Optimal Power Flow of Power Networks with Battery via Distributed Predictive Control

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Outline

- Introduction
- Power Network
- Predictive Control
- Distributed Control
- Simulation

Introduction

Power Network

a network to send from various power supplies to each consumer

Optimal Power Flow(OPF)

optimize a certain objective over power network variables

■**constraints**

(bounds on voltages, line loading, etc.)

⇨ we must optimize under certain constraints

■**renewable energy**

(solar, or wind power, etc.)

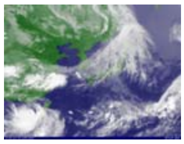
output fluctuates widely and randomly, but predictably

⇨ formulate an optimal power flow problem with battery

Introduction

OPF has been studied static optimization, but…

Introduction of battery
⇨ **Dynamics of battery in OPF**

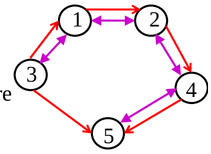


Desired Control

formulate a OPF of power network with storage across multiple time periods

⇩

distributed predictive control



- applicable { —predictive information in the future; — constraints
- only by local information

objective of this work

Optimize power consumption by applying the distributed predictive control control to power network

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Power Network - example

Node

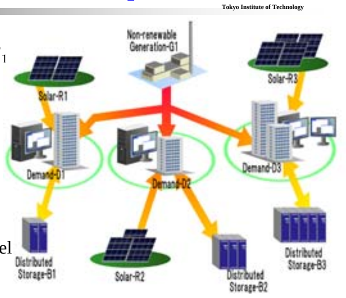
•Demand D_1, D_2, D_3 •Generation G_1
•Battery B_1, B_2, B_3 •Calculator
•Renewable energy R_1, R_2, R_3

Link

$\mathcal{E}_{DG} = \{(G_1, D_1), (G_1, D_2), (G_1, D_3)\}$

$\mathcal{E}_{DR} = \{(R_1, D_1), (R_2, D_2), (R_3, D_3)\}$

$\mathcal{E}_{DB} = \{(B_1, D_1), (B_2, D_2), (B_3, D_3)\}$



Utility

•move battery level closer to target level
•curb costs
•equate supply and demand

Constraints

•Cannot charge battery more than the supply by the others
•Supply is always nonnegative (without battery)
•about battery level
•Upper bound M in the sum of the supply to some demands

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Network structure

be composed of several different links and nodes

■ Node $\mathcal{V} = \mathcal{D} \cup \mathcal{G} \cup \mathcal{R} \cup \mathcal{B} \cup \mathcal{C}$

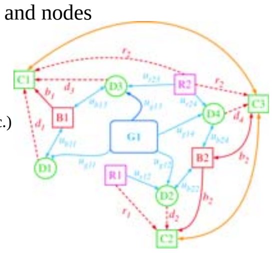
Demand $\mathcal{D}$ — customer

Generation $\mathcal{G}$ — nonrenewable generation (Gas, Coal, Nuclear, Hydro, etc.)

Renewable energy $\mathcal{R}$ — renewable, variable (Solar, Wind, tidal force, etc.)

Battery $\mathcal{B}$ — distributed storage

Calculator $\mathcal{C}$ — scheduling flows of electricity



■ Link $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$

energy flow $\mathcal{E}_E$
- to demand $\mathcal{E}_D$
- to battery $\mathcal{E}_B$

information flow $\mathcal{E}_I$
- battery info $\mathcal{E}_S$
- Predictive info $\mathcal{E}_P$
- communication info $\mathcal{E}_C$

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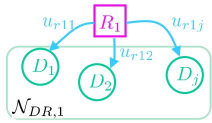
■ Signal $k = 0, 1, 2, 3, \cdots$

•Renewable energy $u_{rij}, D_j \in \mathcal{N}_{DR,i}$

$\mathcal{N}_{DR,i} = \{D_j \in \mathcal{D} | (R_i, D_j) \in \mathcal{E}_{DR}\}$

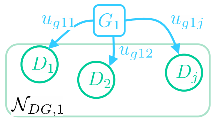
: D_j set that can be supplied by Renewable energy



