Collective Motion of Omnidirectional and Oscillating Robot Groups

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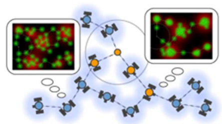
Introduction

Cooperative Control

- A distributed control strategy that achieves specified tasks in multi-agent system

Application

- Search and rescue
- Exploration and mapping



In this presentation

- Attitude and pose synchronization from [2]
- Phase and spatial control of oscillating robots from [1]
  - Oscillating means that movement is periodical
  - Study [1] is based on movement of real fish
- A more complicated model for the fish from [1]

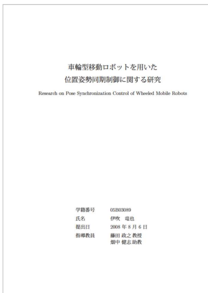
Outline

T. Ibuki, “Research on Pose Synchronization Control of Wheeled Mobile Robots” [2]

- Omnidirectional robots

D. A. Paley, N. E. Leonard and J. K. Parrish, ”Oscillator Models and Collective Motion” [1]

- Oscillating robots, PCOD model
- Oscillating robots, Collective Behaviour model



車輪型移動ロボットを用いた
位置姿勢同期制御に関する研究

Research on Pose Synchronization Control of Wheeled Mobile Robots

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Theory

Problem setting

Omnidirectional robot

- Movement and attitude are decoupled
- Can move all directions without turning
- Dynamics

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & -\sin\theta_i & 0\\ \sin\theta_i & \cos\theta_i & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}v_i\\ \omega_i\end{bmatrix}$$

- Visibility graph $G = (V, E)$ fixed and strongly connected

Used symbols

$v \in \mathbb{R}^2$

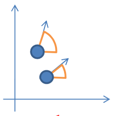
n number of robots

p position, $[x\ y]^T$

$\theta_{ij} = \theta_i - \theta_j$

$V \triangleq \{1, \dots, n\}$ nodes

$E \subset V \times V$ edges



Attitude synchronization

- Angles and speed equal
- $v_i = v_j\ \forall i, j \in \{1, \dots, n\}$
- $\lim_{t\to\infty}(\theta_{ij}) = 0\ \forall i, j$

Pose synchronization

- Angles and position equal
- $\lim_{t\to\infty}(p_{ij}) = 0\ \forall i, j$
- $\lim_{t\to\infty}(\theta_{ij}) = 0\ \forall i, j$

Theory

Control law

Attitude synchronization

- Linear velocity common constant
- Angular velocity based on neighbours
- $\begin{bmatrix}v_i\\ \omega_i\end{bmatrix} = \begin{bmatrix}v\\ k_{\alpha i}\sum_{j\in N_i}\sin(\theta_j - \theta_i)\end{bmatrix}$

Pose synchronization

- Angular velocity ω_i like in attitude synchronization
- Linear velocity is transformed to relative coordinates
- $\begin{bmatrix}v_i\\ \omega_i\end{bmatrix} = K_i \sum_{j\in N_i}\begin{bmatrix}\cos\theta_i & \sin\theta_i & 0\\ -\sin\theta_i & \cos\theta_i & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_j - x_i\\ y_j - y_i\\ \sin(\theta_j - \theta_i)\end{bmatrix}$

Used symbols

N_i neighbours of robot i

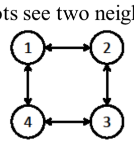
K_i matrix of gains

Theorem Using the vehicle dynamics described and the attitude and pose control laws, the n robots achieve attitude and pose synchronization respectively.

