Basic Cases Study on Limited-Range Anisotropic Sensor in Coverage Control Mobile Networks

Risvan Dirza
09R12111
June 14, 2010
(was written since March)

Outline Report

- Brief Introduction and Previous Work
- Selecting and Manipulating Energy Consumption Model to reduce unnecessary motion (****)
- Considering Communication Cost (****)
- Avoiding Obstacles (***)
- Considering Uneven Surfaces (*)
- Vehicle Model (**)
- 3D Works (**)
- Optimum Multiple Target Achievement – Game theory approach might be one solution (*)

**** : Need to be checked
*** : Need to be proven
** : Has some problems in Simulation
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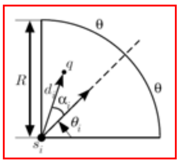
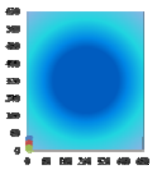
Brief Introduction & Previous Work (1)

Background

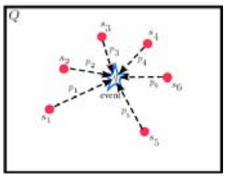
1. Search and Recovery Operations.
2. Manipulation in hazardous environments.
3. Surveillance
4. Environmental Monitoring.

$$P_i(q) = \frac{(d_i - R(Es))^2(\alpha_i - \theta)^2}{R(Es)^2\theta^2} \quad if\ q \in Q_i$$





Field

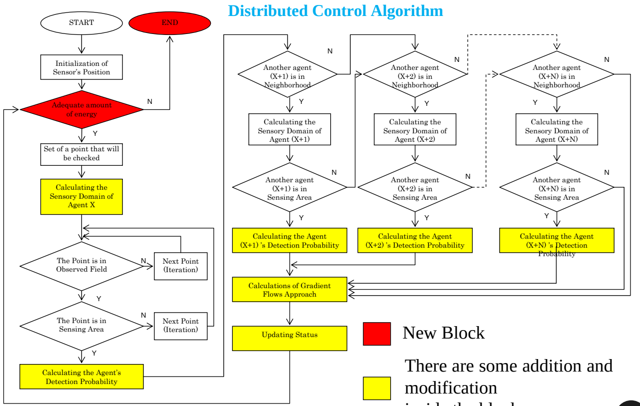


$$P(q,p,\theta) = 1 - \prod_{i=1}^{N}[1 - P_i(q)]$$

$$F(p,\theta) = \int_Q \phi(q).P(q,p,\theta)dq$$

Brief Introduction & Previous Work (2)

Distributed Control Algorithm



New Block

There are some addition and modification inside the block

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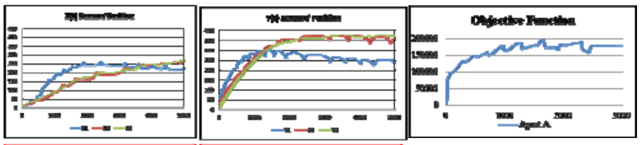
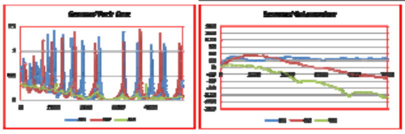
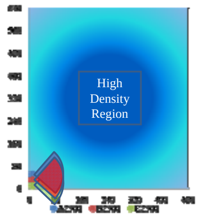
Problem Clarification (1)

Previous Work







Problem :
- Sensors' movement are ineffective,
- Objective Function is not maximized

Simulation Video

Problem Clarification (2)

Previous Work

$$E(t) = -k_e \frac{\|dp_i(t)\|}{dt}$$

Assume :
- The energy will be decreasing proportional to the trajectory

Effect :
- The sensing radius will be proportional to the energy contained

$$R(Es) = \frac{Es}{E_{max}} R_{max}$$

$$P_i(q) = \frac{(d_i - R(Es))^2(\alpha_i - \theta)^2}{R(Es)^2\theta^2} \quad \text{if } q \in Q_i$$

There are 3 solutions that might overcome :
1.Modifying Energy Consumption Model
2.Modifying Sensor Model
3.Designing Suitable Controller

FROM NOW
Assumptions : Finite Energy Supplies

Theorem & Definition (1)

