Study on Pose Synchronization of Multiple Robots

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Introduction

Robotic Network

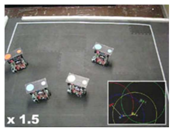
A network consisting of multiple robots, for example:

- mobile sensors
- unmanned vehicles

Advantages of a network are *better performance* or *robustness against failure.*

Application

- Environment monitoring
- Search
- Exploration and Mapping
- Rescue



Operation of Robotic Network

Each robot has to a act *cooperatively* while using only *limited information*.

➡ Cooperative Control

Cooperative Control Problems can be formulated as Pose Coordination Problems.

Outline

- Setting
- Attitude Synchronization [2]
- Attitude Synchronization with Leader [2]
- Pose Synchronization [1]
- Conclusion and Future Works

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- Attitude Synchronization with Leader
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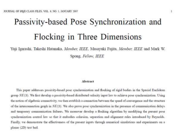
Setting

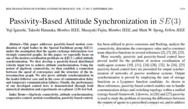
This presentation is a brief introduction of the main results of the following papers:

Paper:

T. Hatanaka, Y. Igarashi, M. Fujita and M.W. Spong, "Passivity-Based Pose Synchronization and Flocking in Three Dimensions", *IEEE Trans. on Automatic Control*, 2010 (conditionally accepted).

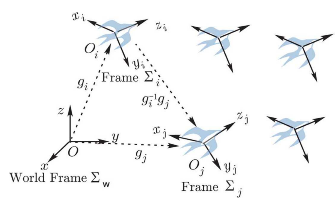


Y. Igarashi, T. Hatanaka, M. Fujita and M.W. Spong, "Passivity-Based Attitude Synchronization in SE(3)", *IEEE Trans. on Control Systems Technology*, Vol. 17,No. 5, pp. 1119-1134, 2009.



The results are transferred from 3D to 2D.

Setting



- We consider n robots in an inertial coordinate frame Σ_w.
- Each robot has a body fixed frame Σ_i.
- The coordinate frames are all Cartesian and right-handed.

Robot Model

The robots are considered as rigid bodies:

- the configuration of the robots can be written as
$$g_i = \begin{bmatrix} e^{\hat{\xi}_i\theta_i} & q_i \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^4$$
with the orientation $e^{\hat{\xi}_i\theta_i}$ and the virtual position $q_i := p_i + d_i$, where d_i is a bias and $p_i \in \mathcal{R}^3$ is the position of the rigid body and
$$\hat{\xi}_i = \widehat{\begin{bmatrix} \xi_{i1} \\ \xi_{i2} \\ \xi_{i3} \end{bmatrix}} = \begin{bmatrix} 0 & -\xi_{i3} & \xi_{i2} \\ \xi_{i3} & 0 & -\xi_{i1} \\ -\xi_{i2} & \xi_{i1} & 0 \end{bmatrix}.$$
- the body velocity $V_i^b = (v_i, w_i)^T \in \mathcal{R}^6$ can also be written as
$$\hat{V}_i^b = \begin{bmatrix} \hat{\omega}_i & v_i \\ 0 & 0 \end{bmatrix} \in \mathcal{R}^4$$

- so the kinematic model is
$$\dot{g}_i = g_i \hat{V}_i^b$$

Robot Model

Model simplifications for 2D

The following simplifications hold:

- $e^{\hat{\xi}_i\theta_i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} := R(\theta_i) \in \mathcal{R}^{2\times 2}$
- $\omega_i \in \mathcal{R} \to \hat{\omega}_i = \begin{bmatrix} 0 & -\omega_i \\ \omega_i & 0 \end{bmatrix} \in \mathcal{R}^{2\times 2}$
- $q_i, v_i \in \mathcal{R}^2$

- so the kinematic model becomes
$$\begin{bmatrix} \dot{q}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} R(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$

Network

The interconnections of the robotic network are described by the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$:

- $\mathcal{V} := \{1, \ldots, n\}$ is the set of all robots
- $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set, which contains pairs of robots, representing the communication
- $\mathcal{W}$ is the set of the weight $w_{ij} > 0$ for each edge, which represents reliability.

