Visual Feedback Attitude Synchronization: Tracking Performance Analysis

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Introduction

Cooperative Control of Robotic Network
A distributed control strategy using only local information so that the robotic network achieves specified tasks
Cooperative Control Problems for Mobile Sensor Network
formulated as
Pose Coordination Problems
Attitude Synchronization
To lead all agents' attitudes to a common one by utilizing distributed control strategies
Available Information
Only Visual Information: each agent has vision
Objective of Our Work
To present vision-based attitude synchronization control laws for robotic networks in 3-dimensional space, prove the convergence mathematically and analyze the performance.

Visual Robotic Network: Rigid Body Motion

Kinematics of Rigid Bodies
\[ \mathbf{p}_{01} \in \mathbb{SE}(3) \]
\[ \mathbf{q}_{01} = [\mathbf{R}_{01} ; \mathbf{p}_{01}] \in \mathbb{R}^{3 \times 4} \]
Exponential Coordinate for Rotation
\[ \begin{bmatrix} \mathbf{e}_{01} \end{bmatrix} = [\mathbf{I} ; \mathbf{P}_{01}] \mathbf{q}_{01} \]
Body Velocity
\[ \mathbf{v}_{0} = \frac{d}{dt} \mathbf{p}_{0} \]
\[ \mathbf{w}_{0} = \frac{d}{dt} \mathbf{q}_{0} \]
Rigid Body Motion
\[ \mathbf{q}_{01} = \mathbf{q}_{0} \mathbf{q}_{01} \]

Outline

- Introduction
- Visual Robotic Network
  - Rigid Body Motion
  - Visibility Structure
  - Measured Output
- Visual Feedback Attitude Synchronization
- Tracking Performance
- Simulation
- Conclusion and Future Works

Visual Robotic Network: Visibility Structure and Measured Output

Visibility Structure among Rigid Bodies
\[ \mathcal{N}_i := \{ j \in \mathcal{V} \mid (j,i) \in \mathcal{E} \} \]
Visible Bodies
\[ \mathcal{V}_j := \{ i \in \mathcal{V} \mid (i,j) \in \mathcal{E} \} \]
Measured Output
\[ \mathbf{f}_j = \{ f_{j1}, \ldots, f_{jn} \} \in \mathbb{R}^{n} \]
Vision Model (Perspective Projection)
\[ f_{ji} = \frac{f_{ji}}{z_{ji}} \]
\[ \mathbf{A}_{ji} = \begin{bmatrix} z_{ji} & 0 & -f_{ji} \end{bmatrix} \]
\[ \begin{bmatrix} x_{ji} \\ y_{ji} \end{bmatrix} = \begin{bmatrix} x_{ji} \\ y_{ji} \\ z_{ji} \end{bmatrix} \]

Cooperative Control
**Visual Robotic Network**

\[ \Sigma \]

- **Visibility Structure**
  \[ N_i := \{ j \in V \mid (j, i) \in \mathcal{E} \}, \ i \in V \]

- **Rigid Bodies**
  \[ \dot{q}_{vi} = g_{vi} \dot{v}_{vi}, \ \ i \in V \]

- **Measured Output**
  \[ f_i = f_{vi} \in \mathcal{N}_i, \ i \in V \]

**Assumption 1 (Leader-following Type Visibility Structure)**
- There exists a leader which has no visible body \( N_1 = \emptyset \)
- The other bodies have a fixed visible body \( (|N_i| = 1, \ i \neq 1) \)
- There exists a visibility path from each body to the leader

\[ G := (V, \mathcal{E}) : \text{Graph} \]

**Definition: Visual Feedback Attitude Synchronization**

A visual robotic network \( \Sigma \) is said to achieve visual feedback attitude synchronization, if

\[ \dot{q}_{vi} = g_{vi} \dot{v}_{vi}, \ \ i \in V \]

\[ \lim_{t \to \infty} \phi(e^{\dot{q}_{vi}+e^{\dot{v}_{vi}}}) = 0, \ \forall i, j \in V \]

