# Market Models in Power Systems

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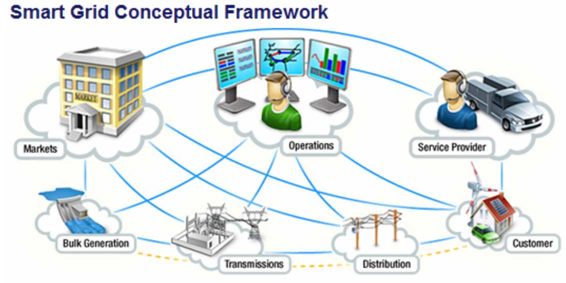
## Introduction

**Smart Grid Conceptual Framework**




http://smartgrid.ieee.org/nist-smartgrid-framework

The Smart Grid : a next-generation electrical power system

– use of communication and information technology in generation, delivery and consumption of electrical energy



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## Introduction

- **Generation**
  conventional (ex. thermal)
  ↓
  renewable (ex. solar, wind)
  – available worldwide
  – sustainability (fuel savings)
- **Efficiency**
  – minimizing energy loss (optimal power flow)
- **Demand response**
  – Two-way communication between customers and utilities
  – help reduce peak-energy demand
  – good for your wallet




smart meter



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## Review - Integration of Renewable Energy

K. Mani Candy, Steven H. Low, Ufuk Topcu and Huan Xu
"A Simple Optimal Power Flow Model with Energy Storage"
In *IEEE Conference on Decision and Control*, 2010.

$$I[A] = Y[S]\cdot V[V]$$
$$Z[\Omega] = Y^{-1}$$
$$q = VI^*[W]$$

Problem : Renewable energy is intermittent
⇨ with storage

Power flow

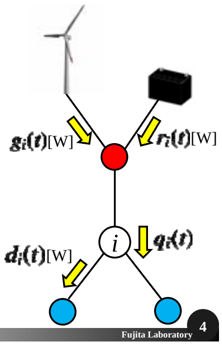
$$q_i(t) = \sum_{j\in N} V_i V_j Y_{ij}(\theta_i(t) - \theta_j(t))\ [W]$$

$$V_i V_j Y_{ij}(\theta_i(t) - \theta_j(t)) \le \bar{q}_{ij}(t)\ [W]$$

Battery level

$$b_i(t) = b_i(t-1) - r_i(t)\ [W]$$

$$0 \le b_i(t) \le B_i\ [W]$$




Optimal power flow problem ⇨ $g^*(t)$ , $b^*(t)$



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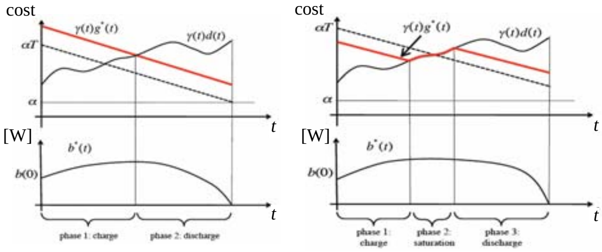
## Review - Integration of Renewable Energy

Single generator and single load

Assumption: Demand $d(t)$ doesn't decrease too rapidly

⇨ $g^*(t)$ cross $d(t)$ at once, from above
$b^*(t)$ : charge → discharge




Network case is under current study…



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## Demand response

Total generation capacity is sized to corresponding to peak demand.

➡ lowering peak demand reduces overall plant and capital cost requirement

smart meter

**Demand response**

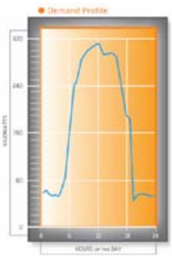
– manage customer consumption of electricity in response to supply condition

Emergency demand response (reliability-based)

ex. demand > supply ⇨ load shedding (avoid blackouts)

Economic demand response (price-based)

ex. Reduction of consumption during high price events



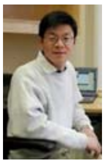
DoE, Smart Grid Intro, 2008

## Market Models for Demand Response



Lijun Chen, Na Li, Steven H. Low and John C. Doyle
"On Two Market Models for Demand Response in Power Networks"
In *IEEE SmartGridComm*, 2010.

Lijun Chen



Steven H. Low

John C. Doyle

Demand response
- matching the supply (load shedding)
- shaping the demand

Auction market?



