Introduction to Transportation Network

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FL10-06-01

5th, July,2010

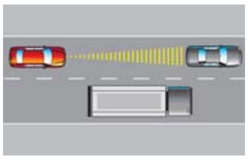
Introduction

Highway congestion

Highway congestion is imposing an intolerable burden on may urban residents.

Congestion occurs when the demand for travel exceeds highway capacity.

Action



There are some approaches to reducing congestion such that building more highways, raising tolls or taxes.

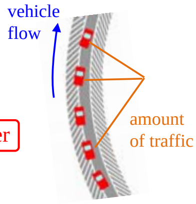
In these approaches, the best way to reducing congestion is that vehicle itself judges the circumstance and change it's behavior automatically to avoid congestion.

P. Varaiya, "Smart Cars on Smart Roads: Problems of Control", *IEEE Transactions on Automatic Control*, Vol. 38, No. 2, Feb. 1993

Congestion avoid

Highway capacity

To improve Highway capacity,
- decrease amount of traffic
- increase vehicle flow



⇨ Control theory can apply to the latter

Improve vehicle flow

Longitudinal control

Vehicles keep a narrow distance of the precede vehicle.

⇨ Simple problem, Many research

Lateral control

Vehicles seek to change a less crowed lane.

⇨ Complex problem, few research

Longitudinal control

Specific

- Keep a desired distance of the precede vehicle
- Disturbance attenuation

Disturbance

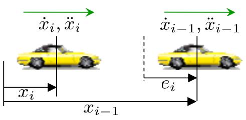
Velocity change of precede vehicle
caused by tunnel, slope or merging etc..

Design controller that satisfies
tracking and disturbance attenuation

Definition

i th vehicle model

Position: x_i
Velocity: $\dot{x}_i = v_i$
Acceleration: $\ddot{x}_i = a_i$



Error input

Position error between i th and i-1th vehicle

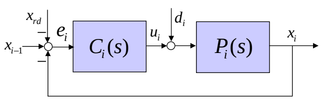
$$e_i = x_{i-1} - x_i - x_{rd}$$

x_{rd}:Desired relative position

Block Diagram

Closed loop transfer function

$$x_i = \frac{P_i(s)C_i(s)}{1+P_i(s)C_i(s)} x_{i-1} = T_i(s) x_{i-1}$$



String stability

Disturbance attenuation

Consider d_0,
Transfer function d_0 to e_i

$$e_i = H_{e_i d_0}(s) d_0$$

⇨ Attenuation condition
$\|H_{e_i d_0}(s)\|_\infty \leq 1 \ \forall i$

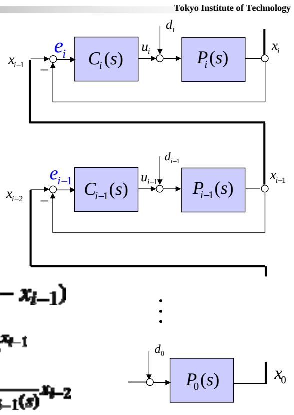
Error input

$$e_i = x_{i-1} - x_i - x_{rd}$$

⇔ $e_i - e_{i-1} = (x_{i-1} - x_i) - (x_{i-2} - x_{i-1})$

⇩ $x_i = \frac{P_i(s)C_i(s)}{1+P_i(s)C_i(s)} x_{i-1}$

$$e_i - e_{i-1} = \frac{1}{1+P_i(s)C_i(s)} x_{i-1} - \frac{1}{1+P_{i-1}(s)C_{i-1}(s)} x_{i-2}$$



String Stability

Tokyo Institute of Technology

$$e_i - e_{i-1} = \frac{1}{1+P_i(s)C_i(s)}x_{i-1} - \frac{1}{1+P_{i-1}(s)C_{i-1}(s)}x_{i-2}$$

