Cooperative Control of Mobile Inverted Pendulum Robot with Underactuated Mechanical Systems

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Outline

- Introduction , Previous Works
- Modeling
  - Mobile Inverted Pendulum
  - Derivation of Nonlinear Model
- Input-Output Feedback Linearization
- Conclusion, Future Works

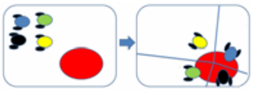
Introduction

Cooperative Control

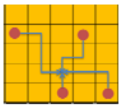
A distributed control law that achieves specified tasks in multi-agent systems.

Example :
- Formation control
- Mobile sensor networks
- coverage
- consensus

Fomulation Control



Coverage[4]



Consensus[4]

Wheeled Inverted Pendulum

Advantages

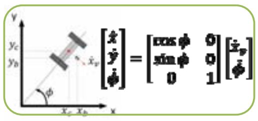
•Easy to rotate
•Take little space to move
•Smaller friction force (Compared with Omnidirectional Robot)

Previous Works

Previous Work : Reserch of Pose Synchronization Control Problem[1]

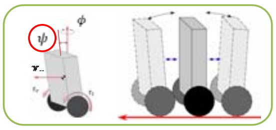
Verification experiment

•Use mobile inverted pendulum robot
•Based on two-wheel-robot's kinematic model without inverting dynamics etc.



Problems

- Inverting dynamics is not considerd in pose synchronization
- Swinging distracts move of the cart



Objective

- Designing local controller with inverting dynamics
- Comparing the designed controller with the existing LQ controller by verification experiment

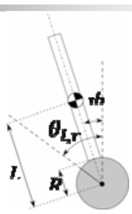
Mobile Inverted Pendulum

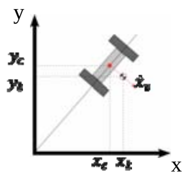
State (6)

$$x' = [\theta_l\ \theta_2\ \psi\ \dot\theta_l \dot\theta_r \dot\psi]^T$$: Angle

T

$$x = [x_v\ \phi\ \psi\ \dot x_v\ \dot\phi\ \dot\psi]^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$$: Attitude





Input (2)

$$(u_1, u_2) = (\tau_l, \tau_r) = a * (u_l, u_r)$$

Real Input to Plant

Underactuate

Output (3)

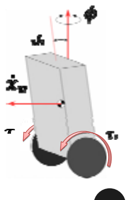
$$(\dot\theta_l, \dot\theta_r, \dot\psi)$$

T

$$(\dot x_v, \dot\phi, \dot\psi)$$

Coordinate Transformation

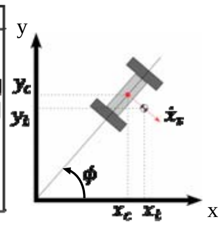
$$T = \begin{bmatrix} R/2 & R/2 & 0 \\ R/W & -R/W & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

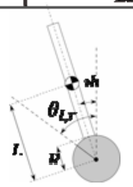


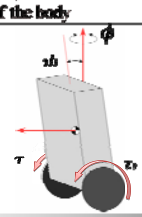
Variable

Variables

Symbol	Variable	Unit
x_b, y_b	Coordinate of the gravity center	[m]
x_c, y_c	Coordinate of the wheel's center	[m]
$\dot x_v$	Velocity of the gravity center of the body	[m/s]
$\tau_{l,r}$	Input torques to each wheels	[Nm]
$u_{l,r}$	Input electrical current	[A]
$\theta_{l,r}$	Rotational angle of each wheel	[rad]
ϕ	Angle of the body around z axis	[rad]
ψ	Lean of the body	[rad]



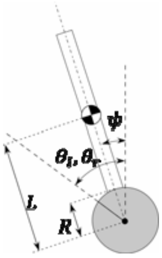




Parameter

Parameters of the model

Symbol	Parameter	Value
M_b	Mass of the cart	0.6113[kg]
M_w	Mass of a wheel	0.0363[kg]
L	Length between the axle and the gravity center of the body	0.0986[m]
R	Radius of the wheel	0.02485[m]
W	Length between the two wheels	0.175[m]
J_ψ	Moment of inertia of the body	$2.276\cdot10^{-3}$[kgm²]
J_ϕ	Moment of inertia of the cart	$1.653\cdot10^{-3}$[kgm²]
J_w	Moment of inertia of the wheels	$4.694\cdot10^{-6}$[kgm²]
J_m	Moment of inertia of the motor rotor	$1.3035\cdot10^{-8}$[kgm²]
ρ	Reduction ratio of the gear	30
β_g	Friction coefficient of viscosity of the gear	$1.0\cdot10^{-4}$[kgm²/s]
β_t	Friction coefficient of viscosity of the tire	0[kgm²/s]
α	Gain from current to torque of the tire	0.0627[kgm²/sA]
g	Gravity acceleration	9.8[m/s²]



