Study on Pose Synchronization of Multiple Robots

Gunter Heppeler
FL10-19-2
25th, November, 2010

Introduction

Robotic Network
A network consisting of multiple robots, for example:
- mobile sensors
- unmanned vehicles

Advantages of a network are better performance or robustness against failure.

Application
- Environment monitoring
- Search
- Exploration and Mapping
- Rescue

Operation of Robotic Network
Each robot has to act cooperatively while using only limited information.

Cooperative Control
Cooperative Control Problems can be formulated as Pose Coordination Problems.

Outline

• Setting
• Attitude Synchronization [2]
• Attitude Synchronization with Leader [2]
• Pose Synchronization [1]
• Conclusion and Future Works

Setting

This presentation is a brief introduction of the main results of the following papers:

Paper:


The results are transferred from 3D to 2D.
The robots are considered as rigid bodies:

- the configuration of the robots can be written as
  \[ \theta_i = [\theta_{i1}, \theta_{i2}, \theta_{i3}, \theta_{i4}] \in \mathbb{R}^4 \]
- with the orientation \( e^{i\theta_i} \) and the virtual position \( q_i = p_i + d_i \), where \( d_i \) is a bias and \( p_i \in \mathbb{R}^3 \) is the position of the rigid body and
  \[ \dot{\theta}_i = \begin{bmatrix} \dot{\theta}_{i1} \\ \dot{\theta}_{i2} \\ \dot{\theta}_{i3} \\ \dot{\theta}_{i4} \end{bmatrix} = \begin{bmatrix} 0 & -\epsilon_{i2} & \epsilon_{i3} & \epsilon_{i4} \\ \epsilon_{i2} & 0 & \epsilon_{i1} & -\epsilon_{i4} \\ -\epsilon_{i3} & -\epsilon_{i1} & 0 & 0 \\ \epsilon_{i4} & \epsilon_{i4} & 0 & 0 \end{bmatrix} \]
- the body velocity \( \dot{q}_i^b = (\dot{v}_i, \omega_i)^T \in \mathbb{R}^6 \) can also be written as
  \[ \dot{q}_i^b = \begin{bmatrix} \dot{v}_i & \omega_i \end{bmatrix} \]
- so the kinematic model is
  \[ \dot{\theta}_i = \theta_i \Omega_i \]

The robots are considered as rigid bodies:

- the configuration of the robots can be written as
  \[ \theta_i = [\theta_{i1}, \theta_{i2}, \theta_{i3}, \theta_{i4}] \in \mathbb{R}^4 \]
- with the orientation \( e^{i\theta_i} \) and the virtual position \( q_i = p_i + d_i \), where \( d_i \) is a bias and \( p_i \in \mathbb{R}^3 \) is the position of the rigid body and
  \[ \dot{\theta}_i = \begin{bmatrix} \dot{\theta}_{i1} \\ \dot{\theta}_{i2} \\ \dot{\theta}_{i3} \\ \dot{\theta}_{i4} \end{bmatrix} = \begin{bmatrix} 0 & -\epsilon_{i2} & \epsilon_{i3} & \epsilon_{i4} \\ \epsilon_{i2} & 0 & \epsilon_{i1} & -\epsilon_{i4} \\ -\epsilon_{i3} & -\epsilon_{i1} & 0 & 0 \\ \epsilon_{i4} & \epsilon_{i4} & 0 & 0 \end{bmatrix} \]
- the body velocity \( \dot{q}_i^b = (\dot{v}_i, \omega_i)^T \in \mathbb{R}^6 \) can also be written as
  \[ \dot{q}_i^b = \begin{bmatrix} \dot{v}_i & \omega_i \end{bmatrix} \]
- so the kinematic model is
  \[ \dot{\theta}_i = \theta_i \Omega_i \]

The interconnections of the robotic network are described by the graph \( G = (V, E, W) \):

- \( V = \{1, ..., n\} \) is the set of all robots
- \( E \subset V \times V \) is the edge set, which contains pairs of robots, representing the communication
- \( W \) is the set of the weight \( w_{ij} > 0 \) for each edge, which represents reliability.

