i}}) - \frac{a}{1-a}\phi(e^{-\hat{\xi}\theta_{wo_i}}, e^{\hat{\xi}\bar{\theta}_{wo_i}}) \quad \forall a \in (0,1)$$

$$\|x-y\|^2 \ge a\|x-z\|^2 - \frac{a}{1-a}\|y-z\|^2$$

$$(1-a)\|x\|^2 + \frac{1}{1-a}\|y\|^2 + \frac{a^2}{1-a}\|z\|^2 - 2x^Ty + 2ax^Tz - \frac{2a}{1-a}y^Tz$$

$$= (1-a)\Big(x - \frac{1}{1-a}y + \frac{a}{1-a}z\Big)^2 \ge 0$$

$$\dot{V} \le \sum_{i\in\mathcal{V}} \Big\{ k_e \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - (1-\epsilon)\beta a\phi(e^{-\hat{\xi}\theta_{wo_i}}, e^{\hat{\xi}\bar{\theta}_{wo_i}})$$

$$+ \frac{a(1-\epsilon)\beta}{1-a}\phi(e^{-\hat{\xi}\theta_{wo_i}}, e^{\hat{\xi}\bar{\theta}_{wo_i}})\big) \Big\} - a_3$$

Proof of Theorem 1

$$\dot{V} \le \sum_{i\in\mathcal{V}} \Big\{ k_e \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - (1-\epsilon)\beta a\phi(e^{-\hat{\xi}\theta_{wo_i}}, e^{\hat{\xi}\bar{\theta}_{wo_i}})$$

$\beta > 0$

$$+ \frac{a(1-\epsilon)\beta}{1-a}\phi(e^{-\hat{\xi}\theta_{wo_i}}, e^{\hat{\xi}\bar{\theta}_{wo_i}})\big) \Big\} - a_3 \quad \forall a \in (0,1)$$

Lemma 4

$$\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_{wo_i}} e^{\hat{\xi}\bar{\theta}_{wo_i}}) \le \frac{L^2 k\rho_R}{\beta} \ \forall i \in \mathcal{V}$$

If $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}_3(k,\varepsilon)$ and at least after the time $\tau(\varepsilon)$

$$\dot{V} \le k_e \Big\{ \sum_{i\in\mathcal{V}} \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - (1-\epsilon)\beta a\phi(e^{-\hat{\xi}\theta_{wo_i}}, e^{\hat{\xi}\bar{\theta}_{wo_i}})$$

$$+ \frac{a(1-\epsilon)k_eL^2}{1-a}\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}})\big) \Big\} - a_3$$

$$\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) \le \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta} e^{\hat{\xi}\bar{\theta}_{wo_i}}) \ \forall e^{\hat{\xi}\theta}$$

$e^{\hat{\xi}\theta^*}$ **is an average !**

Proof of Theorem 1

$$\dot{V} \le k_e \Big\{ \sum_{i\in\mathcal{V}} \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) - (1-\epsilon)\beta a\,\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}})$$

$$+ \frac{a(1-\epsilon)k_eL^2}{1-a}\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}})\big) \Big\} - a_3$$

$$\dot{V} \le k_e \Big\{ \sum_{i\in\mathcal{V}} \big(\phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}}) - \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}})\big)\Big(1 - (1-\epsilon)\Big(a\beta - \frac{k_eL^2a}{1-a}\Big)\Big) \Big\} - a_3$$

Since $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \mathcal{S}_3(k,\varepsilon_R)$ with $\varepsilon_R = 1 - (1-\epsilon)(\sqrt{\beta} - \sqrt{kL})^2$

we have $\sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\bar{\theta}_{wo_i}}) \le \big(1 - (1-\epsilon)(\sqrt{\beta} - \sqrt{kL})^2\big) \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta^*} e^{\hat{\xi}\theta_{wo_i}})$

Thus, $\dot{V} \le k_e\rho_R(1-\epsilon)\Big(a\beta - \frac{k_eL^2a}{1-a} - (\sqrt{\beta} - \sqrt{kL})^2\Big) - a_3$

it is possible to prove that this term is not positive

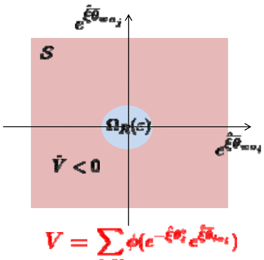
Proof of Theorem 1

The function V is strictly decreasing except for the case $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \Omega_R(\varepsilon_R)$

There is no escape other than $\Omega_R(\varepsilon_R)$

In addition, the time derivative of V is **strictly negative**, the trajectories enter $\Omega_R(\varepsilon_R)$ in a finite time and remain in the set for all subsequent time

→ ε_R **- level averaging accuracy**



$$V = \sum_{i\in\mathcal{V}} \phi(e^{-\hat{\xi}\theta_i} e^{\hat{\xi}\bar{\theta}_{wo_i}})$$

Summary

Visual Sensor Networks

Communication

Communication Graph : $G = (\mathcal{V}, \mathcal{E})$

$(i,j) \in \mathcal{E}$ ⇔ i gets some info. of j

Neighbor Set: $\mathcal{N}_i = \{j \in \mathcal{V} | (i,j) \in \mathcal{E}\}$

Number of Cameras and Targets

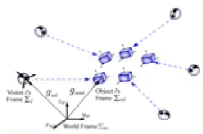
$\mathcal{V} := \{1, \cdots, n\}$

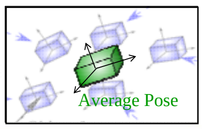
Relative Rigid Body Motion

$\dot{g}_{io_i} = -\hat{V}^b_{wi} g_{io_i} + g_{io_i}\hat{V}^b_{wo_i}, \ i \in \mathcal{V}$

Visual Measurement

$f_i := [f_{i1}^T \cdots f_{im}^T]^T \ m \ge 4$





Definition

The estimates $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}}$ are said to achieve **ε - level averaging accuracy** if there exists a finite T such that

$(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}} \in \Omega_R(\varepsilon) \ \forall t \ge T$

$$u_{ei} = k_e e_{ei} + k_s \sum_{j\in\mathcal{N}_i} E_R(\bar{g}_{io_i}^{-1}\bar{g}_{io_j})$$

VMO[1] Synchronization[2]

Suppose that all assumptions in Lemma 2 hold. Then, for any $\epsilon \in (0,1)$ position estimates $(\bar{p}_{io_i})_{i\in\mathcal{V}}$ achieves ε_p - **level averaging accuracy** with

$$\varepsilon_p = \begin{cases} 1 - (1-\epsilon)(1 - \sqrt{kL})^2 & \text{if } k \le 1/L^2 \\ 1 & \text{otherwise} \end{cases}$$

In addition, the orientation estimates $(e^{\hat{\xi}\bar{\theta}_{wo_i}})_{i\in\mathcal{V}}$ achieves ε_R - **level averaging accuracy** with

$$\varepsilon_R = \begin{cases} 1 - (1-\epsilon)(\sqrt{\beta} - \sqrt{kL})^2 & \text{if } k \le \beta/L^2, \ \beta > 0 \\ 1 & \text{otherwise} \end{cases}$$