Pose Synchronization with Omni-directional Robot

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Introduction

Robotic Network

A network consisting of multiple robot

- Mobile Sensor Network
- Intelligent Transport Systems
- Multiple Unmanned Vehicles System

Fig. 1 Searching Robot[4]

advantages ↓

Network: Performance or Robustness against failure

Unmanned : No risks to human operator

Fig. 2 Camera-Robot Links[5]

Application

- Environment Monitoring
- Search
- Rescue

especially in danger

Fig. 3 ITS[6]

Operation of Robotic Network

Each robot is required to act cooperatively using only limited information ⇨ Cooperative Control

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Introduction

Cooperative Control of Robotic Network

A distributed control law using only local information so that the robotic network achieves specified tasks

Example

- Coverage
- Consensus
- Synchronization
- Formation Control
- Motion Coordination

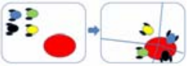


Fig. 4 Coverage[7]

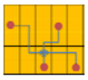


Fig. 5 Consensus[7]

Fig. 6 Resource Allocation[7]

Cooperative Control Problems for Robotic Network

⇩ to localize robots in desired positions and attitudes

Pose Coordination Problems

Pose Synchronization

To lead all agents' position and attitude to a common one by utilizing distributed control strategies

Available Information

Relative pose of neighbor agents

Introduction

Previous Research[1]

- Pose Synchronization of wheel robots[1]

In previous research, theorem is verified in experiment with two-wheel robot

Motivation

Two-wheel robot has non-holonomic constraint

Fig. 6 Two-wheel robot[1]

Fig. 7 Previous experiment[1]

different condition, not suitable to verify

→ more detailed experiment is needed

Objective of this work

To consider pose synchronization on a plane and experiment with omni-directional robots

without non-holonomic constraint

Outline

- Introduction
- Robotic Network
- Pose Synchronization
- Implementation
- Experiment
- Summary

Robotic Network: Rigid Body Motion

Kinematics of Rigid Body

Pose (State X_i)

$(x_{wi}, y_{wi}, \theta_{wi})$ $i \in \{1, \cdots, n\}$

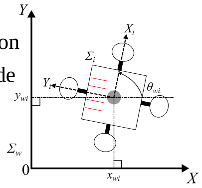
$x_{wi}, y_{wi} \in R$: position

$\theta_{wi} \in R$: attitude

Body Velocity (Input U_i)

$v_{xi}^b, v_{yi}^b \in R$: linear velocity

$\omega_i^b \in R$: angular velocity



Σ_w : world frame

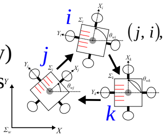
Rigid Body Motion

transformation from Σ_i to Σ_w

$$\begin{bmatrix} \dot{x}_{wi} \\ \dot{y}_{wi} \\ \theta_{wi} \end{bmatrix} = \begin{bmatrix} \cos\theta_{wi} & -\sin\theta_{wi} & 0 \\ \sin\theta_{wi} & \cos\theta_{wi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{xi}^b \\ v_{yi}^b \\ \omega_i^b \end{bmatrix} \qquad \left(\dot{X}_i = f_i(X_i, U_i)\right)$$

Robotic Network: Information Structure

Information Structure among Rigid Bodies [8], [9]



$(j,\ i),\ (k,\ j),\ (i,\ k) \in E$

Graph $G := (V,\ E)$

Rigid Body Set (n : number of rigid body)

$V := \{1,\ \cdots,\ n\}$: A set of unique identifiers

Neighbor Set

$E \subseteq V \times V$ $\big((j,\ i) \in E$: body i can get information of body $j\big)$

: A set of edges represent the neighboring relations

Neighbor Body Set

$N_i := \{j \in V \mid (j,\ i) \in E\}$: A set of bodies whose information is available to body i

Available Information

$[x_{ij}\quad y_{ij}\quad \theta_{ij}]^T,\ j \in N_i$: pose of body j relative to body i in Σ_i

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Robotic Network

Robotic Network $S(V,\ R,\ E)$ [8]

Unique Identifiers

$V = \{1,\ \cdots,\ n\}$

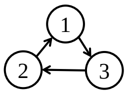
Information Structure

$E \subseteq V \times V$

A Set of Mobile Robots

$R = \{R_i\}_{i \in V} = \{(X_i,\ U_i,\ X_{i0},\ f_i)\}_{i \in V}$ X_{i0} : initial state of robot i

Assumption 1



strongly connected

1. $|\theta_{wi}(0)| < \frac{\pi}{2},\ \forall i \in V$
2. $G := (V, E)$
   - graph is fixed (a topology of a graph does not change)
   - strongly connected (there is a directly path connecting any two distinct nodes)