Definition 1

According to [1] : $u = \frac{\partial F}{\partial p_i}$

Definition 2

According to [2] : the motion of each sensor can be fully controlled as $\dot{p}_i = u$

Theorem 1 (A. Kwok et. all 2007)

Consider general vector field $X_i = (x_1 \ldots x_n)$,

where $X_i = (p_i, E_{s_i})$

Lie Derivative :

$$\mathcal{L}_X F = \dot{p}_i \frac{\partial F}{\partial p_i} + \dot{E}_{s_i} \frac{\partial F}{\partial E_{s_i}}$$

where,

F as the objective function : $F(p, E_s) = \int_Q \phi(q) P(p, q, \theta, E_s) dq$

Theorem & Definition (2)

Theorem 2

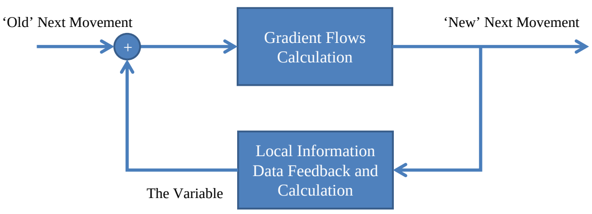
La Salle Principle

Let $Q \subset D$ be a compact set
Let $F : D \to R$ be a continuously differentiable function such that $\dot{F} \leq 0$ in Q
Let X be the set of all points in Q where $\dot{F} = 0$
Let M be the largest invariant set in X.
Then every solution starting in Q approaches M as $t \to \infty$

(H. K. Khalil, *Nonlinear Systems*)

Hypothesis (1)

Related to the problem occurred (explained before), There will be two hypothesis that should be tested :
1.There is at least one variable which should be well considered to manipulate agent's motion.
2.That variable can be controlled by local information.



Solution (1)

1. Modifying Energy Consumption Model

Previous Model / Proposed New Model

$\dot{E_s}(t) = -\left(\frac{\|dp_i(t)\|}{dt}\right)^{k_e}$ **???? => complicated in mathematic tools**

$$\dot{E_s}(t) = -k_e \frac{\|dp_i(t)\|}{dt} \rightarrow \dot{E_s}(t) = -k_e \left(\frac{E_s}{E_{max}}\right)^2 \left(\frac{\|dp_i(t)\|}{dt}\right)^2$$

Input for this system (u) is the next movement of a mobile sensor.
According to [1] : $u = \frac{\partial F}{\partial p_i}$,
So that, Proposed New Model will be defined as the following :

$$\dot{E}_{s_i}(t) = -k_e \left(\frac{E_s}{E_{max}}\right)^2 \left(\frac{\partial F}{\partial p_i}\right)^2$$

Solution (2)

2. Modifying Sensor Model

Previous Model

$$P_i(q) = \frac{\left(d_i - \frac{E_{s_i}}{E_{max}} R\right)^2 (\alpha_i - \theta)^2}{\left(\frac{E_{s_i}}{E_{max}} R\,\theta\right)^2}$$

Physical Meaning
Too many variables should be considered

Proposed New Model

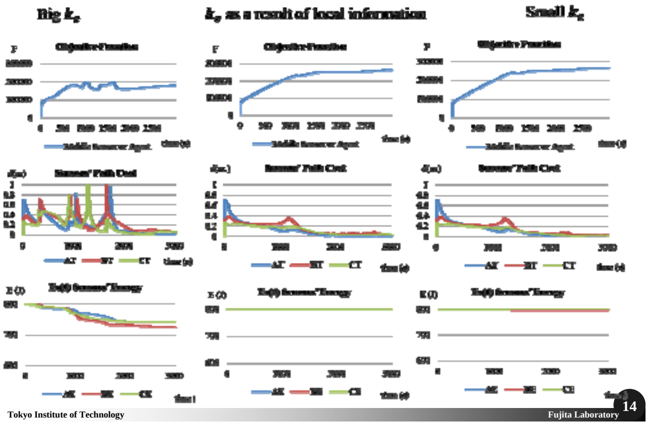
$$P_i(q) = \frac{\left(d_i - \frac{E_{s_i}}{E_{max}} R\right)^2 (\alpha_i - \theta)^2}{(R\,\theta)^2}$$