The set of the neighbors of each robot is defined as \( N_i = \{ j \in V \mid (i, j) \in E \} \).

With this we can define the weighted Laplacian matrix

\[ L_W = \sum_{j \in N_i} w_{ij} \begin{cases} w_{ij}, & \text{if } j = i \\ -w_{ij}, & \text{if } j \in N_i \\ 0, & \text{if } j \notin N_i \end{cases} \]

For this model we use:

\[ V_i^p = \begin{bmatrix} v_i^p \\ \theta_i \end{bmatrix}, \quad \Pi_i = \begin{bmatrix} 1 \\ \sin \theta_i \end{bmatrix} \]

If \( |\theta_i| < \frac{\pi}{2} \), the values of \( \sin \theta_i \) are distinct and we can use \( \Pi_i \) as output of the model.

To show the passivity of the model (in 2D) we use the potential function representing the total energy

\[ \psi(q_i) = \frac{1}{2} \eta_i^T (I - g_i^T (I - g_i)) \frac{1}{2} \eta_i + \frac{1}{2} ||v_i||^2 + \phi(\theta_i) \]

with rotational energy

\[ \phi(\theta_i) = \frac{1}{2} \eta_i^T \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix} = 1 - \cos \theta_i \geq 0 \]

So the time derivative of \( \psi \) is

\[ \dot{\psi}(q_i) = q_i \cdot \dot{q}_i + q_i \cdot \dot{q}_i + \dot{\phi}(\theta_i) = (V_i^p)^T \Pi_i \]

With the input \( V_i^p \), the model is passive.
Attitude Synchronization

Definition:
A group of rigid bodies achieves Attitude Synchronization if \( v_i = v_j \ \forall i, j \in \{1, ..., n\} \) and
\[
\lim_{t \to \infty} \phi(\theta_j - \theta_i) = 0 \ \forall i, j \in \{1, ..., n\}
\]

The proposed control law is
\[
\alpha_i = k_i \sum_{j \neq i} w_{ij} \sin(\theta_j - \theta_i), \quad i \in \{1, ..., n\}
\]
with \( k_i > 0 \) and fixed \( v_i = v_j \ \forall i, j \).

Additional assumptions:
- orientation matrices of rigid bodies are positive \( \left\{ \theta_i \right\} \subset \mathbb{R} \)
- interconnection graph is fixed and strongly connected.

Proof of attitude synchronization:
We define the potential
\[
\phi(\theta_j - \theta_i) = \frac{1}{2} \sum_{j \neq i} w_{ij} (1 - \cos(\theta_j - \theta_i))
\]
with \( y^T = [y_1, ..., y_n]^T, y_i > 0 \ \forall i \in \{1, ..., n\} \) satisfying \( y^T L y = 0 \).

This holds because of the strongly connected graph.
The derivative along the trajectories of the model is
\[
\dot{\phi}_i = \sum_{j \neq i} w_{ij} \sin(\theta_i - \theta_j) \sum_{j \neq i} w_{ij} \sin(\theta_j - \theta_i) - \frac{1}{2} \sum_{j \neq i} w_{ij} (1 - \cos(\theta_j - \theta_i)) \leq 0
\]
with \( \sum_{i=1}^n \sum_{j \neq i} w_{ij} \phi(\theta_j - \phi(\theta_i)) = -y^T L y \phi(\theta_i) \leq 0 \).

With Lasalle’s Invariance Principle Attitude Synchronization is proved.

Outline

- Setting
- Attitude Synchronization
  - Attitude Synchronization with Leader
  - Pose Synchronization
- Conclusion and Future Works

Attitude Synchronization with Leader

Now the rigid body, labeled as 0, acts as leader with constant velocity and constant orientation.