Motivation

Nonlinear Design Tools[3]

- Gain Scheduling
  Linearization about a family of operating points, designing a parameterized family of linear controllers.
  → $\sin(\theta) \approx \theta$, $\cos(\theta) \approx 1$
- Feedback Linearization
  Cancelling a nonlinear terms
  - Coordinate transformation
  - Input-Output Linearization

  → Input : $u = \alpha(x) + \beta(x)v$
- Sliding Mode Control
  Stabilizing by high-frequency switching control

etc..

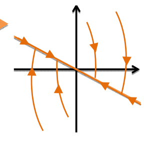


fig.9 A typical phase portrait

Feedback Linearization
by Input-Output Linearization

Input-Output Linearization

Relative degree ρ

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\\ \dot x_5\\ \dot x_6\end{bmatrix} = \begin{bmatrix} x_4\\ x_5\\ x_6\\ f_1(\psi,\dot x_v,\dot\phi,\dot\psi)\\ f_2(\psi,\dot\phi)\\ f_3(\psi,\dot x_v,\dot\phi,\dot\psi)\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&0\\0&g_1(\psi)\\ g_2(\psi)&0\\0&g_3(\psi)\end{bmatrix}\begin{bmatrix}u'_1\\u'_2\end{bmatrix}$$

Output : $x_2 = \varphi$

$$\dot x_2 = x_5$$
$$\ddot x_2 = \dot x_5 = f_2(x) + g_2(x)u'_1$$
$$\rho_1 = 2$$

Output : $x_3 = \psi$

$$\dot x_3 = x_6$$
$$\ddot x_3 = \dot x_6 = f_3(x) + g_3(x)u'_2$$
$$\rho_2 = 2$$

$$\rho = \rho_1 + \rho_2 = 2 + 2 = 4$$

$$\begin{bmatrix}\dot x_2\\ \dot x_3\\ \dot x_5\\ \dot x_6\end{bmatrix} = \begin{bmatrix} x_5\\ x_6\\ f_2(\psi,\dot\phi)\\ f_3(\psi,\dot x_v,\dot\phi,\dot\psi)\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\ g_2(\psi)&0\\0&g_3(\psi)\end{bmatrix}\begin{bmatrix}u'_1\\u'_2\end{bmatrix}$$

$$\begin{bmatrix}\dot x_1\\ \dot x_4\end{bmatrix} = \begin{bmatrix} x_4\\ f_1(\psi,\dot x_v,\dot\phi,\dot\psi)\end{bmatrix} + \begin{bmatrix}0&0\\0&g_1(\psi)\end{bmatrix}\begin{bmatrix}u'_1\\u'_2\end{bmatrix}$$

Partial Feedback Linearization

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\\ \dot x_5\\ \dot x_6\end{bmatrix} = \begin{bmatrix} x_4\\ x_5\\ x_6\\ f_1(\psi,\dot x_v,\dot\phi,\dot\psi)\\ f_2(\psi,\dot\phi)\\ f_3(\psi,\dot x_v,\dot\phi,\dot\psi)\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&0\\ g_1(\psi)&g_1(\psi)\\ g_2(\psi)&-g_2(\psi)\\ g_3(\psi)&g_3(\psi)\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}$$

Input Transformation : $u'_1 = u_1 - u_2$, $u'_2 = u_1 + u_2$

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\\ \dot x_5\\ \dot x_6\end{bmatrix} = \begin{bmatrix} x_4\\ x_5\\ x_6\\ f_1(\psi,\dot x_v,\dot\phi,\dot\psi)\\ f_2(\psi,\dot\phi)\\ f_3(\psi,\dot x_v,\dot\phi,\dot\psi)\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&0\\0&g_1(\psi)\\ g_2(\psi)&0\\0&g_3(\psi)\end{bmatrix}\begin{bmatrix}u'_1\\u'_2\end{bmatrix}$$

$$\dot x_5 = f_2(\psi,\dot\phi) + g_2(\psi)u'_1$$

Linearizable

$$\dot x_4 = f_1(\psi,\dot x_v,\dot\phi,\dot\psi) + g_1(\psi)u'_2$$
$$\dot x_6 = f_3(\psi,\dot x_v,\dot\phi,\dot\psi) + g_3(\psi)u'_2$$