**Energy Function of Rotation**

\[ \phi(e^{\dot{q}_{vi}+e^{\dot{v}_{vi}}}) := \frac{1}{2} tr(I_3 - e^{\dot{v}_{vi}}) \geq 0 \]

**Synchronization**

- All rigid bodies' linear velocity is the same
- All attitudes asymptotically converge to a common value

**Visual Feedback Attitude Synchronization Law**

\[ \dot{q}_{ij} = \left( \dot{q}_{ij} - e^{\dot{q}_{ij}} \right) \]

**Visual Observer to estimate relative pose of neighbors**

\[ \dot{q}_{ij} = \dot{q}_{ij} - e^{\dot{q}_{ij}} \]

**Input (7): calculated only by measured output \( f_{ij} \)**

**Conclusion and Future Works**

- Tracking Performance
- Simulation

**Visual Feedback Attitude Synchronization Law**

\[ \dot{q}_{ij} = \left( \dot{q}_{ij} - e^{\dot{q}_{ij}} \right) \]

**Estimation Error**

\[ g_{ij} = \dot{q}_{ij} - \dot{q}_{ij} \]

**Conclusion and Future Works**

- Tracking Performance
- Simulation

**Sketch of Proof**

- **Lyapunov Function Candidate**
  \[ q_i = [1, 2, \ldots] \]

- **Lyapunov Argument (Asymptotic Stability)**
  \[ \dot{V} := \sum_{i \in V} \sum_{j \in N_i} \left( \phi(e^{\dot{q}_{ij}+e^{\dot{v}_{ij}}}) + \frac{1}{2} |q_{ij}|^2 \right) \geq 0 \]

- **Gain Condition**
  \[ k_{ij} < \frac{2k_{ij} + k_{ij}^2}{k_{ij} + 2k_{ij}^2} \]

- **Tracking Performance based on Input-to-state Stability**
  - Tracking Performance based on \( L_1 \)-gain Performance

**Conclusion and Future Works**

- Tracking Performance
- Simulation
Tracking Performance Analysis

Theorem 1: \( \omega^*_{\text{eq}} = 0 \)

- For a rotating leader \( \omega^*_{\text{eq}} \neq 0 \), is there sufficient tracking performance?

Evaluate attitude errors regarding the leader’s angular velocity as an external disturbance

\( \alpha_{\omega} \): All rigid bodies’ control and estimation errors

Qualitative Evaluation: Input-to-state Stability

For any bounded input, the state will be bounded

\[ |x_e(t)| \leq \beta(|x_0|, t) + \alpha(|\omega^*_{\text{eq}}|) \]

Quantitative Evaluation: \( \mathcal{L}_2 \)-gain Performance Analysis

For any energy bounded input, the output will be bounded

\[ ||x_e||_2 \leq \gamma ||\omega^*_{\text{eq}}||_2 + \delta \]

\( \gamma, \delta \geq 0 \)


Tracking Performance Analysis based on ISS

Theorem 2: Tracking Performance based on ISS

Suppose the leader rotates \( \omega^*_{\text{eq}} \neq 0 \) and Assumption 1 is satisfied.

If \( |\omega_0(0)| < \frac{\pi}{2}, |\gamma_0(0)| < \frac{\pi}{2}, j \in \mathcal{N}_i, i \in \mathcal{V} \) and for any positive scalars \( \epsilon_i, \epsilon_i' > \epsilon_i \), inequality (12) is satisfied, then there exists class-\( \mathcal{K} \)-\( \mathcal{L} \) function \( \beta(\cdot, \cdot) \) and class-\( \mathcal{K} \)-\( \mathcal{L} \) function \( \alpha(\cdot, \cdot) \) satisfying inequality (13).

\[ \begin{align*}
  k_{ii} &> \epsilon_i' \\
  k_{ii} + k_{i(i-1)} &> \epsilon_i
\end{align*} \]