The set of the neighbors of each robot is defined as
$$\mathcal{N}_i := \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}.$$

With this we can define the weighted Laplacian matrix
$$L_w := \begin{cases} \sum_{j\in\mathcal{N}_i} w_{ij}, & if\ j = i \\ -w_{ij}, & if\ j \in \mathcal{N}_i \\ 0, & if\ j \notin \mathcal{N}_i \end{cases}$$

Passivity

Definition of Passivity:

The system is passive if there is a continuous differentiable positive semi definite function ψ, such that:
$$\dot{\psi}(g_i) \leq (V_i^b)^T \Pi_i$$
where V_i^b is the vector of the inputs and Π_i is the vector of outputs.

For this model we use:
$$V_i^b = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}$$
$$\Pi_i := \begin{bmatrix} R^{-1}(\theta_i) q_i \\ \sin\theta_i \end{bmatrix}.$$

If $|\theta_i| < \frac{\pi}{2}$, the values of $\sin\theta_i$ are distinct and we can use Π_i as output of the model.

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Passivity

To show the passivity of the model (in 2D) we use the potential function representing the total energy
$$\psi(g_i) := tr\left(\begin{bmatrix} \frac{1}{2} \cdot I_2 & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}^{\mathrm{T}} (I_3 - g_i)^T (I_3 - g_i) \begin{bmatrix} \frac{1}{2} \cdot I_2 & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \right) = \frac{1}{2} \|p_i\|^2 + \phi(\theta_i)$$
with rotational energy
$$\phi(\theta_i) := \frac{1}{2} tr\left(I_2 - \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} \right) = 1 - \cos\theta_i \geq 0$$

So the time derivate of ψ is
$$\dot{\psi}(g_i) = q_{i,x}\dot{q}_{i,x} + q_{i,y}\dot{q}_{i,y} + \dot{\phi}(\theta_i) = (V_i^b)^T \Pi_i.$$

With the input V_i^b, the model is passive.

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- Setting
- Attitude Synchronization
- Attitude Synchronization with Leader
- Pose Synchronization
- Conclusion and Future Works

Attitude Synchronization

Definition:
A group of rigid bodies achieves Attitude Synchronization if $v_i = v_j\ \forall i, \in \{1, \dots, n\}$ and
$$\lim_{t\to\infty} \phi\left(\theta_j - \theta_i\right) = 0\ \forall i, j \in \{1, \dots, n\}$$

The proposed control law is
$$\omega_i = k_i \sum_{j\in\mathcal{N}_i} w_{ij} \sin(\theta_j - \theta_i), \qquad i \in \{1, \dots, n\}$$
with $k_i > 0$ and fixed $v_i = v_j\ \forall\ i, j$.
Additional assumptions:

- orientation matrices of rigid bodies are positive $\left(|\theta_i| < \frac{\pi}{2}\ \forall\ i\right)$
- interconnection graph is fixed and strongly connected.