Simulation

Simulation setting

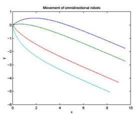
- $n = 4$ robots
- Initial attitudes random
- Robots see two neighbours



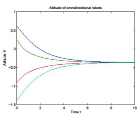
Simulation results

Attitude synchronization

- Initial position (0, 0)



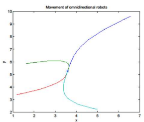
XY Plane



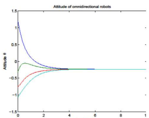
Attitude

Pose synchronization

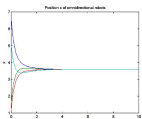
- Initial positions random



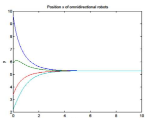
XY plane



Attitude



X



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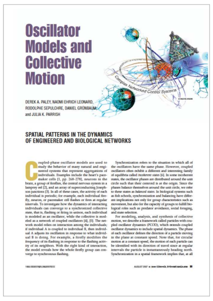


Theory

Particles with Coupled Oscillator Dynamics (PCOD)

- Mathematical framework to describe motion
- Particle model

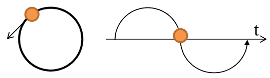
$$\begin{cases} \dot{r}_k = e^{i\theta_k} \\ \dot{\theta}_k = u_k(r,\theta) \end{cases}$$

$$u_k = \omega_0 + u_k^{spac}(r,\theta) + u_k^{ori}(\theta)$$

Used symbols

$r_k = x + iy_k$

ω_0 base angular velocity



Definition The phases θ_j and θ_k are phase locked if $\dot{\theta}_{kj} = 0$. A synchronized phase arrangement θ is a phase-locked arrangement for which $\theta_k = \theta_j$ for all pairs j and k, which implies that the particles are in a parallel formation.

- Simply, phase locking is a new term that means two phases move at the same speed.

Theory

Phase Control

- Objective is to synchronize or balance the phase
- Phase order parameter $p_\theta \triangleq \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$
  - When $|p_\theta| = 1$ phases synchronized
  - When $p_\theta = 0$ phases balanced
- Control law $u_k^{ori} = -\frac{K_1}{N}\langle ie^{i\theta_k}, L_k e^{i\theta}\rangle$, $u_k^{spac} = 0$

Spatial Control

- Objective is to get robots converge on a circle
- Centre of the circle $c_k \triangleq r_k + \omega_0^{-1} ie^{i\theta_k}$
- Control law $u_k^{spac} = -\frac{K_0}{N}\langle e^{i\theta_k}, L_k c\rangle$, $u_k^{ori} = 0$

Phase and Spatial Control

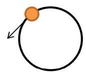
- Phase and spatial control combined
- $u_k = \omega_0 + u_k^{ori} + u_k^{spac}$

Used symbols

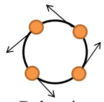
N number of robots

L_k kth row of L

r position



Synchronization $N = 4$



Balancing $N = 4$

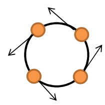
Theory

Symmetric Circular Formations

- Objective is to divide N robots into M groups balanced on a circle → phase control
- Generalized phase order parameter, mth moment of the group $p_{m\theta} \triangleq \frac{1}{mN}\sum_{k=1}^{N} e^{im\theta_k}$
  - When $p_{m\theta} = 0, m = 1, \ldots, M-1$ the phases are balanced
  - When $M|p_{m\theta}| = 1, m = M$ the phases are synchronized
- $u_k^{ori} \triangleq -\sum_{m=1}^{M} \frac{K_m}{mN} \sum_{j=1}^{N} \sin(m\theta_j - m\theta_k)$
  - Results in $2M$ groups instead of M → future work
- u_k^{spac} is the spatial control law

Used symbols

M number of groups



N robots in $M = 4$ groups

Differences and Similarities

Omnidirectional Case

- Attitude synchronization, angular velocity

$\omega_i = k_{\alpha i} \sum_{j \in N_i} \sin(\theta_j - \theta_i)$

Oscillating Case

- Phase synchronization, orientation part

$\omega_k = -\frac{K_1}{N}\langle ie^{i\theta_k}, L_k e^{i\theta}\rangle$

$\Rightarrow \omega_k = i\frac{K_1}{N}\sum_{j \in N_k} 1 - \frac{\cos(\theta_i)+i\sin(\theta_i)}{\cos(\theta_j)+i\sin(\theta_j)}$