•Generator $u_{gij}, D_j \in \mathcal{N}_{DG,i}$

$\mathcal{N}_{DG,i} = \{D_j \in \mathcal{D} | (G_i, D_j) \in \mathcal{E}_{GR}\}$

: D_j set that can be supplied by Generator



•Battery $u_{bij}, D_j \in \mathcal{N}_{DB,i}$

$(B_i, D_j) \in \mathcal{E}_{DB} \Leftrightarrow (D_j, B_i) \in \mathcal{E}_B$

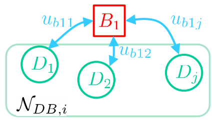
Demand that can be supplied by battery can charge the battery oppositely

$\mathcal{N}_{DB,i} = \{D_j \in \mathcal{D} | (B_i, D_j) \in \mathcal{E}_{DB}\}$

: D_j set that can be supplied by battery

$\mathcal{N}_{BD,j} = \{B_i \in \mathcal{B} | (D_j, B_i) \in \mathcal{E}_B\}$

: B_i set that can be supplied by Generator



$u_{bij} < 0$ means battery charge

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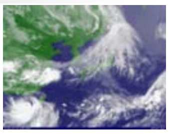
Problem of this system



•Instability of renewable energy

Mainly controlled by the weather

⇨ It is possible to predict at some level

•Dynamics of battery

$$\Sigma_i : b_i(k+1) = b_i(k) - \sum_{j \in \mathcal{N}_{DB,i}} u_{bij}, i \in \mathcal{B}, \quad \cdots (1)$$

Past information

$b_i(k)$: remaining battery level

⇨ There is relativity around the time of a time period

We must use dynamic optimization across multiple time periods

⇩

Predictive control

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predictive control

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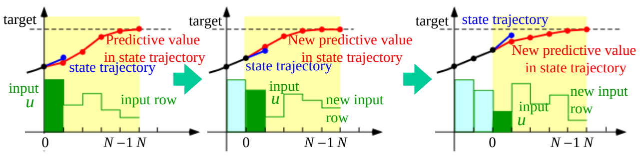
Utility function $\min_{U^N} J = (U^N)^T HU^N + h(b_0) U^N$

Constraints $s.t. \ AU^N \leq B(b_0)$

Predictive control

1) solve optimal control of finite horizon by the state x_0 at the time k
2) input the first step of the obtained input row
3) in the next time step, return 1) and repeat



predictive control

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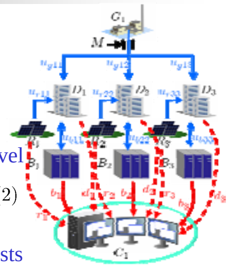
Utility function

utility requested on power network

$$\min_{u(k),\ k=0,\cdots,N} J = q_d J_d + q_b J_b + q_g J_g$$



■Bring remaining battery level close to target level

$$J_b = \sum_{k=0}^{N} \sum_{i \in \mathcal{B}} w_{B,i} (b_i(k) - b_{i,ref})^2 \quad \cdots (2)$$

battery level — target level

■Curb power feeding from generation for the costs

$$J_g = \sum_{k=0}^{N} \sum_{i \in \mathcal{G}} \sum_{j \in \mathcal{N}_{DG,i}} w_{G,ij} u_{gij}^2(k) \quad \cdots (3)$$

■Bring supply close to demand

$$J_d = \sum_{k=0}^{N} \sum_{j \in \mathcal{D}} w_{D,j} \left(\sum_{i \in \mathcal{N}_{GD,j}} u_{gij}(k) + \sum_{i \in \mathcal{N}_{RD,j}} u_{rij}(k) + \sum_{i \in \mathcal{N}_{BD,j}} u_{bij}(k) - d_j(k) \right)^2 \cdots (4)$$