⇒Necessity for Partial Linearization

Linearize $x_6(\dot\psi)$ to stabilize ψ

Input for Linearization

$$u'_1 = \frac{1}{g_2(x)}(-f_2(x) + v_1)$$
$$u'_2 = \frac{1}{g_3(x)}(-f_3(x) + v_2)$$

Use new input v_1, v_2

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\\ \dot x_5\\ \dot x_6\end{bmatrix} = \begin{bmatrix} x_4\\ x_5\\ x_6\\ f_1(x) - \frac{g_1(x)f_3(x)}{g_3(x)}\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&0\\0&\frac{g_1(x)}{g_3(x)}\\ 1&0\\0&1\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix}$$

Input-Output Linearization

Partial Feedback Linearization
Linearization
from input v_1, v_2 to Output $\dot\phi, \dot\psi$

Control law

Linearization of $\dot\phi$, $\dot\psi$

$$u'_1 = \frac{1}{g_2(x)}(-f_2(x) + v_1)$$
$$u'_2 = \frac{1}{g_3(x)}(-f_3(x) + v_2)$$

⇒ $\ddot\phi = v_1$, $\ddot\psi = v_2$

Objective of Control

Synchronization of

- **translational velocity $\dot x_v$**
- **rotational velocity $\dot\phi$**

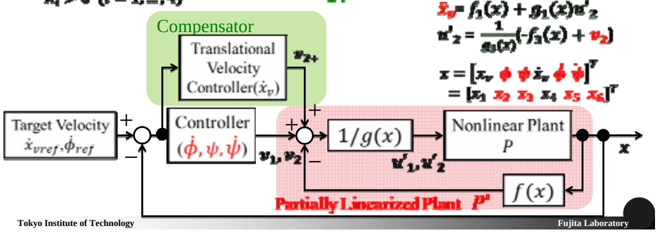
Necessity for not linearized translational velocity $\dot x_v$

$$v_1 = K_1(\dot\phi_{ref} - \dot\phi)$$
$$v_2 = -K_2\psi - K_3\dot\psi + K_4(\dot x_{vref} - \dot x_v)$$
$$K_i > 0 \ (i = 1,\ldots,4)$$

v_{2+}

$$\ddot x_v = f_1(x) + g_1(x)u'_2$$
$$u'_2 = \frac{1}{g_3(x)}(-f_3(x) + v_2)$$

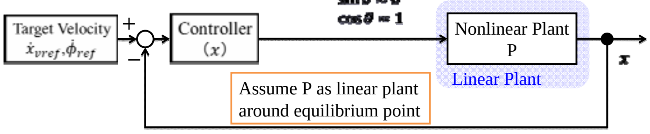
$$x = [x_v\ \phi\ \psi\ \dot x_v\ \dot\phi\ \dot\psi]^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$$



Brock diagrams

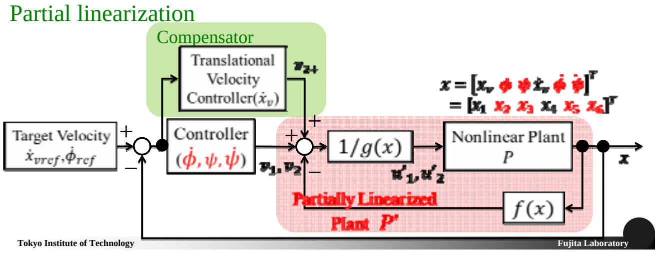
Linearizaion with linear approximation

$$x = [x_v\ \phi\ \psi\ \dot x_v\ \dot\phi\ \dot\psi]^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$$

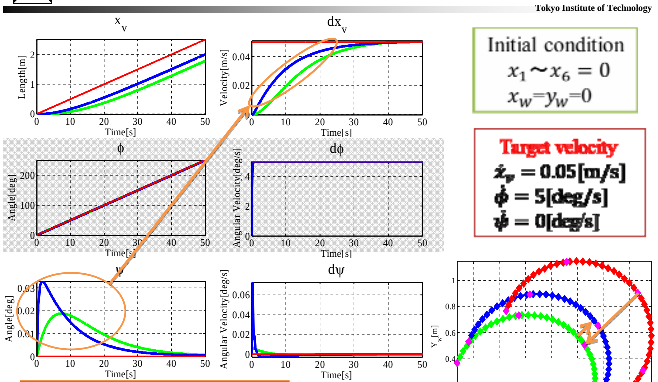


Partial linearization

$$x = [x_v\ \phi\ \psi\ \dot x_v\ \dot\phi\ \dot\psi]^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$$