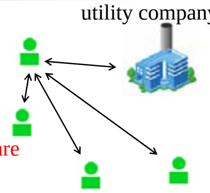
## Overview



➢Market model

Non-cooperative game
- customer, utility company : selfish
- with unique equilibrium
- unique equilibrium satisfies social welfare

∴ Each player maximize its payoff
⇨ maximize social welfare




➢Unique equilibrium conditions

(1) optimality condition 福島雅夫, '非線形最適化の基礎', 朝倉書店, 2009

$f : \mathbf{R}^n \to \mathbf{R}$, continuously differentiable  $X$ : convex set

$x^*$ : optimal solution ⇨ $\nabla f(x^*)^T (x - x^*) \geq 0,\ \forall x \in X$

$f$ : convex ⇨ $x^*$ : global optimal solution, $\nabla f(x^*) = 0$



## Overview



(2) Karush-Kuhn-Tucker condition

$\max_x f(x)$ s.t. $g_i(x) \leq 0$, $h_j(x) = 0$
(concave / convex / linear)

⇨ Lagrange multiplier: $\lambda_i \geq 0$, $\mu_j$

$$\frac{\partial f}{\partial x_k}(x^*) - \sum_i \lambda_i \frac{\partial g_i}{\partial x_k}(x^*) + \sum_j \mu_j \frac{\partial h_j}{\partial x_k}(x^*) = 0, \quad \forall k$$

$$\lambda_i g_i(x^*) = 0, \quad \forall i$$

$x^*$ : global maximum

(3) Nash equilibrium $u_i(b_i^*, b_{-i}^*) \geq u_i(b_i, b_{-i}^*)$, $\forall b_i$

Unique equilibrium conditions
- player's payoff
- social welfare

⇨ (Convex) optimization problem



## Matching The Supply



- supply < demand
- very costly to increase or decrease the supply power ⇨ inelastic
- load $q_i$ : customer $i \in N$ is willing to shed
- one utility company

System model

$$\sum_i q_i = d$$ ← supply deficit $d > 0$

load shedding allocation ⇨ Supply function bidding   price : $p$

Customer's 'supply' function : $q_i(b_i, p) = b_i p$, $b_i \geq 0$

$$\sum_i q_i(b_i, p) = \sum_i b_i p = d \ \Rightarrow\ p(b) = \frac{d}{\sum_i b_i}$$

supply function profile : $b = (b_1, b_2, ..., b_{|N|})$



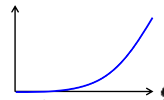
## Matching the Supply - Competitive Market



customers : given price $p$

Each customer $i$:

$$\max_{b_i \geq 0} p q_i(b_i, p) - C_i(q_i(b_i, p))$$
(revenue − cost)




Social welfare:

$$\max_{b_i \geq 0} \sum_i p q_i(b_i, p) - C_i(q_i(b_i, p)) \Rightarrow \max_{q_i \geq 0} pd + \sum_i -C_i(q_i)$$

$\sum_i q_i(b_i, p) = d$, $q_i(b_i, p) = b_i p$

Optimality condition
$\nabla f(x^*)^T (x - x^*) \geq 0$, $\forall x \in X$

Competitive equilibrium (for customers and social welfare) :

$\{(b_i)_{i \in N}, p\}$ s.t.
- $(C_i'(q_i(b_i, p)) - p)(\hat{b}_i - b_i) \geq 0$, $\forall \hat{b}_i \geq 0$
- $\sum_i q_i(b_i, p) = d$



## Matching the Supply - Competitive Market

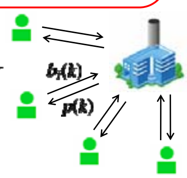


**Theorem 1**
There exists a unique competitive equilibrium for the demand response system. Moreover, the equilibrium is efficient, i.e., it maximizes the social welfare:

$$\max_{q_i \geq 0} \sum_i -C_i(q_i) \quad s.t. \ \sum_i q_i = d$$

Iterative bidding(At $k$-th iteration):

1. Utility company announces $p(k)$
2. Each customer updates $b_i(k) = \left[\frac{(C_i')^{-1}(p(k))}{p(k)}\right]^+$
3. Utility company gathers $b_i(k)$
4. Utility company updates




$$p(k+1) = \left[p(k) - \gamma\left(\sum_i b_i(k) p(k) - d\right)\right]^+$$

'+' : the projection onto $\mathcal{R}^+$

constant stepsize $\gamma > 0$

Customers know that price $p(b) = \frac{d}{\sum_i b_i}$ ➡ anticipate price

strategic!