Generator — Renewable energy — Battery — Demand

predictive control

Constraints

■Supply by generator and renewable energy is always nonnegative

$$u_{gij}(k) \geq 0,\ u_{rij}(k) \geq 0 \qquad \cdots(5)$$

■Cannot charge battery more than the supply by the others

$$\sum_{i\in\mathcal{N}_{BD,j}} u_{bij}(k) \geq -\sum_{i\in\mathcal{N}_{GD,j}} u_{gij}(k) - \sum_{i\in\mathcal{N}_{RD,j}} u_{rij}(k) \qquad \cdots(6)$$

■Renewable energy is distributed by demands in $\mathcal{N}_{DR,i}$

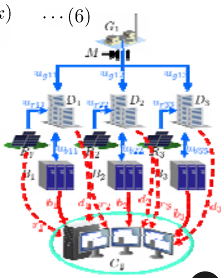
$$r_i(k) = \sum_{j\in\mathcal{N}_{DR,i}} u_{rij}(k) \qquad \cdots(7)$$

■Upper bound in the sum of the supply to some demands

$$\sum_{j\in\mathcal{D}_{sum,l}} u_{gij} \leq M,\ M \geq 0 \qquad \cdots(8)$$

■Maximum battery level

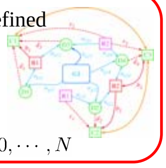
$$b_i(k) \in [0, B_{i,max}] \qquad \cdots(9)$$



predictive control

Predictive control problem

All the utility functions and the constraints are defined by convex function, so the combination is free



$$\min_{u(k),\ k=0,\cdots,N} J = q_d J_d + q_b J_b + q_g J_g$$

subject to (1), (5) − (9),

$$b(0) = b_0, r_i(k) = r_{i,k}, d_j(k) = d_{j,k}, i \in \mathcal{R}, j \in \mathcal{D}, k = 0, \cdots, N$$

Quadratic programming problem

Utility function $\min_{U^N} J = (U^N)^T HU^N + h(b_0)U^N$

Constraints $AU^N \leq B(b_0)$

optimal control of finite horizon

method : Primal-Dual Algorithm

predictive control

Primal-Dual Algorithm

Primal : $\min_{U^N} J = (U^N)^T HU^N + h(b_0)U^N \qquad AU^N \leq B(b_0)$

Lagrange-Dual $L(U^N, \lambda) = (U^N)^T HU^N + h(b_0)U^N - \lambda(AU^N - B(b_0))$

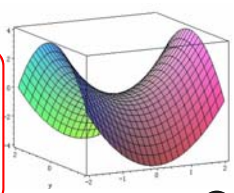
Dual : $\max_{\lambda>0} D(\lambda) = \max_{\lambda>0}\{\min_{U^N} L(U^N, \lambda)\}$

Primal : convex function

Dual : concave function

Optimized solution is the saddle point by both variables

$$\dot{U}^N = \left(\frac{\partial D}{\partial U^N}\right)_{U^N,\lambda} \qquad \text{Update alternately} \qquad \dot{\lambda} = -\left(\frac{\partial D}{\partial \lambda}\right)_{U^N,\lambda}$$



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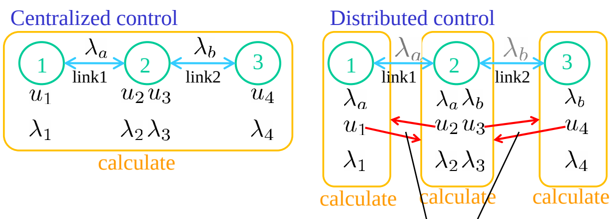
Distributed Control