Control Error Vector:

\[ e_{\text{ctrl}} = \left[ e_{\text{ctrl}}, e_{\text{est}} \right] \]

Estimation Error Vector:

\[ e_{\text{est}} = \left[ e_{\text{est}}, e_{\text{est}} \right] \]

Thus, from [16, pp. 95, Theorem 1], we get the following from ISS definition

\[ ||x_e(t)||_2 \leq \beta(|x_e(0)|, t) + \alpha(|\omega^*_{\text{eq}}|) \]

\( \alpha(\cdot) \in \mathcal{K}, \beta(\cdot, \cdot) \in \mathcal{K} \mathcal{L} \)

Step 2: Chain Type Visibility Structure (\( n \) Rigid Bodies)

Using the ISS property:

1. Cascade connection of ISS systems is also ISS

\[ ||x_e(t)||_2 \leq \beta_e(||x_e(0)||, t) + \alpha_e(||\omega^*_{\text{eq}}||) \]

\( \alpha_e(\cdot) \in \mathcal{K}, \beta_e(\cdot, \cdot) \in \mathcal{K} \mathcal{L} \)

Step 3: Multiple Chain Type Visibility Structures (Assumption 1)

For each \( \Sigma_{\text{eq}i}, i \in \mathcal{V} \), we get the following

\[ ||x_e(t)||_2 \leq \beta(\cdot, \cdot) \]

\( \alpha(\cdot) \in \mathcal{K}, \beta(\cdot, \cdot) \in \mathcal{K} \mathcal{L} \)

For \( ||x_e(0)||_2 \leq \beta(\cdot, \cdot) \), the followings is satisfied

\[ ||x_e(t)||_2 \leq ||x_e(0)||_2 + \alpha(|\omega^*_{\text{eq}}|) \]

Thus, summation of the above inequality for \( i \in \mathcal{V} \) yields

\[ ||x_e(t)||_2 \leq \sum_{i \in \mathcal{V}} \beta(\cdot, \cdot) + \sum_{i \in \mathcal{V}} \alpha(|\omega^*_{\text{eq}}|) \]

\( \beta = \sum_{i \in \mathcal{V}} \beta(\cdot, \cdot), \alpha = \sum_{i \in \mathcal{V}} \alpha(\cdot) \)
**Tracking Performance Analysis based on \( C_2 \)-gain Performance**

Assumption 1: Multiple Chain Type Visibility Structures

For each \( \Sigma_{\text{col}} \), \( i \in V_i \)

Real Symmetric Matrix: \( \Sigma_{\text{col}} \in \mathbb{R}^{n_i, n_i-1} \times \mathbb{R}^{n_i, n_i-1} \)

(refer to the resume for the definition)

number of bodies of \( \Sigma_{\text{col}} \)

\( V_i = [4, 5, 6] \)

Identifier: \([1, 2, \ldots, n_i]\)

Theorem 3: Tracking Performance based on \( C_2 \)-gain Performance

Suppose the leader rotates \( (\omega^o_{\text{col}})^T \neq 0 \) and Assumption 1 is satisfied.

Then, for any \( k_{\Sigma_i}, k_{\Sigma_i-1}, i \in [2, \ldots, n_i] \) and positive scalar \( \gamma_i, i \in V_i \) satisfying LMI \( P_i > 0 \), \( i \in V_i \), there exists a nonnegative scalar \( \delta \) satisfying the following inequality.

\[
\|x_i\|_{L_2} \leq \gamma \|\omega_{\text{col}}^o\|_{L_2} + \delta, \quad \gamma := \sqrt{\sum_{i \in V_i} \gamma_i} \tag{14}
\]

\( \gamma \) evaluates cont. and est. errors

Barometer of Tracking Performance

**Proof of Theorem 3**

For each \( \Sigma_{\text{col}}, i \in V_i \)

Energy Function: \( U_i := \sum_{i \in \Sigma_{\text{col}}} \left( \phi(e^{\Sigma_i}) + \frac{1}{2} \|e^{\Sigma_i-1}\|^2 + \phi(e^{\Sigma_i-1}) \right) \)