  - Also works

$\omega_k = -\frac{K_1}{N}\sum_{j \in N_k} \sin(\theta_j - \theta_k)$

- Symmetric circular formations, orientation part

$\omega_k = -\sum_{m=1}^{M} \frac{K_m}{mN} \sum_{j=1}^{N} \sin(m\theta_j - m\theta_k)$

⇒ All follow similar pattern $\omega_k = c\sum_j f(k,j), \quad c \in \mathbb{C}$

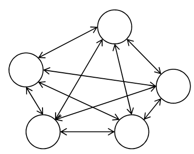
Visibility Graph G

- Different controls require different visibility graphs

Requires complete graph

- Phase balancing

When full information used in simulation, everyone sees everyone

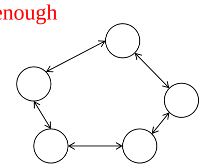


Connected graph is enough

- Phase synchronization
- Spatial synchronization
- Symmetric circular formations

When partial information used in simulation, everyone sees two others

- Generally, any connected graph is enough



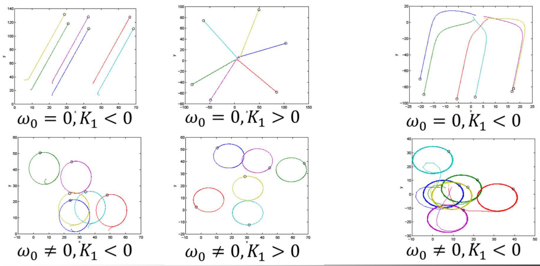
Simulation

Phase Control
- $n = 6$ robots
- Initial positions
- Initial attitudes random

Important points
- $K_1 < 0$, phase synchronization
- $K_1 > 0$, phase balancing

Full information

Partial information
- Only synchronization



$\omega_0 = 0, K_1 < 0$ | $\omega_0 = 0, K_1 > 0$ | $\omega_0 = 0, K_1 < 0$

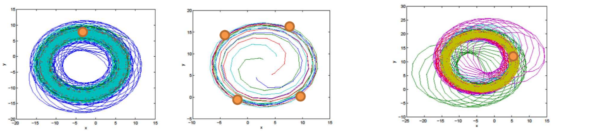
$\omega_0 \neq 0, K_1 < 0$ | $\omega_0 \neq 0, K_1 > 0$ | $\omega_0 \neq 0, K_1 < 0$

Simulation

Spatial Control
- Initial positions random
- Initial attitudes random
- $\omega_0 = 0.1$

Important points
- $K_1 < 0$, phase synchronization
- $K_1 > 0$, phase balancing

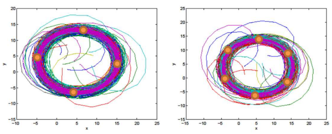
Full information, $n = 4$ | **Partial information, $n = 6$**



$K_1 < 0$ | $K_1 > 0$ | $K_1 < 0$

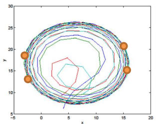
Simulation

Circular Formations
- $n = 6$ robots
- Initial positions random
- Initial attitudes random
- Full information
- $\omega_0 = 0.1$
- Unexpectedly $2M$ groups instead of M groups

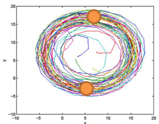


N=12, M=2 | N=12, M=3

Some Less Obvious Points In Phase and Spatial Control



Phase balancing (full information) does not necessarily mean equal intervals



Without full information (only knowledge of 2 neighbours) balancing does not result into equal intervals.