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Pose Synchronization

Goal: Pose Synchronization

Robotic Network S is said to achieve pose synchronization

if $\lim_{t\to\infty}|x_{wi} - x_{wj}| = 0,\ \lim_{t\to\infty}|y_{wi} - y_{wj}| = 0,\ \lim_{t\to\infty}|\theta_{wi} - \theta_{wj}| = 0 \quad \forall i,\ j$

Pose Synchronization Law[1]

$$\begin{bmatrix} v_{xi}^b \\ v_{yi}^b \end{bmatrix} = k_{pi} \sum_{j \in N_i} \begin{bmatrix} \cos\theta_{wi} & \sin\theta_{wi} \\ -\sin\theta_{wi} & \cos\theta_{wi} \end{bmatrix} \begin{bmatrix} x_{wj} - x_{wi} \\ y_{wj} - y_{wi} \end{bmatrix} = k_{pi} \sum_{j \in N_i} \begin{bmatrix} x_{ij} \\ y_{ij} \end{bmatrix}$$

$$\omega_i^b = k_{ai} \sum_{j \in N_i} \sin(\theta_{wj} - \theta_{wi}) = k_{ai} \sum_{j \in N_i} \sin\theta_{ij}$$

this input is calculated only by relative information

body velocity input

Theorem 1: Pose Synchronization [1]

Robotic Network S with the above control law satisfying Assumption 1 achieves pose synchronization

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Simulation

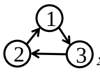
by theorem 1, getting this result

Pose

$$P_i(t) = \begin{bmatrix} x_{wi}(t) \\ y_{wi}(t) \\ \theta_{wi}(t) \end{bmatrix}$$

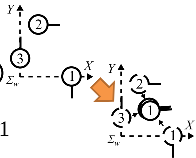
$i \in V$

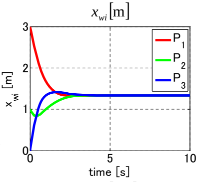
Graph



Gain

$k_{pi}, k_{ai} = 1$



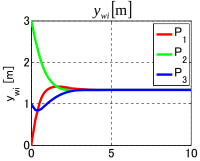


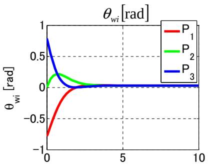
Initial State

$$P_1(0) = \begin{bmatrix} 3 & 0 & -\frac{\pi}{2} \end{bmatrix}^T$$

$$P_2(0) = [1\quad 3\quad 0]^T$$

$$P_3(0) = \begin{bmatrix} 0 & 1 & \frac{\pi}{2} \end{bmatrix}^T$$





simulation result

achieve synchronization

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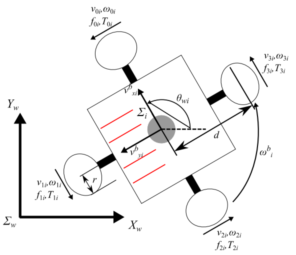
Variable

Variable definition for Omni-directional robot

motor number: $m \in \{0,\ 1,\ 2,\ 3\}$

variable	explanation
v_{mi}[m/s]	moving velocity of the wheel
ω_{mi}[rad/s]	rotation velocity of the wheel
f_{mi}[N]	moving force of the wheel
T_{mi}[N・m]	rotation torque of the wheel
d[m]	distance from center of robot to center of the wheel
r[m]	radius of the wheel



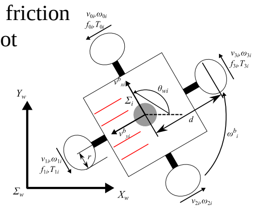
Objective

To generate $\begin{bmatrix} v^b_{xi} & v^b_{yi} & \omega^b_i \end{bmatrix}^T$ by input voltage $\begin{bmatrix} e_{0i} & e_{1i} & e_{2i} & e_{3i} \end{bmatrix}^T$

Robot Kinematics

Kinematics of Omni-directional robot

Assume
- wheels touch the ground at the single points and don't slip
- not consider gear and ground friction
- not consider dynamics of robot



Kinematics[2]

previous input

$$\begin{bmatrix} v_{0i}(t) \\ v_{1i}(t) \\ v_{2i}(t) \\ v_{3i}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & d \\ -1 & 0 & d \\ 0 & -1 & d \\ 1 & 0 & d \end{bmatrix} \begin{bmatrix} v^b_{xi}(t) \\ v^b_{yi}(t) \\ \omega^b_i(t) \end{bmatrix}$$