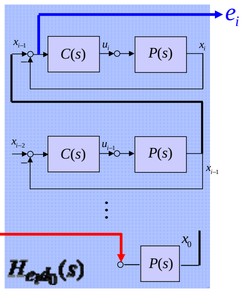
Suppose homogeneous vehicle system,

$$e_i - e_{i-1} = \frac{1}{1+P(s)C(s)}e_{i-1}$$

$$e_i = \frac{P(s)C(s)}{1+P(s)C(s)}e_{i-1} = T(s)e_{i-1}$$

$$\Rightarrow e_i = H_{e_i d_0}(s)d_0 = T(s) * T(s) * \cdots * d_0$$

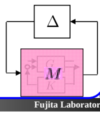
$$\therefore \|T(s)\|_\infty < 1$$



Sufficient condition ⇒ String Stability $\left\| \frac{e_i}{e_{i-1}} \right\|_\infty < 1$

Small Gain Theorem

$\|M\Delta\|_\infty < 1$



Bode's integral formula

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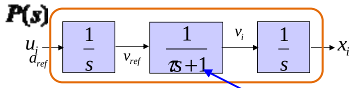
Vehicle's model

$$P(s) = \frac{1}{s^2(\tau s+1)}$$

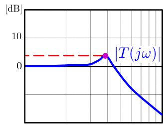


Bode's integral formula

$$\int_0^\infty \ln|T(j\omega)|\frac{d\omega}{\omega^2} \geq 0$$

$\Rightarrow \|T(s)\|_\infty > 1$ String Stability ×

Any controller can't achieve String Stability



What's behind

$$e_i = x_{i-1} - x_i - x_{rd}$$

Only uses position data ⇒ Increase more feedback information

Survey

Tokyo Institute of Technology

S. Klinge and R. H. Middleton,
"Time Headway Requirements for String Stability of Homogeneous Linear Unidirectionally Connected Systems,"
Proc. of the 48th IEEE CDC, pp. 1992-1997, 2009.

Richard H. Middleton
Hamilton Institute,
National University of Ireland Maynooth

Research:
Trade-offs & Performance limitations in Feedback Control Systems
Applications of Systems, Control and Optimisation

Collaborative study with Braslavsky

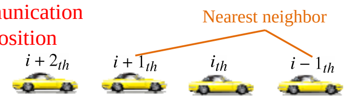
Julio H. Braslavsky
University of Newcastle

Richard H. Middleton

Tokyo Institute of Technology

Argument

It is not possible to achieve String Stability with
- nearest neighbor communication
- fixed desired relative position



Error input

$$e_i = x_{i-1} - x_i - x_{rd}$$

Fixed desired relative position

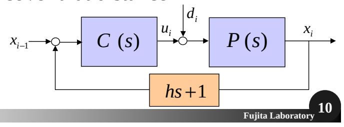
⇒ Change a speed dependent inter-vehicle spacing policy

Time headway

The required time of a vehicle to cover that distance

$$e_{vi} = x_{i-1} - x_i - \dot{x}_i h$$

h_i :Time headway



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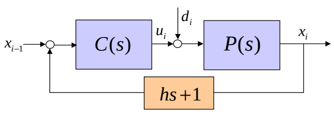
Error input

$$e_{vi} = x_{i-1} - x_i - \dot{x}_i h$$

h_i :Time headway

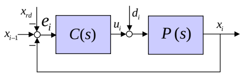


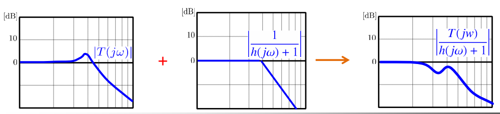
Closed loop transfer function

$$x_i = \frac{P(s)C(s)}{1+(hs+1)P(s)C(s)}x_{i-1}$$

$$= \frac{(hs+1)P(s)C(s)}{1+(hs+1)P(s)C(s)}\frac{1}{hs+1}x_{i-1}$$

$$= T'(s)\frac{1}{hs+1}x_{i-1} \Leftrightarrow x_i = T(s)x_{i-1}$$





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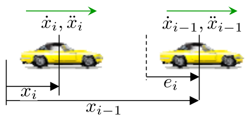
String Stability condition expression