Time Derivative of \( U_i \)

\[
\dot{U}_i = -\frac{\gamma_i}{2} \|\omega_{\text{col}}^o\|^2 - \frac{\gamma_i}{2} \|x_{\text{col}}^o\|^2 - \frac{\gamma_i}{2} \|x_{\text{col}}^o\|^2
\]

Integrating from 0 to \( T \) yields

\[
U_i(T) - U_i(0) \leq \frac{\gamma_i}{2} \int_0^T \|\omega_{\text{col}}^o\|^2 dt - \frac{\gamma_i}{2} \int_0^T \|x_{\text{col}}^o\|^2 dt
\]

**Analysis of Gain Condition**

3 Rigid Bodies

\( P_3 > 0 \)

\[
P_3 = \begin{bmatrix}
-k_{31} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Affine on \( k_{(i-1)}, k_{(i-1)} \)

Schur Complements

\( x \in \mathbb{R}^n, \dot{Q}(x) = Q(x)^T, R(x) = R(x)^T \) and \( S(x) \) depend affinely on \( x \).

\[
\begin{cases}
Q(x) & S(x) \\
R(x) & R(x) \end{cases} > 0 \quad \iff \begin{cases}
Q(x) > 0 & S(x) > 0 \\
R(x) > 0 & R(x) \end{cases}
\]

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Simulation

Visibility Structure

Initial Condition

\[
\begin{align*}
 p_{c1} &= [5 - 5 5]^T \\
 p_{c2} &= [5 0 5]^T \\
 p_{c3} &= [0 0 5]^T \\
 p_{c4} &= [5 - 5 5]^T \\
 p_{c5} &= [-5 0 0 ]^T
\end{align*}
\]

\[
\begin{align*}
 \theta_{c1}(0) &= [0 \pi/4 \pi]^T \\
 \theta_{c2}(0) &= [0 0 0]^T \\
 \theta_{c3}(0) &= [0 0 0]^T \\
 \theta_{c4}(0) &= [0 -\pi/4 \pi]^T \\
 \theta_{c5}(0) &= [0 0 0]^T
\end{align*}
\]

Tracking Performance

\[
\|\dot{x}_e\| \leq \gamma \|\omega_{\theta}^d\| \|z_2\| + \delta
\]

(i) \( \gamma = 2 \)

\[
\begin{align*}
 k_{c1} &= 2.73, k_{c2} = 4.45, k_{c3} = 5.69, k_{c4} = 8.38, k_{c5} = 3.37, k_{c32} = 3.94, k_{c52} = 6.65, k_{c32} = 7.72
\end{align*}
\]

(ii) \( \gamma = 0.49 \) (better performance)

\[
\begin{align*}
 k_{c1} &= 11.28, k_{c2} = 19.93, k_{c3} = 18.99, k_{c4} = 19.99, k_{c5} = 19.98, k_{c32} = 19.92, k_{c52} = 19.99
\end{align*}
\]

Simulation Results

Rotation Angle Errors

\[
\begin{align*}
 |\dot{\theta}_1| & \leq 0.01 \text{ (i)} \\
 |\dot{\theta}_2| & \leq 0.01 \text{ (ii)}
\end{align*}
\]

Better Tracking !

2-norm of \( x_e \)

\[
\begin{align*}
 |x_e| & \leq 0.01 \text{ (i)} \\
 |x_e| & \leq 0.01 \text{ (ii)}
\end{align*}
\]

Synchronization !

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Conclusion and Future Work

Conclusions

- Proposal of Visual Robotic Network
- Proposal of vision-based attitude synchronization law
- Proof of visual feedback attitude synchronization
- Analysis of tracking performance
- Simulation and experiment for verification

Future Works

- Bidirectional visibility (too difficult: local report)
- Tracking performance analysis of visual feedback pose synchronization
- Panoramic camera model
- Rigid bodies with actuators (dynamics)