Physical Meaning

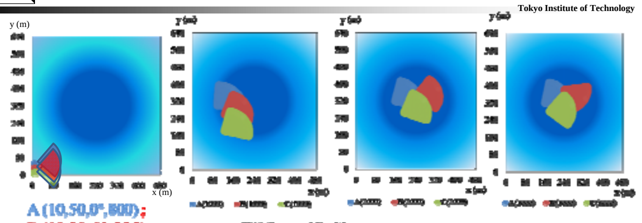
$E_{s_i} \gg, P_i \gg$

$E_{s_i} \ll, P_i \ll$

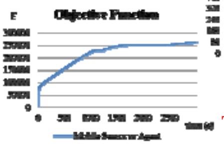
Simulation I(1)

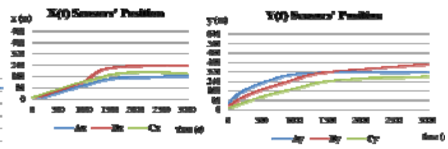


Simulation II (1)



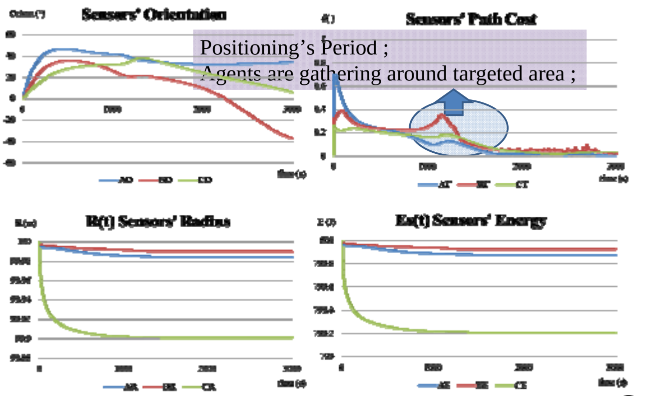
A (10,50,0°,800);
B (10,30, 0°,800);
C (10,10, 0°,800);



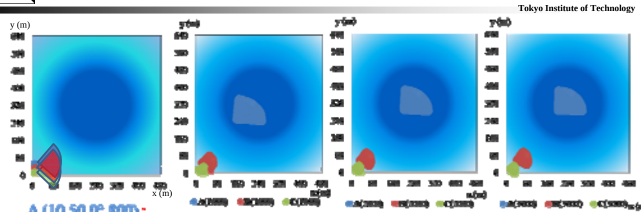


The motions are pretty smooth (No Ripple!!)

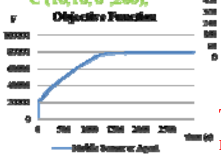
Simulation II (2)

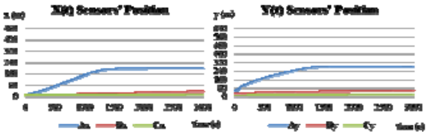


Simulation III(1)



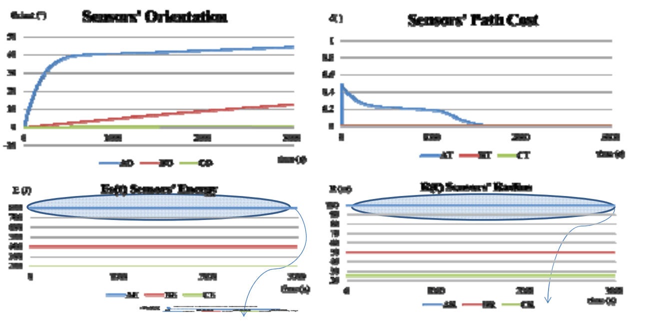
A (10,50,0°,800);
B (10,30, 0°,400);
C (10,10, 0°,200);





The area result are almost the as the previous report. But,
The motions are pretty smooth (No Ripple!!)