Supply function for all customers except $i$ :

$$b_{-i} = (b_1, b_2, \cdots, b_{i-1}, b_{i+1}, \cdots, b_{|N|}) \Rightarrow b = (b_i, b_{-i})$$

Each customer $i \in N$ maximizes:

$$u_i(b_i, b_{-i}) = p(b) q_i(p(b), b_i) - C_i(q_i(p(b), b_i))$$

payoff

$$= \frac{d^2 b_i}{(\sum_j b_j)^2} - C_i\left(\frac{d b_i}{\sum_j b_j}\right) \quad \because q_i(p(b), b_i) = b_i p(b) = \frac{d b_i}{\sum_j b_j}$$

➡ Demand response game among customers

Game-theoretic equilibrium

➡ Nash equilibrium
$u_i(b_i^*, b_{-i}^*) \geq u(b_i, b_{-i}^*)$ , $\forall b_i \geq 0$, $i \in N$





Calculate equilibrium:

$$B_{-i} = \sum_{j \neq i} b_j \Rightarrow u_i(b_i, b_{-i}) = \frac{d^2 b_i}{(B_{-i} + b_i)^2} - C_i\left(\frac{d b_i}{B_{-i} + b_i}\right)$$

$$\Rightarrow \frac{\partial u_i(b_i, b_{-i})}{\partial b_i} = \frac{d^2 (B_{-i} - b_i)}{(B_{-i} + b_i)^3} - \frac{d B_{-i}}{(B_{-i} + b_i)^2} C_i'\left(\frac{d b_i}{B_{-i} + b_i}\right)$$

$$= \frac{d^2}{(B_{-i} + b_i)^2}\left[\frac{B_{-i} - b_i}{B_{-i} + b_i} - \frac{B_{-i}}{d} C_i'\left(\frac{d b_i}{B_{-i} + b_i}\right)\right]$$

decreasing in $b_i$ increasing in $b_i$

If $\frac{B_{-i}}{d} C_i'(0) \geq 1$ , then $\frac{\partial u_i(b_i, b_{-i})}{\partial b_i} \leq 0$ ➡ $b_i^* = 0$

If $\frac{B_{-i}}{d} C_i'(0) < 1$ , then $\frac{\partial u_i(b_i, b_{-i})}{\partial b_i} = 0$ only at one point $b_i > 0$

➡ $b_i^*$ satisfies $\frac{B_{-i}^* - b_i^*}{B_{-i}^* + b_i^*} - \frac{B_{-i}^*}{d} C_i'\left(\frac{d b_i^*}{B_{-i}^* + b_i^*}\right) = 0$





Lemma

If $b^*$ is a Nash equilibrium of the demand response game, then $b_i^* < B_{-i}^* = \sum_{j \neq i} b_j^*$ for any $i \in N$ , i.e., each customer will shed a load of less than $d/2$ at the equilibrium.

➡ No Nash equilibrium exists when $|N| = 2$

Social welfare

$$b_i^* = 0 \ , \quad \frac{B_{-i}^* - b_i^*}{B_{-i}^* + b_i^*} - \frac{B_{-i}^*}{d} C_i'\left(\frac{d b_i^*}{B_{-i}^* + b_i^*}\right) = 0$$

$$\Rightarrow \left(\frac{d}{B_{-i}^* + b_i^*} - \frac{B_{-i}^*}{B_{-i}^* - b_i^*} C_i'\left(\frac{d b_i^*}{B_{-i}^* + b_i^*}\right)\right)(b_i - b_i^*) \leq 0 \ , \ \forall b_i \geq 0 \quad \cdots (1)$$

at Nash equilibrium: $p^* = \frac{d}{\sum_i b_i^*} = \frac{d}{B_{-i}^* + b_i^*}$ , $q_i^* = b_i^* p^*$ $\cdots (2)$





Substitute (2) to (1)

➡ Nash equilibrium condition

$$\left(p^* - \left(1 + \frac{q_i^*}{d - 2q_i^*}\right) C_i'(q_i^*)\right)(b_i p^* - q_i^*) \leq 0 \ , \ \forall q_i \geq 0$$