$$\left\| \frac{e_{vi}}{e_{v(i-1)}} \right\|_\infty < 1$$



i th error input: $e_{vi} = x_{i-1} - x_i - \dot{x}_i h$

i-1th error input: $e_{v(i-1)} = x_{i-2} - x_{i-1} - \dot{x}_{i-1} h$

Closed loop transfer function

$$x_i = T(s)\frac{1}{hs+1}x_{i-1}$$

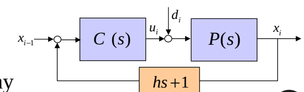
$$\Rightarrow e_i = T(s)\frac{1}{hs+1}e_{i-1}$$

String Stability $\left\| T(s)\frac{1}{hs+1} \right\|_\infty < 1$

Constraint

h needs to be larger than constant h_0

⇒ There is a limitation by using time headway



Survey

(*)P. Seiler, A. Pant and K. Hedrick, "Disturbance Propagation in Vehicle Strings," *IEEE Transactions on Automatic Control*, vol. 49, No. 10, pp. 1835-1841, 2004.

J. Karl Hedrick
University of California, Berkley
Research:
Automotive Control
Autonomous Unmanned Vehicle Control

Relevant Researchers

Pravin Varaiya
University of California, Berkley

Darbha Swaroop
Texas A&M University

J. Karl Hedrick

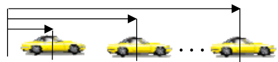
Argument

It is not possible to achieve String Stability with predecessor following.

Though, fixed relative desired position may be required in some situation. (such that fuel saving)

⇨ Don't use Time Headway but, achieve String Stability

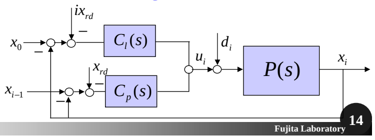
Leader following



$$u_i = C_p(s)(x_{i-1} - x_i - x_{rd}) + C_l(s)(x_0 - x_i - ix_{rd})$$

predecessor following — leader following

With Communicating to leader vehicle, Both predecessor and leader information are used.



J. Karl Hedrick

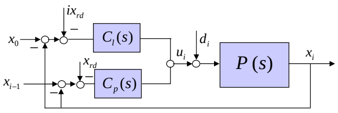
Input

$$u_i = C_p(s)(x_{i-1} - x_i - x_{rd}) + C_l(s)(x_0 - x_i - ix_{rd})$$

predecessor following — leader following

Closed loop transfer function

$$x_i = \frac{P(s)C_p(s)}{1 + P(s)(C_p(s) + C_l(s))} x_{i-1}$$



$$⇨ \; e_i = \frac{P(s)C_p(s)}{1 + P(s)(C_p(s) + C_l(s))} e_{i-1}$$

$$= T\frac{C_p(s)}{C_p(s) + C_l(s)} e_{i-1}$$

String Stability

$$T\frac{C_p(s)}{C_p(s) + C_l(s)} < 1$$

Example

$C_p(s) = p$
$C_l(s) = l$
$l + p = 1$

$$e_i = T\frac{C_p(s)}{C_p(s) + C_l(s)} e_{i-1} = pTe_{i-1}$$

$p \rightarrow large$ (desired)
Less communication, hard SS
$p \rightarrow small$
High communication, easy SS

Drawback of communication

X. Liu, S. S. Mahal, A. Goldsmith and K. Hedrick, "Effects of Communication Delay on String Stability in vehicle Platoons," *IEEE Intelligent Transportation Systems Conference Proceeding*, Oakland, USA, August 25-29, 2001.

Communication Delay

String stability cannot hold even infinitesimal communication delay.

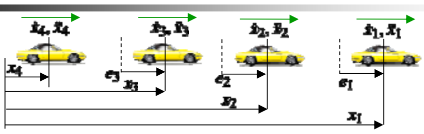
⇨ All vehicle controllers update simultaneously used by time jitter(such that token).