Simulation III(2)



Proof (1-1)

Suppose we can not design any suitable controller. But we still can find 'the variables':

$\mathcal{L}_X F \leq 0$

$$1 + \left(-k_e \left(\frac{E_s}{E_{max}}\right)^2\right) \frac{\partial F}{\partial E_{s_i}} \leq 0 \qquad \boxed{\text{Assume : } \frac{\partial F}{\partial E_{s_i}} \left(\frac{E_s}{E_{max}}\right)^2 = \gamma}$$

$$\boxed{\gamma \geq \frac{1}{k_e}}$$

Smaller k_e, give better performance

Proof (2-1)

$$\boxed{E_s^{\cdot}(t) = -k_e \left(\frac{E_s}{E_{max}}\right)^2 \left(\frac{\partial F}{\partial p_i}\right)^2}$$

Need to be defined Physical Meaning

Fact :
Assigning Constant Value will decrease the objective function's value (based on the result of the simulation)

Using Theorem I

$$\mathcal{L}_X F = \dot{p}_i \frac{\partial F}{\partial p_i} + \dot{E}_{s_i} \frac{\partial F}{\partial E_{s_i}}$$

Definition I and Definition II

$$u = \dot{p}_i = \frac{\partial F}{\partial p_i}$$

$$\mathcal{L}_X F = \left(\frac{\partial F}{\partial p_i}\right)^2 + \left(-k_e \left(\frac{E_s}{E_{max}}\right)^2 \left(\frac{\partial F}{\partial p_i}\right)^2\right) \frac{\partial F}{\partial E_{s_i}}$$

Proof (2-2)

Using Theorem II

Since Q is compact set, by LaSalle's invariance principle; the proposed new model will approach the largest invariant set contained in $\mathcal{L}_X F = 0$; Invariant Set $\{(p_1, E_1, \dots, p_n, E_n) \in Q^n\}$
(H. K. Khalil, *Nonlinear Systems*)

$\mathcal{L}_X F = 0$

$$\left(\frac{\partial F}{\partial p_i}\right)^2 + \left(-k_e \left(\frac{E_s}{E_{max}}\right)^2 \left(\frac{\partial F}{\partial p_i}\right)^2\right) \frac{\partial F}{\partial E_{s_i}} = 0$$

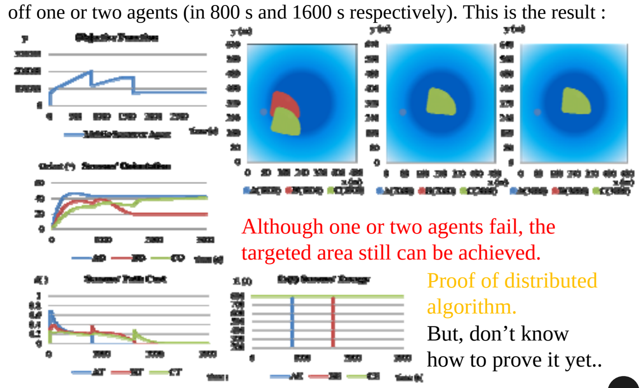
$$\boxed{k_e = \frac{1}{\frac{\partial F}{\partial E_{s_i}} \left(\frac{E_s}{E_{max}}\right)^2}}$$

Based on these theorem,
the objective will be achieved
If the energy of sensor is supplied
based on the value of $\frac{\partial F}{\partial E_{s_i}}$ and $\left(\frac{E_s}{E_{max}}\right)$

It means : 3. Designing Suitable Controller to produce k_e

Failure Cases

During writing the report etc., I'm just curious of the condition of turning off one or two agents (in 800 s and 1600 s respectively). This is the result :



Although one or two agents fail, the targeted area still can be achieved.

Proof of distributed algorithm.

But, don't know how to prove it yet..

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Considering Communication Cost (1)

Modeling

Communication Cost mainly comes from the power/energy consumption for wireless transmission !!