Attitude Synchronization

Proof of attitude synchronization:
We define the potential
$$U_A := \sum_{i=1}^{n} \frac{\gamma_i}{k_i} \phi(\theta_i) = \sum_{i=1}^{n} \frac{\gamma_i}{k_i} (1 - \cos\theta_i)$$
with $\gamma^T = [\gamma_1, \dots, \gamma_n]^T, \gamma_i > 0\ \ \forall i \in \{1, \dots, n\}$ satisfying $\gamma^T L_w = 0$.
This holds because of the strongly connected graph.
The derivative along the trajectories of the model is
$$\dot{U}_A = \sum_{i=1}^{n} \frac{\gamma_i}{k_i} \sin\theta_i\, \omega_i = \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij} \sin\theta_i \sin(\theta_j - \theta_i)$$
$$= \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \gamma_i \left(\Big(\underbrace{1 - \cos\theta_j}_{\phi(\theta_j)}\Big) - \Big(\underbrace{1 - \cos\theta_i}_{\phi(\theta_i)}\Big) - \underbrace{\cos\theta_i}_{>0} \Big(\underbrace{1 - \cos\left(\theta_j - \theta_i\right)}_{\geq 0}\Big) \right) \leq 0$$
with $\sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij} \left(\phi\left(\theta_j\right) - \phi(\theta_i)\right) = -\gamma^T L_w \begin{bmatrix} \phi(\theta_1) \\ \vdots \\ \phi(\theta_n) \end{bmatrix} = 0.$
With Lasalle's Invariance Principle Attitude Synchronization is proved.

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Attitude Synchronization with Leader

Now the rigid body, labeled as 0, acts as leader with constant velocity and constant orientation.

So now Attitude Synchronization is defined as
$$\lim_{t\to\infty} \phi(\theta_0 - \theta_i) = 0 \quad \forall i \in \{1, \dots, n\}.$$

The new control law is
$$\omega_i = k_i \left(\sum_{j\in\mathcal{N}_i} w_{ij} \sin(\theta_j - \theta_i) + c_i w_{i0} \sin(\theta_0 - \theta_i) \right), k_i, w_{i0} > 0$$
where $c_i = \begin{cases} 1, & \text{if the leader is a neighbor of } i \\ 0 & \text{otherwise.} \end{cases}$

To achieve synchronization the following conditions have to be fulfilled:

- relative orientation matrices between leader and agents are positive definite
- the interconnection graph excluding leader is fixed and strongly connected
- there exists at least one $c_i = 1$.

Attitude Synchronization with Leader

Proof for Synchronization:
Use of the potential function
$$U_l = \sum_{i=1}^{n} \frac{\gamma_i}{k_i} (1 - \cos(\theta_0 - \theta_i)) \geq 0.$$
So the derivate along the trajectories of the model is
$$\dot{U}_l = -\sum_{i=1}^{n} \frac{\gamma_i}{k_i} \sin(\theta_o - \theta_i)\, \omega_i$$
$$= -\sum_{i=1}^{n} \gamma_i \sin(\theta_0 - \theta_i) \left(\sum_{j\in\mathcal{N}_i} w_{ij} \sin\left(\theta_j - \theta_i\right) + c_i w_{i0} \sin(\theta_0 - \theta_i) \right)$$
$$= -\sum_{i=1}^{n} \gamma_i \left(\sum_{j\in\mathcal{N}_i} w_{ij} \left(\Big(\underbrace{\cos\left(\theta_j - \theta_0\right) - 1}_{= -\phi(\theta_j - \theta_0)}\Big) + \Big(\underbrace{1 - \cos(\theta_i - \theta_0)}_{= \phi(\theta_i - \theta_0)}\Big) \right.\right.$$
$$\left.\left. + \cos\left(\theta_0 - \theta_j\right) \left(1 - \cos\left(\theta_j - \theta_i\right)\right) \right) + \underbrace{c_i}_{\geq 0} \underbrace{w_{i0}}_{>0} \underbrace{(\sin(\theta_0 - \theta_i))^2}_{\geq 0} \right) \leq 0.$$
Again with Lasalle's Invariance Principle we prove synchronization.

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Pose Synchronization

Definition of Pose Synchronization of n rigid bodies:

$$\lim_{t\to\infty} \psi(g_i^{-1} g_j) = 0 \;\forall\, i,j\; (i \neq j) \in \{1,\dots,n\}.$$

This implies that the orientations and virtual positions converge to a common value.