Problem

Communication method and error
leader and last vehicle Decision
Update at the same time

⇨ Hard to implementation

Simulation –R. H. Middleton–



Plant model

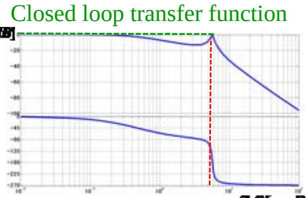
$$P(s) = \frac{1}{s^2(s+1)}$$

$h = 2.05$

Controller

$C = 16$

Closed loop transfer function



Initial condition

$x_{1_0} = 30[m]$ $x_{3_0} = 10[m]$
$x_{2_0} = 20[m]$ $x_{4_0} = 0[m]$
$v_{1_0} = v_{2_0} = v_{3_0} = v_{4_0} = v_0 = 13.8[m/s]$
$= 50[km/h]$

String Stability ○ $\omega = 7.0[rad]$

Disturbance

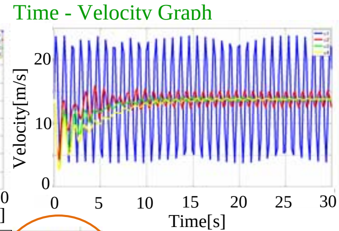
$v_1 = v_0 + 10sin(7t)$

Simulate x_i , v_i , e_i in existing disturbance

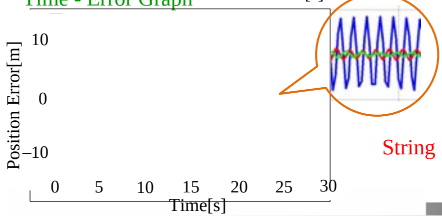
Simulation Result

Time - Position Graph

Time - Velocity Graph



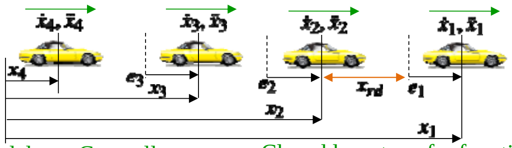
Time - Error Graph



Error is more attenuated to successive vehicle

String Stability is approved

Simulation −J. Karl Hedrick−



Plant model

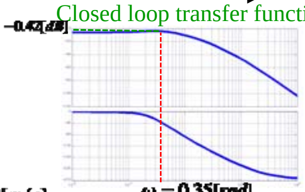
$$P(s) = \frac{1}{s^2(s+1)}$$

Controller

$$C = \frac{5.33s+1}{0.383s+1}$$

$$x_{rd} = 10[m]$$

Closed loop transfer function



String Stability ○

Initial condition

$x_{1_0} = 30[m]$ $x_{3_0} = 10[m]$

$x_{2_0} = 20[m]$ $x_{4_0} = 0[m]$

$v_{1_0} = v_{2_0} = v_{3_0} = v_{4_0} = v_0 = 13.8[m/s]$
$= 50[km/h]$

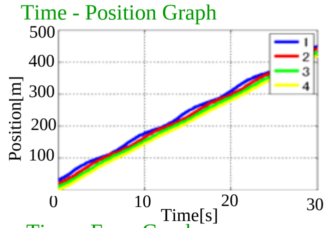
Disturbance

$v_1 = v_0 + 5sin(0.35t)$

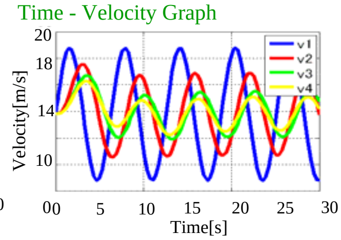
Simulate x_i , v_i , e_i in existing disturbance