We consider, there are two key energy parameters :

$E_{tx} = \alpha_{11} + \alpha_2 d^n$ $\qquad e(d) = \alpha_{11} + \alpha_2 d^n + \alpha_{12}$

$E_{rx} = \alpha_{12}$

E_{tx}= Transmission Energy/bit $\qquad e(d) = \alpha_1 + \alpha_2 d^n$

E_{rx}= Receiving Energy/bit

α_{11}= Energy/bit consumed by transmitter

α_2 = Energy Dissipated/bit in the transmit op-amp (*10pJ/bit/m^2 for n=2*)

α_{12}= Energy/bit consumed by receiver

$\alpha_1 = \alpha_{12} + \alpha_{11}$ = (*180nJ/bit*)

So that, **Where j is the closest agent or point from i**

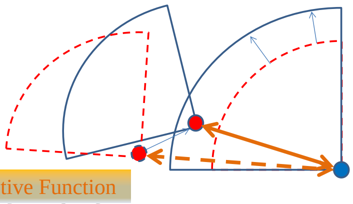
$$\boxed{E_{s_i}^{\cdot}(t) = -w_p k_e \left(\frac{E_{s_i}}{E_{max}}\right)^2 \left(\frac{\partial F}{\partial p_i}\right)^2 - w_c n \alpha_2 \|p_i - p_j\|^{(n-2)} (p_i - p_j)}$$

w_p= Weight for path cost energy consumption **Assume : k_e <<<**

w_c= Weight for communication cost energy consumption

Considering Communication Cost (2)

Hypothesis



New Joint Objective Function

$J(p,\theta) = F(p,\theta) - L(p,\theta)$

$= w_1 \int_Q \phi(q)\, P(q,p,\theta)\,dq - w_2 \sum_{i=1}^{N} r_i(p_i,\theta_i) E_{s_i}(p_i,\theta_i)$

w_1 and w_2 is weighting factor ; can be interpreted into several meanings.

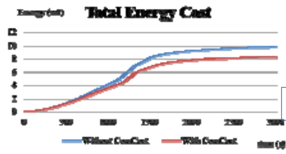
$r_i(p_i,\theta_i) = \int_Q \phi(q) P_i(q,p_i,\theta_i)\,dq$ is event detected frequency of agent i

Considering Communication Cost (3)

Simulation

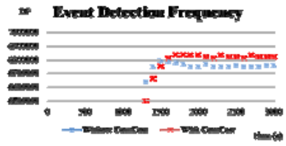
$W_p = W_c$;

$n = 2 ; k_e = 0.1 ; \alpha_1 = 18.10^{-8} ; \alpha_2 = 10.10^{-12} ; w_1 + w_2 = 1$



$$E_{s(Total)} = \sum_{i=1}^{N} \int_0^t E_{s_i}(t)\,dt$$

Reducing around 20% of Energy Cost

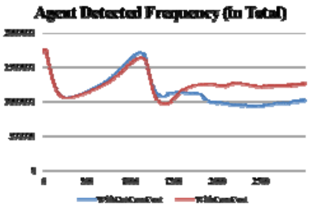


$$EDF_{(Total)} = \sum_{i=1}^{N} r_i(t)$$

Detecting more events

Considering Communication Cost (4)

$$ADF_{(Total)} = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \int_Q \phi(q) P_{ij}(q, p_i, \theta_i, p_j, \theta_j)\,dq$$



Minimizing the communication cost makes the sensing radius performs better for longer time. This condition make the events detection frequency is increasing.

One Agent detect other existing agents more.

In spite of that, in order to perform the best position for minimizing cost. The probability of an agent to be detected by other agent is bigger. Because minimizing communication cost means indirectly push agents to be closer among them.

Considering Communication Cost (4)

Proof

$E_{s_i}\grave{}(t) \approx -\|p_i(t) - p_j(t)\|$;

$0 < \|p_i(t) - p_j(t)\| \leq R_i(E_{s_i}) + R_j(E_{s_j})$;

Optimum condition depends on agents direction.

Since, $\dot{p(t)} = \Delta \frac{\partial J}{\partial p(t)} = \Delta w_1 \frac{\partial F}{\partial p(t)} - \Delta w_2 \frac{\partial L}{\partial p(t)}$ and $\dot{\theta(t)} = \Delta \frac{\partial J}{\partial \theta(t)} = \Delta w_1 \frac{\partial F}{\partial \theta(t)} - \Delta w_2 \frac{\partial L}{\partial \theta(t)}$ and

E_{s_i} is a function of $p_i(t)$,

Then reducing distance d by modifying agent's next movement (as control input) $\dot{p(t)}$ and $\dot{\theta(t)}$ will decrease the amount of energy consumption. And, this condition will make sensing area still well wide. $(q \in Q)$;