$q_i$

gradient of objective function

optimality condition: $\nabla f(x^*)^T (x - x^*) \geq 0$

$$f(q_i) := \int \left(p - \left(1 + \frac{q_i}{d - 2q_i}\right) C_i'(q_i)\right) dq_i$$

$$= p q_i - \left(1 + \frac{q_i}{d - 2q_i}\right) C_i(q_i) + \int \frac{d}{(d - 2q_i)^2} C_i(q_i) dq_i$$

objective function: $\max_{0 \leq q_i < d/2} \sum_i f(q_i)$ ※ $\sum_i p q_i = pd$





Theorem 2

Assume $|N| > 2$. The demand response game has a unique Nash equilibrium. Moreover, the equilibrium solves the following convex optimization problem:

$$\max_{0 \leq q_i < d/2} \sum_i -D_i(q_i) \quad s.t. \ \sum_i q_i = d$$

with

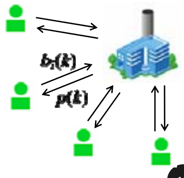
$$D_i(q_i) = \left(1 + \frac{q_i}{d - 2q_i}\right) C_i(q_i) - \int_0^{q_i} \frac{d}{(d - 2x_i)^2} C_i(x_i) dx_i$$

Iterative bidding(At $k$-th iteration):

Each customer updates $b_i(k) = \left[\frac{(D_i')^{-1}(p(k))}{p(k)}\right]^+$

Utility company updates

$$p(k+1) = \left[p(k) - \gamma\left(\sum_i b_i(k) p(k) - d\right)\right]^+$$








realtime spot prices
incentivize customers to shift or reduce their loads ➡ elastic
power load $q_i(t)$ at time $t$ ←※

System model

customer:

$$\sum_{t=1}^{T} q_i(t) \geq \underline{Q}_i \ , \quad \sum_{t=1}^{T} q_i(t) \leq \bar{Q}_i$$

$t \in T = \{1, 2, \cdots, |T|\}$

supply: $Q(t)$

utility company :

$$\max_{Q(t) \geq 0} \sum_{t \in T} Q(t) p(t) - C(Q(t), t) \Rightarrow$$

solution
$C'(Q(t), t) = p(t), \ t \in T$

cost(strictly convex)

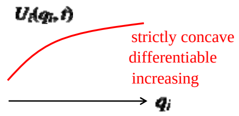
supply = demand: $\sum_{i \in N} q_i(t) = Q(t)$ , $t \in T$

## Shaping The Demand - Competitive Market



customers : given realtime spot price $p(t)$

Customer $i$ allocates its energy usage:




$$\max_{q_i(t)\geq 0} \sum_{t\in T} U_i(q_i(t),t) - q_i(t)p(t)$$

customer's utility

$$\text{s.t.} \ \sum_{t=1}^{T} q_i(t) \geq \underline{Q}_i \ , \ \sum_{t=1}^{T} q_i(t) \leq \bar{Q}_i \ , \ i \in N$$

➡ realtime pricing , demand shifting

Social welfare:

$$\max_{q_i(t)\geq 0} \sum_{t\in T} \{\sum_{i\in N} (U_i(q_i(t),t) - q_i(t)p(t))\}$$

$$\text{s.t.} \ \sum_{t=1}^{T} q_i(t) \geq \underline{Q}_i \ , \ \sum_{t=1}^{T} q_i(t) \leq \bar{Q}_i \ , \ i \in N$$



## Shaping The Demand - Competitive Market



Karush-Kuhn-Tucker condition

Lagrange multiplier: $\lambda_i \geq 0$

$$\frac{\partial f}{\partial x_k}(x^*) - \sum_i \lambda_i \frac{\partial g_i}{\partial x_k}(x^*) = 0 \ , \forall k \qquad \lambda_i g_i(x^*) = 0 \ , \ \forall i$$

Competitive equilibrium(for customers and social welfare):