Simulation −J. Karl Hedrick−

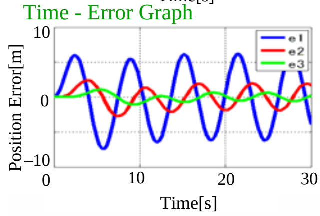
Time - Position Graph



Time - Velocity Graph



Time - Error Graph



Error is more attenuated to successive vehicle

String Stability is approved

Other Researches

Heterogeneous Vehicles

(**) E. Shaw and J. K. Hedrick, "Controller Design for String Stable Heterogeneous Vehicle Strings," *Proc. of IEEE Conference on Decision and Control*, pp. 2868-2875, 2007

G. Naus, R. Vugts, J. Ploeg, R. V. Molengraft and M. Steinbuch , "Towards on-the-road Implementation of Cooperative Adaptive Cruise Control," *Proc. of The 16th World Congress and Exhibition on Intelligent Transport Systems and Services*, 2009

I. Lestas and G. Vinnicombe, "Scalability in heterogeneous vehicle platoons," *Proc. of 2007 American Control Conference*, pp 4678-4683, 2007.

String Instability

P. Seiler, A. Pant, and K. Hedrick, "Disturbance propagation in vehicle strings," *IEEE Transactions on Automatic Control*, vol. 49, pp. 1835-1841, 2004.

P.Barroah and J. P. Hespanha, "Error Amplification and Disturbance Propagation in Vehicle Strings with Decentralized Linear Control," *Proc. of the 44th IEEE CDC and ECC*, pp. 4964-4969, 2005.

Other Researches

Communication delay

G. Orosz, J. Moehlis and F. Bullo, "Robotic Reactions: Delay-induced patterns in autonomous vehicle systems," *Physical Review*, E811, 025204(R) pp. 1-4, 2010.

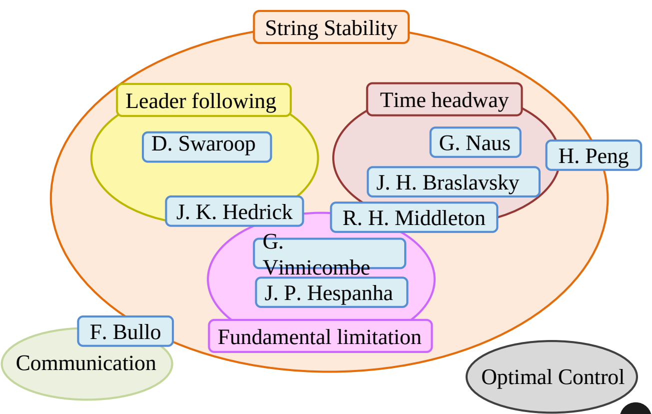
Improve time headway

J. Zhou and H. Peng, "Range Policy of Adaptive Cruise Control Vehicles for Improved Flow Stability and String Stability," *IEEE Transactions on Intelligent Transportation Systems*, Vol. 6, No. 2, pp. 229- 237, 2005

Optimal Control

W. S. Levine and M. Athans, "On the optimal error regulation of a string of moving vehicles," *IEEE Transactions on Automatic Control*, vol. AC-11, no. 3, pp. 355-361, 1966.

B. Shu and B. Bamieh, "Robust *H2* Control of Vehicular Strings," *ASME Journal on Dynamics systems, Measurement and Control*, 1996