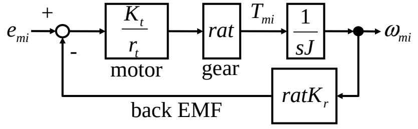
$$\omega_{mi} = \frac{v_{mi}}{r} \longrightarrow \begin{bmatrix} \omega_{0i}(t) \\ \omega_{1i}(t) \\ \omega_{2i}(t) \\ \omega_{3i}(t) \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 0 & 1 & d \\ -1 & 0 & d \\ 0 & -1 & d \\ 1 & 0 & d \end{bmatrix} \begin{bmatrix} v^b_{xi}(t) \\ v^b_{yi}(t) \\ \omega^b_i(t) \end{bmatrix}$$

input is voltage ⇨ Next, voltage equation

Actuator

Derive relation between voltage and wheel rotation velocity

Actuator Model[3] (no load torque and friction)



$e_{mi}[\mathrm{V}]$: input voltage

$J[\mathrm{kg \cdot m^2}]$: inertial of wheel

rat : gear

$K_t[\mathrm{Nm/A}]$: torque constant

$r_t[\Omega]$: inner resistor

$K_r[\mathrm{V \cdot s}]$: back EMF constant

$$J\dot{\omega}_{mi}(t) + \frac{K_r K_t}{r_t}\omega_{mi}(t) = \frac{K_t}{rat \cdot r_t} e_{mi}(t) [\mathrm{N \cdot m}]$$

$$\omega_{mi} = \frac{K_t}{rat \cdot (J r_t s + K_r K_t)} e_{mi} [\mathrm{rad/s}]$$

not considering dynamics delay $Jr_t \to 0$

e_{mi} response $\omega_{mi} = \dfrac{e_{mi}}{ratK_r}$ ⇨ $\omega_{mi}(t) = \dfrac{1}{ratK_r} e_{mi}(t)$

assumption

Robot Representation

Rigid Body *i*, $i \in V$

Kinematics

$$\begin{bmatrix} \omega_{0i}(t) \\ \omega_{1i}(t) \\ \omega_{2i}(t) \\ \omega_{3i}(t) \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 0 & 1 & d \\ -1 & 0 & d \\ 0 & -1 & d \\ 1 & 0 & d \end{bmatrix} \begin{bmatrix} v^b_{xi}(t) \\ v^b_{yi}(t) \\ \omega^b_i(t) \end{bmatrix},\ i \in V$$

Actuator

$$\omega_{mi}(t) = \frac{1}{ratK_r} e_{mi}(t),\ m := \{0,\ 1,\ 2,\ 3\},\ i \in V$$

⇩

Rigid Body *i*, $i \in V$

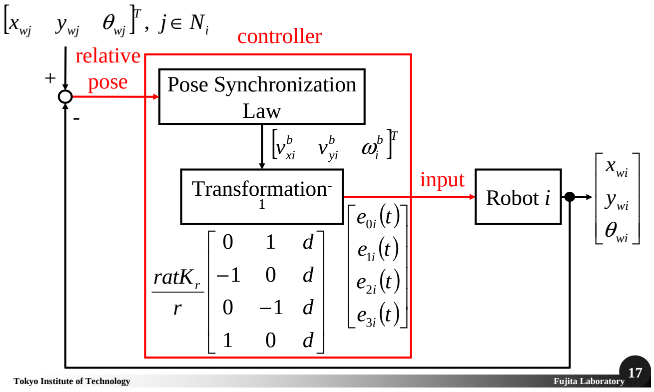
$$\begin{bmatrix} v^b_{xi}(t) \\ v^b_{yi}(t) \\ \omega^b_i(t) \end{bmatrix} = \frac{r}{ratK_r} \begin{bmatrix} 0 & -1/2 & 0 & 1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/4d & 1/4d & 1/4d & 1/4d \end{bmatrix} \begin{bmatrix} e_{0i}(t) \\ e_{1i}(t) \\ e_{2i}(t) \\ e_{3i}(t) \end{bmatrix},\ i \in V$$

using this representation,
we make robot generate
body velocity

$$e_i \in R^4 \longrightarrow \boxed{\text{Transformation}} \longrightarrow \begin{bmatrix} v^b_{xi}(t) \\ v^b_{yi}(t) \\ \omega^b_i(t) \end{bmatrix}$$

Overview

Input and Output



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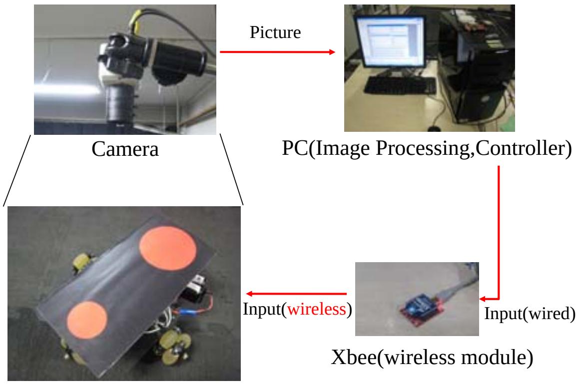
Experimental Environment