Lagrange multiplier: $\underline{\lambda}_i \geq 0$ , $\bar{\lambda}_i \geq 0$

$$\{(q_i(t))_{i\in N, t\in T}, (Q(t))_{t\in T}, (p(t))_{t\in T}\}$$

$$\text{s.t.} \begin{cases} U_i'(q_i(t),t) = p(t) + \bar{\lambda}_i - \underline{\lambda}_i \\ \underline{\lambda}_i(\underline{Q}_i - \sum_{t=1}^{T} q_i(t)) = 0 \ , \quad \bar{\lambda}_i(\sum_{t=1}^{T} q_i(t) - \bar{Q}_i) = 0 \end{cases}$$



## Shaping The Demand - Competitive Market



Social welfare $\quad \because \sum_{i\in N} q_i(t) = Q(t)$ supply

$$\sum_{i\in N} (U_i(q_i(t),t) - q_i(t)p(t)) \Rightarrow \sum_{i\in N} U_i(q_i(t),t) - Q(t)p(t)$$

$$C'(Q(t),t) = p(t) \Rightarrow C(Q(t),t) = Q(t)p(t) + \alpha$$

integral constant

**Theorem 3**

There exists a unique competitive equilibrium for the demand response system. Moreover, the equilibrium is efficient, i.e., it maximizes the social welfare:

$$\max_{q_i(t)\geq 0} \sum_{t\in T} \{\sum_{i\in N} U_i(q_i(t),t) - C(\sum_{i\in N} q_i(t),t)\}$$

$$\text{s.t.} \ \sum_{t=1}^{T} q_i(t) \geq \underline{Q}_i \ , \ \sum_{t=1}^{T} q_i(t) \leq \bar{Q}_i \ , \ i \in N$$



## Shaping The Demand - Competitive Market



decide on power loads and supply for each time $t$

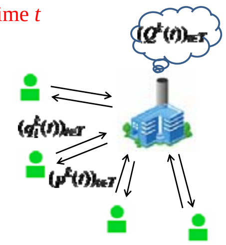
Distributed Algorithm(at $k$-th iteration):

1. Utility company collects $(q_i^k(t))_{t\in T}$
2. Utility company calculates the total demand $(Q^k(t))_{t\in T}$ and marginal cost
$$p^k(t) = C'(Q^k(t),t)$$
3. Utility company announces $(p^k(t))_{t\in T}$
4. Each customer $i$ updates
$$q_i^{k+1}(t) = [q_i^k(t) - \gamma(U_i'(q_i^k(t),t) - p^k(t))]^{S_i}$$

constant stepsize $\gamma > 0$

'$S_i$' : the projection onto the set

$$\begin{cases} \sum_{t=1}^{T} q_i(t) \geq \underline{Q}_i \\ \sum_{t=1}^{T} q_i(t) \leq \bar{Q}_i \end{cases}$$






## Shaping The Demand - Oligopoly Market



Customers know $C'(Q(t),t) = p(t)$ strategic!

positive, increasing

Given Other customer power loads:

$$(q_{-i}(t))_{t\in T} = \{(q_j(t))_{t\in T}, j \in N/\{i\}\}$$

➡ Each customer $i$ maximizes:

$$\sum_{t\in T} u_i(q_i(t), q_{-i}(t)) = \sum_{t\in T} U_i(q_i(t),t) - q_i(t)C'(\sum_{i\in N} q_i(t),t)$$

Social welfare:

$$\max_{q_i(t)} \sum_{t\in T} \{\sum_{i\in N} U_i(q_i(t),t) - q_i(t)C'(\sum_{i\in N} q_i(t),t)\}$$

Game-theoretic equilibrium:

optimality condition $\nabla f(x^*)^T(x - x^*) \geq 0$

$$(q_i^*(t))_{t\in T} \ \text{s.t.} \ \sum_{t\in T} \frac{\partial u_i(q_i^*(t), q_{-i}^*(t))}{\partial q_i^*(t)}(q_i(t) - q_i^*(t)) \leq 0 \ , \ (q_i(t))_{t\in T} \in S_i$$



## Shaping The Demand - Oligopoly Market



**Theorem 4**

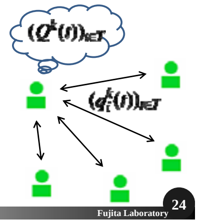
The demand response game has a unique Nash equilibrium. Moreover, it solves the convex problem:

$$\max_{q_i(t)} \sum_{t\in T} \{\sum_{i\in N} U_i(q_i(t),t) - q_i(t)C'(\sum_{i\in N} q_i(t),t)\}$$

$$\text{s.t.} \ \sum_{t=1}^{T} q_i(t) \geq \underline{Q}_i \ , \ \sum_{t=1}^{T} q_i(t) \leq \bar{Q}_i \ , \ i \in N$$