So now Attitude Synchronization is defined as
\[
\lim_{t \to \infty} \phi(\theta_j - \theta_i) = 0 \ \forall i \in \{1, ..., n\}.
\]

The new control law is
\[
\alpha_i = k_i \left( \sum_{j \neq i} w_{ij} \sin(\theta_j - \theta_i) + c_i w_{0i} \sin(\theta_0 - \theta_i) \right), \quad k_i, w_{0i} > 0
\]
with \( c_i \equiv \begin{cases} 1, & \text{if the leader is a neighbor of } i \\ 0, & \text{otherwise.} \end{cases} \)

To achieve synchronization the following conditions have to be fulfilled:
- relative orientation matrices between leader and agents are positive definite
- the interconnection graph excluding leader is fixed and strongly connected
- there exists at least one \( c_i = 1 \).

Outline

- Setting
- Attitude Synchronization
  - Attitude Synchronization with Leader
  - Pose Synchronization
- Conclusion and Future Works
Definition of Pose Synchronization of n rigid bodies:
\[
\lim_{t \to \infty} \varphi(g^{-1} g_i) = 0 \ \forall \ i, j, (i \neq j) \in \{1, \ldots, n\}.
\]
This implies that the orientations and virtual positions converge to a common value.

To achieve the synchronization the following control law is used:
\[
\nu_i^p = - \frac{\mu_i}{2} \sum_{j=1}^{n} \left[ \frac{1}{2} \nabla_\theta \varphi(g^{-1} g_j) \right] \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j)
\]
\[
\nu_i^p + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j)
\]
with \(k_i, \kappa_i > 0\) and the desired linear and angular velocities \(v_{d_i} = \frac{\theta_{d_i}}{\omega_{d_i}}\).

Theorem:
If there exists a \(R(\theta_k)\) such that \(R(\theta_k) = R(\theta_k - \theta_k), \forall \ i\) are positive definite and interconnection graph \(g\) is fixed and strongly connected, the control law achieves pose synchronization.

Proof of the Theorem:
For this we use the potential function
\[
U_i = \sum_{j=1}^{n} \frac{\gamma_i}{2} \| \varphi(g^{-1} g_j) - \varphi(g^{-1} g_j) \|^2 + \frac{1}{2} \gamma_i^p (\varphi(g^{-1} g_j) - \varphi(g^{-1} g_j)) = 0,
\]
with \(\gamma_i = q_i - \int v_{d_i} dt\).

Definition for Pose Synchronization of Leader:
\[
\lim_{t \to \infty} \varphi(g^{-1} g_i) = 0 \ \forall \ i, \ \text{if the leader is a neighbor of} \ i \ \text{or otherwise}.
\]

The control law is
\[
v_i^p = - \frac{\mu_i}{2} \sum_{j=1}^{n} \left[ \frac{1}{2} \nabla_\theta \varphi(g^{-1} g_j) \right] \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j)
\]
\[
\nu_i^p + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j) + \frac{1}{2} \sum_{j=1}^{n} \kappa_i^p \varphi(g^{-1} g_j)
\]
with \(k_i, \kappa_i > 0\) and \(\gamma_i = \frac{1}{2}, \text{if the leader is a neighbor of } i\) otherwise.

Proof and Theorem is the same as for Attitude Synchronization with Leader.
Outline

• Setting
• Attitude Synchronization
• Attitude Synchronization with Leader
• Pose Synchronization
• Conclusion and Future Works

Summary and Future Works

Summary
• Passivity based 2D control laws were presented for
  • Attitude Synchronization
  • Attitude Synchronization with Leader (constant linear and angular velocity)
  • Pose Synchronization
  • Pose Synchronization with Leader (constant linear and angular velocity)

Future Works
• Pose Synchronization with leader (variable velocities) based on Contraction Theory [3]

Paper:
S.-J. Chung and J.-J. E. Slotine,

References