To achieve the synchronization the following control law is used

$$V_i^b = -\begin{bmatrix} k_{pi} I_2 & 0 \\ 0 & k_{ei} \end{bmatrix} \sum_{j\in\mathcal{N}_i} \left(w_{ij} \begin{bmatrix} R^{-1}(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_i - q_j \\ \sin(\theta_i - \theta_j) \end{bmatrix} \right) + \begin{bmatrix} R^{-1}(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_d \\ \omega_d \end{bmatrix}, i \in \{1,\dots,n\}$$

with $k_{pi}, k_{ei} > 0$ and the desired linear and angular velocities v_d and $\omega_d \coloneqq \dot{\theta}_d$.

Pose Synchronization

Theorem:

If there exists a $R(\theta_\alpha)$ such that $R(\bar{\theta}_i) \coloneqq R(\theta_i - \theta_\alpha - \theta_d), \forall\, i$ are positive definite and interconnection graph $\mathcal{G}$ is fixed and strongly connected, the control law achieves pose synchronization.

Proof of the Theorem:
For this we use the potential function

$$U_0 := \sum_{i=1}^{n} \gamma_i \left(\frac{1}{2k_{pi}} \|\bar{q}_i\|^2 + \frac{1}{k_{ei}} \phi(\bar{\theta}_i) \right) \geq 0,$$

with $\bar{q}_i \coloneqq q_i - \int_0^t v_d \, dt$.

Pose Synchronization

Differentiating of U_0 w.r.t. time results in

$$\dot{U}_0 = \sum_{i=1}^{n} \gamma_i \begin{bmatrix} \bar{q}_i \\ \sin(\bar{\theta}_i) \end{bmatrix} \begin{bmatrix} \frac{1}{k_{pi}} I_2 & 0 \\ 0 & \frac{1}{k_{ei}} \end{bmatrix} \left(\begin{bmatrix} R^{-1}(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} V_i^b - \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} \right) = -\sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij} \left(\bar{q}_i^T \left(\bar{q}_i - \bar{q}_j \right) + \sin(\bar{\theta}_i) \sin(\theta_i - \theta_j) \right).$$

We can write $\bar{q}_i^T \left(\bar{q}_i - \bar{q}_j \right) = -\frac{1}{2} \|\bar{q}_j\|^2 + \frac{1}{2} \|\bar{q}_i\|^2 + \frac{1}{2} \|\bar{q}_j - \bar{q}_i\|^2$ by completing the square.

Pose Synchronization

Also we can use

$$\sin\bar{\theta}_i \sin(\theta_i - \theta_j) = \sin\bar{\theta}_i \sin(\bar{\theta}_i - \bar{\theta}_j) = -\phi(\bar{\theta}_j) + \phi(\bar{\theta}_i) + \cos\bar{\theta}_i \left(1 - \cos(\bar{\theta}_i - \bar{\theta}_j)\right).$$

We define

$$\bar{U}_i \coloneqq \frac{1}{2} \|\bar{q}_i\|^2 + \phi(\bar{\theta}_i) \geq 0.$$

Now we use these formulas in $\dot{U}_0$:

$$\dot{U}_0 = \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij} \left(\frac{1}{2} \|\bar{q}_j\|^2 - \frac{1}{2} \|\bar{q}_i\| - \frac{1}{2} \|\bar{q}_i - \bar{q}_j\|^2 + \phi(\bar{\theta}_j) - \phi(\bar{\theta}_i) - \cos\bar{\theta}_i (1 - \cos(\bar{\theta}_i - \bar{\theta}_j)) \right)$$

$$= \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij} \left(\bar{U}_j - \bar{U}_i - \frac{1}{2} \|\bar{q}_i - \bar{q}_j\|^2 - \cos\bar{\theta}_i (1 - \cos(\bar{\theta}_i - \bar{\theta}_j)) \right)$$