Simulation



Initial condition
$x_1 \sim x_6 = 0$
$x_w = y_w = 0$

Target velocity
$\dot{x}_v = 0.05$[m/s]
$\dot{\phi} = 5$[deg/s]
$\dot{\psi} = 0$[deg/s]

By considering the inverting ,
運動学モデルの再現精度が向上

PFL
LQ
Kinematic Model

Conclusion

Summary

- Designing of the control system with the dynamics of inverting by using partial feedback linearization
- Verification of the effect at the simulation of velocity following with single robot

Future Works

- Programing of the Built-in controller
- Comparison with existing LQ controller by an experiment
  - Velocity following with single robot
  - Cooperative control (Pose synchronization)

Appendix

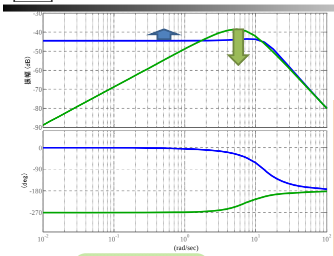
各状態量の導出と扱い

各状態量の導出と扱い

θ :ロータリーエンコーダによる測定値
$\dot{\theta}$:θの疑似微分 : ①
x_v :θから計算 $\frac{R}{2}(\theta_l + \theta_r)$
$\dot{x}_v$: $\dot{\theta}$から計算 $\frac{R}{2}(\dot{\theta}_l + \dot{\theta}_r)$: ②
ϕ :θから計算 $\frac{R}{2}(\dot{\theta}_l + \dot{\theta}_r)$
$\dot{\phi}$: $\dot{\theta}$から計算 $\frac{R}{W}(\dot{\theta}_l - \dot{\theta}_r)$: ②
ψ : $\dot{\psi}$を積分
$\dot{\psi}$:ジャイロセンサによる測定値
高周波ノイズを含むため，ローパスフィルタの必要

①か②でローパスフィルタの必要

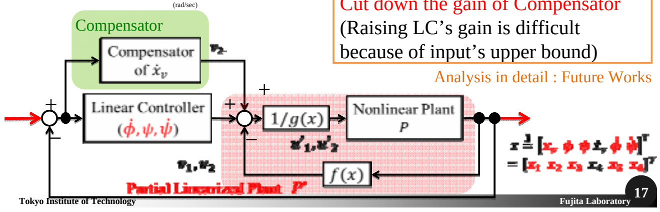
Compensator’s Influence on Stability



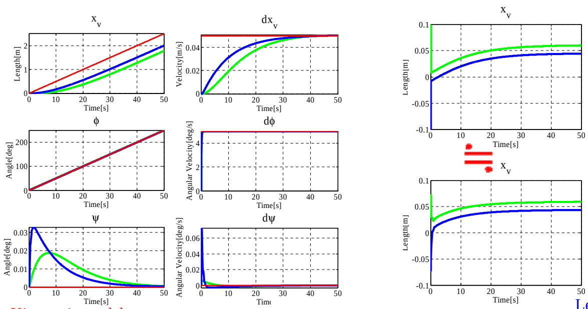
— : Linear Controller($\psi, \dot{\psi}$)
— : Compensator($\dot{x}_v$)

Securing of stability
Compensator and Linear controller share almost the same frequency band
Unable to cut high-frequency band
Cut down the gain of Compensator
(Raising LC’s gain is difficult because of input’s upper bound)

Analysis in detail : Future Works

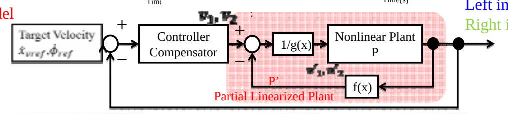


Simulation Result (Comparison of Input)

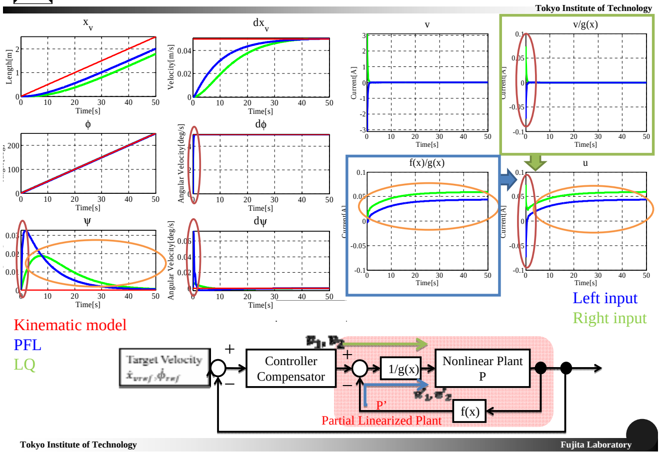


Kinematic model
PFL
LQ

Left input
Right input



Simulation Result (Input of the part of PFL)