Power network tends to become large-scale

⇒ The amount of calculation becomes huge

Distributed Control

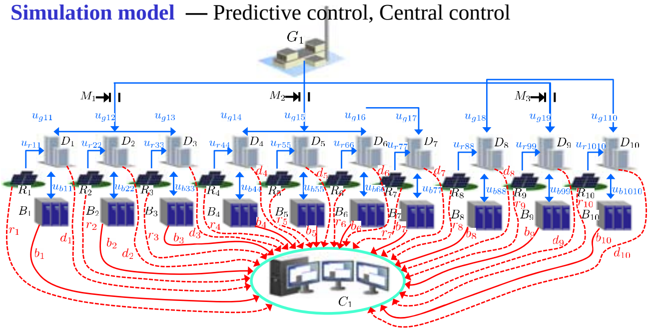
Each grid controls only by local information



communicate relevant information to decide λ_a, λ_b

Distributed Control

Simulation model — Predictive control, Central control

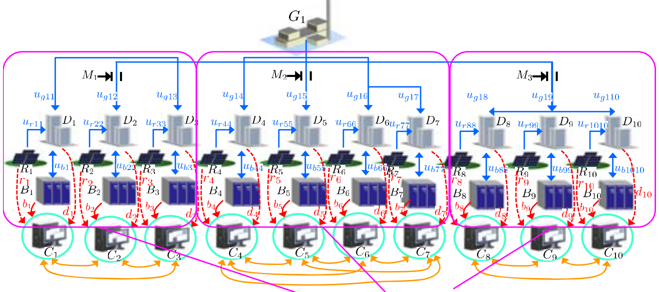


Need information on all variables in power network

⇒ When the network becomes large-scale, amount of calculation becomes huge.

Distributed Control

Block diagram — Distributed predictive control

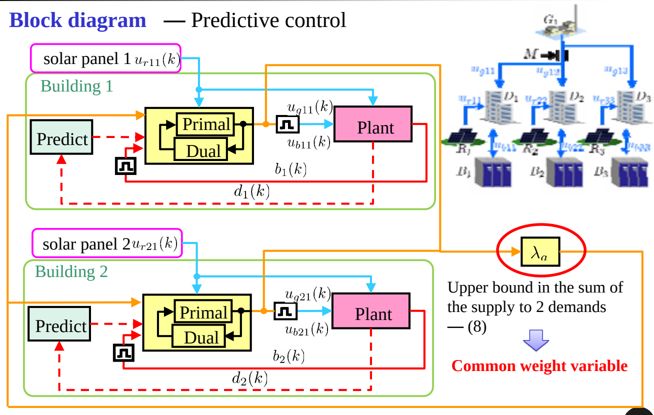


each calculator needs only local information

even if the network becomes large-scale,
amount of calculation of each calculator don't becomes huge.

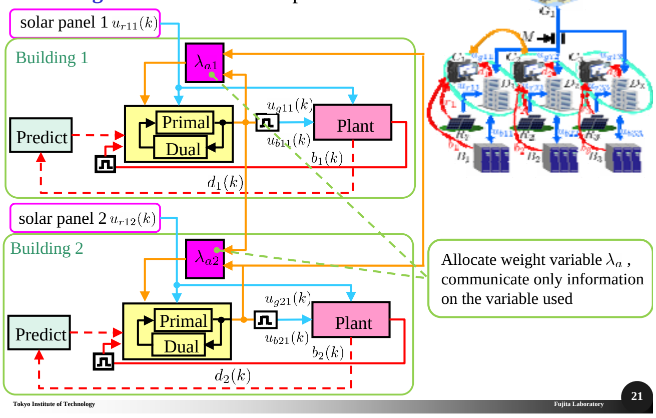
Distributed Control - Simulation block

Block diagram — Predictive control



Distributed Control - Simulation block

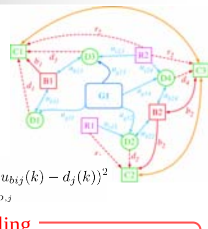
Block diagram — Distributed predictive control



Distributed Control

In power network, for a certain demand j

Utility function



(2) → $J_{b,i} = \sum_{k=0}^{N} \sum_{i\in\mathcal{N}_{BD,j}} w_{B,i}(b_i(k) - b_{i,ref})^2$

(3) → $J_{g,j} = \sum_{k=0}^{N} \sum_{i\in\mathcal{N}_{GD,j}} w_{G,ij} u_{gij}^2$