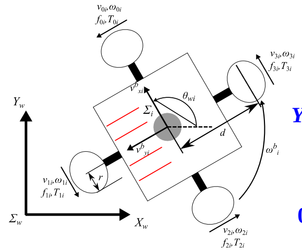
Camera

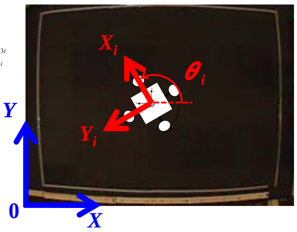
PC(Image Processing,Controller)

Xbee(wireless module)

Robot

Frame and Parameter





Overhead Camera

constant		parameter	
d	wheel distance from center	0.085	[m]
r	radius of wheel	0.024	[m]
K_r	reverse voltage constant	0.00247	[V･s/rad]
rat	torque gain of gear	64.8	[]

Simulation

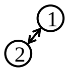
Cooperation with two robot

Pose

$$P_i(t)=\begin{bmatrix} x_{wi}(t) \\ y_{wi}(t) \\ \theta_{wi}(t) \end{bmatrix}$$

$i \in V$

Graph



Error

estimated
measurement error
Pose: 0.003 [m](1[pixel])
Attitude: 0.05[rad]

Gain $k_{pi} = 0.7$

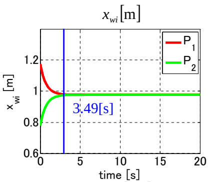
$k_{ai} = 2.1$

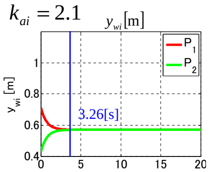
Initial State

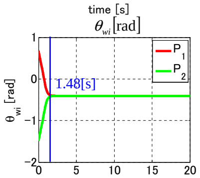
$$P_1(0)=\begin{bmatrix} 1.1756 \\ 0.7128 \\ 0.6786 \end{bmatrix}$$

$$P_2(0)=\begin{bmatrix} 0.7790 \\ 0.4279 \\ -1.4914 \end{bmatrix}$$

$$P_1(20), P_2(20) = \begin{bmatrix} 0.9773 \\ 0.5704 \\ -0.4064 \end{bmatrix}$$







simulation result

achieve synchronization

Experiment Data

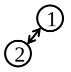
Cooperation with two robot

Pose

$$P_i(t)=\begin{bmatrix} x_{wi}(t) \\ y_{wi}(t) \\ \theta_{wi}(t) \end{bmatrix}$$

$i \in V$

Graph



Error

estimated
measurement error
Pose: 0.003 [m](1[pixel])
Attitude: 0.05[rad]

Gain $k_{pi} = 0.7$

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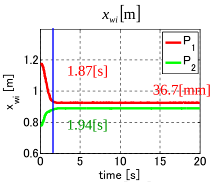
Initial State

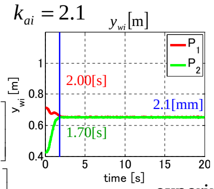
$$P_1(0)=\begin{bmatrix} 1.1756 \\ 0.7128 \\ 0.6786 \end{bmatrix}$$

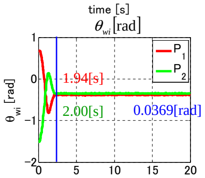
$$P_2(0)=\begin{bmatrix} 0.7790 \\ 0.4279 \\ -1.4914 \end{bmatrix}$$

$$P_1(20)=\begin{bmatrix} 0.9262 \\ 0.6527 \\ -0.3885 \end{bmatrix}$$

$$P_2(20)=\begin{bmatrix} 0.8895 \\ 0.6506 \\ -0.3516 \end{bmatrix}$$







experiment result

difference remains

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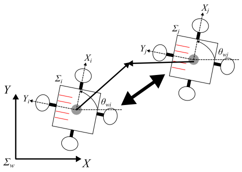
Summary

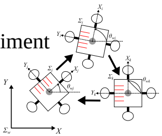
Achievement

- Derivation input of four-wheeled omni-directional robot to achieve pose synchronization
- Pose synchronization experiment with two robot
- Verification is not sufficient by a remarkable difference



Future Works

- Increasing the number of robots in experiment
- Detailed analysis of sampling data
- Difference reduction



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[2] H. P. Oliveira, A. J. Sousa, A. P. Moreira and P. J. Costa, “Precise Modeling of a Four Wheeled Omni-directional Robot,” *Proc. of the 8th Conference on Autonomous Robot Systems and Competitions*, pp. 57-62, 2008.

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