Feedback linearization

Example

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}a\sin(x_2)\\ -x_1^2\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}u \qquad u = \left(a\left(\cos\left(\frac{z_2}{a}\right) - \frac{1}{\cos(\frac{z_2}{a})}\right)\right)^{-1} v + z_1^2$$

$$\begin{bmatrix}z_1\\ z_2\end{bmatrix} = \begin{bmatrix}x_1\\ a\sin(x_2)\end{bmatrix}$$

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix} = \begin{bmatrix}z_2\\ 0\end{bmatrix} + a\left(\cos\left(\frac{z_2}{a}\right) - \frac{1}{\cos(\frac{z_2}{a})}\right)\begin{bmatrix}0\\ -z_1^2 + u\end{bmatrix} \qquad \begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix} = \begin{bmatrix}z_2\\ 0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}v$$

A nonlinear system

$$\dot{x} = f(x) + G(x)u \quad (x \in R^n, f: R^n \to R^n, G: R^n \to R^{n\times p})$$

$z = T(x)$ (T:diffeomorphism, $R^n \to R^n$) T, T^{-1}: continuously differentiable

$$\dot{z} = Az + B\gamma(x)[u - \alpha(x)]$$

(A, B):controllable
$\gamma(x)$:nonsingular

$$u = \gamma^{-1}(x)v + \alpha(x) \Rightarrow \dot{z} = Az + Bv$$

Input-Output Linearization(SISO)

Relative degree ρ

A nonlinear system(SISO)

$$\begin{cases}\dot{x} = f(x) + g(x)u\\ y = h(x) \ (h: R^n \to R)\end{cases} \quad (*)$$

f(x),g(x),h(x) are smooth function

Lie derivative

$$L_f h(x) := \frac{\partial h}{\partial x} f(x)$$

$$L_f^i h(x) = L_f L_f^{i-1} h(x) = \frac{\partial L_f^{i-1} h}{\partial x} f(x)$$

$$\dot{y} = \frac{\partial h(x)}{\partial x}\dot{x} = \frac{\partial h(x)}{\partial x}(f(x) + g(x)u) \quad \text{If } \frac{\partial h(x)}{\partial x}g(x) = 0, \dot{y} = \frac{\partial h(x)}{\partial x}f(x)$$

$$\ddot{y} = \frac{\partial L_f h(x)}{\partial x}(f(x) + g(x)u) \quad \text{If } \frac{\partial L_f h(x)}{\partial x}g(x) = 0, \ddot{y} = \frac{\partial L_f h(x)}{\partial x}f(x)$$

$$\vdots$$

$$y^{(\rho)} = \frac{\partial L_f^{\rho-1} h(x)}{\partial x}(f(x) + g(x)u) \quad \text{If } \frac{\partial L_f^{\rho-1} h(x)}{\partial x}g(x) \neq 0$$

$$u = \frac{1}{L_g L_f^{\rho-1} h(x)}(-L_f^{\rho} h(x) + v)$$

→ Relative degree

$$L_g L_f^{i-1} h(x) = 0, i = 1, 2, \cdots, \rho - 1 \qquad L_g L_f^{\rho-1} h(x) \neq 0$$

Relative degree ρ

Case: $\rho < n$

$i = 1, \cdots, \rho$ Observable from y
$i = \rho + 1, \cdots, n$ Unobservable from y ⇒ Feedback linearizable Unlinearizable Partial linearization

Case: $\rho = n$

$i = 1, \cdots, n$ Observable from y ⇒ Full linearization

Theorem1

Consider the system , and suppose it had relative degree ρ. Then for every $x_0 \in N$, a neighborhood N of x_0 exists such that the map

$$T(x) = \left[h(x), L_f h(x), \cdots, L_f^{\rho-1} h(x), \phi_1(x), \cdots, \phi_{n-\rho}(x)\right]^T$$

restricted to N, is a diffeomorphism on N.

$$\frac{\partial \phi_j}{\partial x}g(x) = 0 \ (j = 1, 2, \cdots, n - \rho)$$ & *T(x)* has rank n.

Comparison with Omnidirectional Robot

Advantage of Mobile Inverted Pendulum

- Small-footprint
  - Because of fewer wheels and inverting
- Ability to turn in a small radius
- Small Frictional Force
  - Smaller ground contact area & Difference of mechanism and the way of moving
    - Small static friction
      ⇒Ability to move in low speed
    - Small kinetic friction
      ⇒Less energy loss