(4) → $J_d = \sum_{k=0}^{N} \sum_{j\in\mathcal{D}} w_{D,j}\left(\sum_{i\in\mathcal{N}_{GD,j}} u_{gij}(k) + \sum_{i\in\mathcal{N}_{RD,j}} u_{rij}(k) + \sum_{i\in\mathcal{N}_{RD,j}} u_{bij}(k) - d_j(k)\right)^2$

Constraints

coupling: communicate these information to decide common variable

(5) → $u_{gij}(k) \geq 0,\ u_{rij}(k) \geq 0$

(6) → $\sum_{i\in\mathcal{N}_{BD,j}} u_{bij}(k) \geq -\sum_{i\in\mathcal{N}_{GD,j}} u_{gij}(k) - \sum_{i\in\mathcal{N}_{RD,j}} u_{rij}(k)$

(7) → $r_i(k) = \sum_{j\in\mathcal{N}_{DR,i}} u_{rij},\ i\in\mathcal{N}_{RD,j}$

(8) → $\sum_{j\in\mathcal{D}_{s,l}} u_{gij} + x_l = M_l,\ x_l \geq 0$

(9) → $b_i(k) \in [0, B_{i,max}],\ i\in\mathcal{N}_{BD,j}$

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Simulation model

Constant

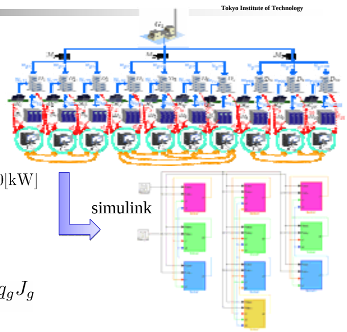
$\tau = 6[\text{min}]$

$q_b = 0.0005,\ q_d = 10,\ q_g = 0.1$

$B_{1,max} = 50[\text{kW}], b_{1,ref} = B_{1,max}/2$

$b_1(0) = 10[\text{kW}]$

$M_1 = 6.0[\text{kW}], M_2 = 9.0[\text{kW}], M_3 = 10.0[\text{kW}]$



Utility function

use (2), (3), (4)

$$\min_{u(k),\ k=0,\cdots,N} J = q_d J_d + q_b J_b + q_g J_g$$

Constraints

$u_{gij}(k) \geq 0,\ u_{rij}(k) \geq 0$

$u_{bij}(k) \geq u_{gij}(k) - u_{rij}(k)$

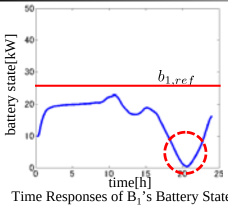
$b_i(k) \in [0, B_{i,max}]$

$\sum_{j\in\mathcal{D}_{s,l}} u_{gij} + x_l = M_l,\ x_l \geq 0$

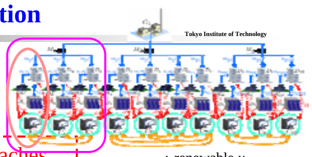
$r_i(k) = \sum_{j\in\mathcal{N}_{DR,i}} u_{rij},\ i\in\mathcal{N}_{RD,j}$

$j\in\mathcal{D}, k = 0,\cdots,N$

Simulation

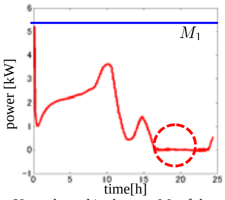


Time Responses of B_1's Battery State

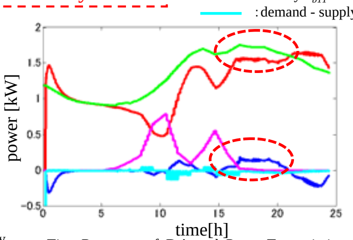


When it reaches the upper bound, the battery is used

: renewable u_{r11}
: demand d_1
: generation u_{g11}
: battery u_{b11}
: demand - supply



Upper bound in the sum M_1 of the supply



Time Responses of D_1's each Power Transmission

Conclusion

- Introduction of generalization power network
- Introduction of distributed predictive control
- Application of distributed predictive control for a dynamic OPF
- validation of the utility of distributed predictive control by simulation