Distributed Algorithm(At $k$-th iteration):

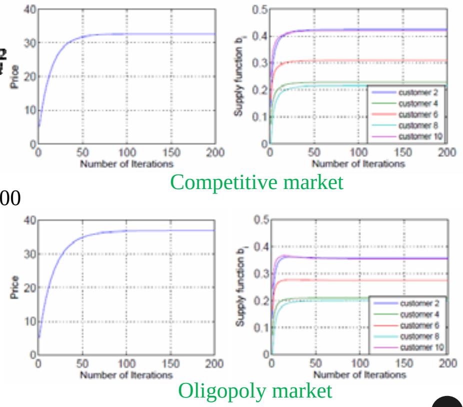
1. customers exchange $(q_i^k(t))_{t\in T}$
2. each customer calculates $(Q^k(t))_{t\in T}$
3. each customer $i$ updates
$$q_i^{k+1}(t) = [q_i^k(t) + \gamma(U_i'(q_i^k(t),t) - C'(Q^k(t),t) - p^k(t)C''(Q^k(t),t))]^{S_i}$$



# Simulation



- 5 customers
- $C_i(q_i) = a_i q_i + h_i q_i^2$
  - $a_i \in [1,2]$
  - $h_i \in [2,6]$
- $\gamma = 0.02$
- supply deficit $d$ =100

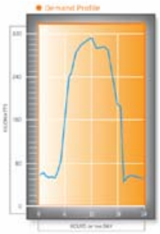





# Conclusion



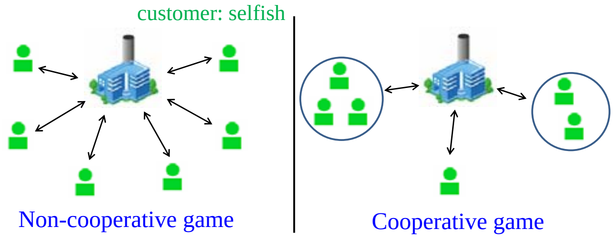
- Non-cooperative game
- 2 market models for demand response
  - to match electricity supply
  - to shape electricity demand
- Characterizing the unique equilibrium
  - competitive market
  - oligopolistic market
- Distributed demand response schemes and algorithms





# Future Works






- Non-cooperative game ➡ Cooperative game
  - coalition , bargaining , transfer
- Robust equilibrium





# Appendix



# How to decide $b_i$



Optimal condition

$$\nabla f(x^*)^T(x - x^*) \geq 0,\ \forall x \in X$$

$f$ : convex ➡ $x^*$ : global optimal solution , $\nabla f(x^*) = 0$

$$(C_i'(q_i(b_i, p)) - p)(\hat{b}_i - b_i) \geq 0$$

➡ $C_i'(q_i(b_i, p)) = p$

$$q_i(b_i, p) = (C_i')^{-1}(p)$$

$$b_i p = (C_i')^{-1}(p) \qquad \because q_i(b_i, p) = b_i p$$

$$b_i = \frac{(C_i')^{-1}(p)}{p}$$



# Matching the Supply - Oligopoly Market



$$u_i(b_i, b_{-i}) = \frac{d^2 b_i}{(\sum_j b_j)^2} - C_i\left(\frac{d b_i}{\sum_j b_j}\right)$$

**Nash equilibrium**

$$u_i(b_i^*, b_{-i}^*) \geq u(b_i, b_{-i}^*),\ \forall b_i \geq 0,\ i \in N$$

**Lemma**

If $b^*$ is a Nash equilibrium of the demand response game, then $\sum_{j \neq i} b_j^* > 0$ for any $i \in N$.

Proof:

$$\sum_{j \neq i} b_j^* = 0,\ b_i^* > 0 \Rightarrow u_i(b_i^*, b_{-i}^*) = \frac{d^2}{b_i^*} - C_i(d) < u_i(b_i, b_{-i}^*) \quad (b_i < b_i^*)$$

■

➡ at the Nash equilibrium at least 2 customers have $b_i^* > 0$