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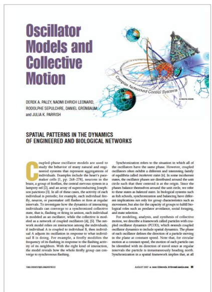


Oscillator Models and Collective Motion

SPATIAL PATTERNS IN THE DYNAMICS OF ENGINEERED AND BIOLOGICAL NETWORKS

Collective Behaviour Model – Exact Theory

- More realistic model, incl. blind spot
- Discrete model

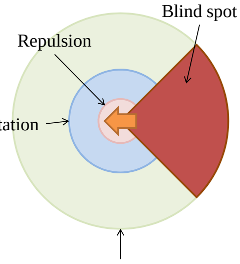
$$\begin{cases} r_k(q+1) = r_k(q)e^{i\theta_k(q)} \\ \theta_k(q+1) = \theta_k(q) + T u_k(q) \end{cases}$$

  - $T \ll 1$ is response latency
- Three behaviour zones
  - Repulsion – get away
  - Orientation – synchronize attitude
  - Attraction – get closer

$$\begin{cases} v_k^{rep}(q) = -\sum_{j \in N_k^{rep}(q)} \hat{r}_{kj}(q) \\ v_k^{ori}(q) = e^{i\theta_k(q)} + \sum_{j \in N_k^{ori}(q)} e^{i\theta_j(q)} \\ v_k^{att}(q) = \sum_{j \in N_k^{att}(q)} \hat{r}_{kj}(q) \end{cases}$$

- Control input (dropping q notation)

$$u_k = \omega_k + \langle i e^{i\theta_k}, K_{rep} v_k^{rep} + K_{ori} v_k^{ori} + K_{att} v_k^{att} \rangle$$



Collective Behaviour Model – Applied Theory

- If repulsion zone is assumed empty and input is written in terms of directed graph Laplacian L

$$u_k = \omega_k - K_{ori} \langle i e^{i\theta_k}, L_k^{ori} e^{i\theta} \rangle + K_{att} \langle i e^{i\theta_k}, L_k^{att} r \rangle$$

⇒ Collective Behaviour Model can be approximated with PCOD as

$$u_k = \omega_0 - \frac{K_1 - K_0}{N} \langle i e^{i\theta_k}, L_k^{ori} e^{i\theta} \rangle + \omega_0 \frac{K_0}{N} \langle i e^{i\theta_k}, L_k^{att} r \rangle$$

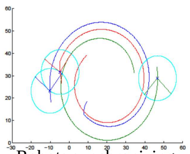
- For comparison, phase and spatial control

$$u_k = \omega_0 - \frac{K_1}{N} \langle i e^{i\theta_k}, L_k e^{i\theta} \rangle - \frac{K_0}{N} \langle e^{i\theta_k}, L_k c \rangle$$

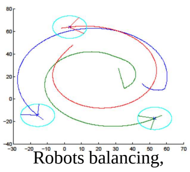
- Graph Laplacians for different zones still recalculated all the time

Simulations

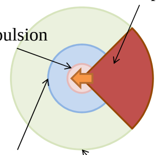
- Initial positions and angles random, $\omega_0 = 0.1$, blind spot 90°
- Repulsion zone empty, orientation zone drawn in picture, attraction zone $\gg$ orientation zone (all robots see other robots)
- Simulation very time consuming
  - Simulating bifurbications (changing number of robots but keeping the same control law) left out → future work



Robots synchronizing, one robot is catching up



Robots balancing, equal intervals



Future Works and References

Future works

- Understanding why control law for circular formations produces $2M$ groups instead of M groups.
- Reprogramming the algorithm for simulation collective behaviour model and studying bifurbications

References

[1] D. A. Paley, N. E. Leonard and J. K. Parrish, ”Oscillator Models and Collective Motion,” *IEEE Control Systems Magazine*, Vol. 27, No. 4, Aug., 2007, pp. 89-105.

[2] T. Ibuki, “Research on Pose Synchronization Control of Wheeled Mobile Robots,” *Tokyo Institute of Technology Bachelor Thesis*, 2008.