Appendix

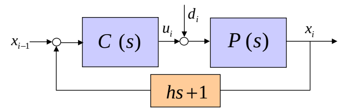
Richard H. Middleton Parameter Design

Plant analysis

Plant model

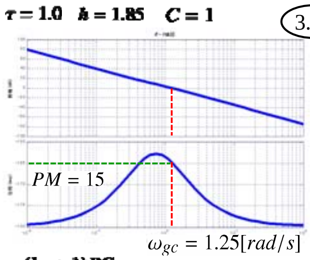
$$P(s) = \frac{1}{s^2(\tau s + 1)}$$

zero: non
pole: 0,- τ



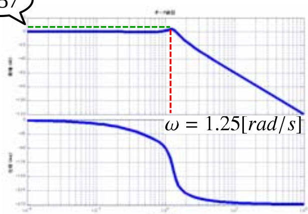
Open loop diagram

$\tau = 1.0 \quad h = 1.85 \quad C = 1$



$(hs+1)PC$

Closed loop diagram $T\frac{1}{hs+1}$

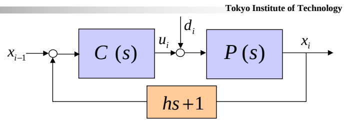


$$\left\| T\frac{1}{hs+1} \right\|_\infty > 1$$

Design controller

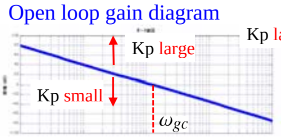
Kp controller

$C = K_p$

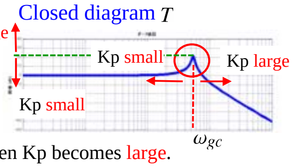


Bode analysis

Open loop gain diagram

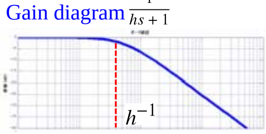


Closed diagram T



ω_{gc} becomes large when Kp becomes large.

Gain diagram $\frac{1}{hs+1}$



ω_{gc} must be larger than h^{-1}

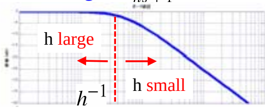
⇨ find a controller that satisfies String Stability

Design controller

Constraint

The controllers that satisfy String Stability exist if $h > 2.0$

Gain diagram $\frac{1}{hs+1}$



Large h makes String Stability easier to accomplish.

But, h is desired to be small for efficiency.

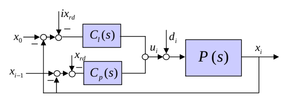
J. K. Hedrick Parameter Design

Plant analysis

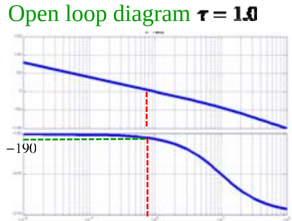
Plant model

$$P(s) = \frac{1}{s^2(\tau s + 1)}$$

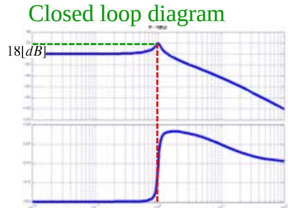
zero: non
pole: 0,- τ



Open loop diagram $\tau = 1.0$



Closed loop diagram



Controller design

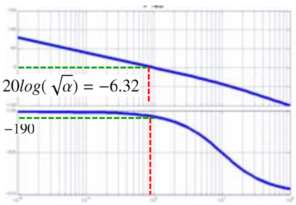
Controller

$C_p(s) = p * K(s)$

$C_i(s) = i * K(s) \quad p + i = 1$

Open loop diagram

$\tau = 1.0 \quad p = 1$



$\omega_{gc} = 0.70[rad/s]$

Phase lead compensation

$K(s) = \frac{K_p(Ts+1)}{\alpha Ts+1}$

Specification

$p = 0.5 \quad K_p = 1 \quad \tilde{PM} = 50$

Parameter

$\phi = \tilde{PM} - PM = 60$

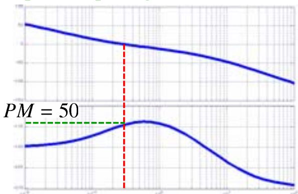
$\alpha = \frac{1-sin\phi}{1+sin\phi} = 0.0718 \quad \sqrt{\alpha} = 0.268$

$\omega_{max} = 0.70 \quad T = \frac{1}{\omega_{max}\sqrt{\alpha}}$

$C = \frac{5.33s+1}{0.383s+1}$

Phase lead compensation

Open loop diagram



$\omega_{gc} = 0.30[rad/s]$

$P(s) = \frac{1}{s^2(\tau s+1)} \quad C = \frac{5.33s+1}{0.383s+1}$

$\tau = 1.0 \quad p = 0.5$