Pose Synchronization

With

$$\sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij} (\bar{U}_j - \bar{U}_i) = -\gamma^T L_w \begin{bmatrix} \bar{U}_1 \\ \vdots \\ \bar{U}_n \end{bmatrix} = 0$$

we get

$$\dot{U}_0 = \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \gamma_i w_{ij} \left(-\frac{1}{2} \|\bar{q}_i - \bar{q}_j\|^2 - \cos\bar{\theta}_i (1 - \cos(\bar{\theta}_i - \bar{\theta}_j)) \right) \leq 0.$$

Now we define the set

$$E \coloneqq \left\{ y_i \in SE(2), \forall i \mid \|\bar{q}_i - \bar{q}_j\|^2 = 0, \quad (1 - \cos(\bar{\theta}_i - \bar{\theta}_j)) = 0 \;\forall i,j \right\} = \{ y_i \in SE(2), \forall i \mid \psi(y_i^{-1} y_j) = 0 \;\forall i,j \}$$

With Lasalle's Invariance Principle pose synchronization can be shown.

Pose Synchronization with Leader

Definition for Pose Synchronization of n rigid bodies and a leader with constant linear and angular velocity, labeled 0:

$$\lim_{t\to\infty} \psi(g_i^{-1} g_0) = 0 \;\forall\, i \in \{1,\dots,n\}.$$

The control law is

$$V_i^b = -\begin{bmatrix} k_{pi} I_2 & 0 \\ 0 & k_{ei} \end{bmatrix} \left(\sum_{j\in\mathcal{N}_i} \left(w_{ij} \begin{bmatrix} R^{-1}(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} q_i - q_j \\ \sin(\theta_i - \theta_j) \end{bmatrix} - \begin{bmatrix} v_0 \\ \omega_0 \end{bmatrix} \right) \right) + c_i w_{i0} \begin{bmatrix} R^{-1}(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_i - q_0 \\ \sin(\theta_i - \theta_0) \end{bmatrix} \right), i \in \{1,\dots,n\}$$

with $k_{pi}, k_{ei}, w_{i0} > 0$ and

$$c_i = \begin{cases} 1, & \text{if the leader is a neighbor of } i \\ 0 & \text{otherwise.} \end{cases}$$

Proof and Theorem is the same as for Attitude Synchronization with Leader.

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Summary and Future Works

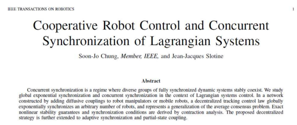
Summary

- Passivity based 2D control laws were presented for
  - Attitude Synchronization
  - Attitude Synchronization with Leader (constant linear and angular velocity)
  - Pose Synchronization
  - Pose Synchronization with Leader (constant linear and angular velocity)

Future Works

- Pose Synchronization with leader (variable velocities) based on Contraction Theory [3]

Paper:
S.-J. Chung and J.-J. E. Slotine, "Cooperative Robot Control and Concurrent Synchronization of Lagrangian Systems", *IEEE Trans. on Robotics*, Vol. 25, No. 3, pp. 686-700, 2009.


IEEE TRANSACTIONS ON ROBOTICS

Cooperative Robot Control and Concurrent Synchronization of Lagrangian Systems

Soon-Jo Chung, *Member, IEEE,* and Jean-Jacques Slotine

Abstract

References

[1] T. Hatanaka, Y. Igarashi, M. Fujita and M.W. Spong, "Passivity-Based Pose Synchronization and Flocking in Three Dimensions", *IEEE Trans. on Automatic Control*, 2010 (conditionally accepted).

[2] Y. Igarashi, T. Hatanaka, M. Fujita and M.W. Spong, "Passivity-Based Attitude Synchronization in SE(3)", *IEEE Trans. on Control Systems Technology*, Vol. 17,No. 5, pp. 1119-1134, 2009.

[3] S.-J. Chung and J.-J. E. Slotine, "Cooperative Robot Control and Concurrent Synchronization of Lagrangian Systems", *IEEE Trans. on Robotics*, Vol. 25, No. 3, pp. 686-700, 2009.