Because $r_i(t)$ and $ADF_{(Total)}$ is a function of q then the value of both of them will also increase

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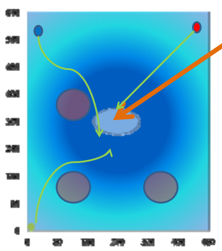
Avoiding Obstacles (1)

Hypothesis

Agents will avoid any obstacles. As the compensation, it needs longer time or trajectory to maximizes the objective function

Field Model



High Density Area : Target

Obstacles

Idea : Defining Obstacles $\zeta_i(q)$ as Density Function

$$\phi_n(q) = \phi(q) - \sum_{i=1}^{N} \zeta_i(q)$$

Avoiding Obstacles (2)

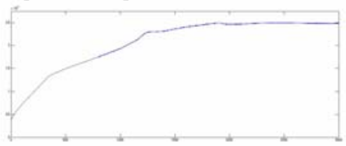
Solution : New Objective Function

$$\boxed{F_R(p,\theta) = w_1 \int_Q \phi_R(q)\, P(q,p,\theta)\, dq}$$

If we want to consider the communication cost and energy also than

The Objective Function will be :

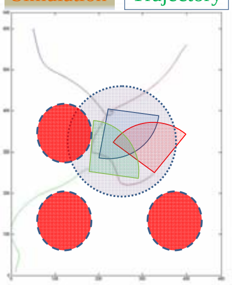
$$J(p,\theta) = F_R(p,\theta) - L(p,\theta)$$



Objective Function

To Evaluate this solution, we will show the trajectory of each agent.

Simulation Trajectory



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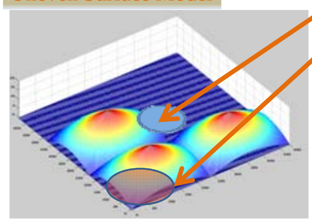
Considering Uneven Surfaces (1)

Hypothesis

Agents will avoid any obstacles (hills or valleys) in order to optimize the motion. As the compensation, it needs longer time or more step to maximizes the objective function. Control Input k_R is needed

Uneven Surface Model



High Density Area : Target

Agents Initial Position

Idea :
Defining Uneven Surfaces $\zeta_i(q)$ as Density Function

$$\boxed{\phi_R(q) = \phi(q) - k_R \sum_{i=1}^{N} \zeta_i(q)}$$

k_R : Control Input

Considering Uneven Surfaces (2)

Solution :

New Objective Function

$$\boxed{F_R(p,\theta) = w_1 \int_Q \phi_R(q)\, P(q,p,\theta)\, dq}$$

To Evaluate this solution, we will consider the cost of path that all of agents through.
This parameter can be noticed by observing the energy status since the cost of path will decrease the amount of energy.

$$E_{S(Total)} = \sum_{i=1}^{N} \int_0^t E_{S_i}(t)\, dt$$

If we want to consider the communication cost and energy also than

The Objective Function will be :

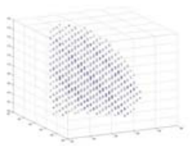
$$J(p,\theta) = F_R(p,\theta) - L(p,\theta)$$

Considering Uneven Surfaces (3)

Simulation

I was thinking to use the idea of representing hills/valleys as negative density function in order to simplify the technique to optimize the path cost by finding the best value of k_R
But,
That idea did not work well. Because, in fact, agent would move like avoiding obstacles only. Agents did not optimize the path cost.

3D Works



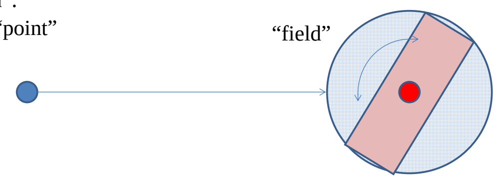
I have problem in selecting step-size.

Considering Simplified Vehicle Model (1)

Hypothesis :

An Agent will not hit other agents by expanding the “point” into “field”.

“point” “field”



Considering Agent’s Orientation/Direction, then this model has some advantages :
1.Avoiding any collision related to rotating motion (orientation)
2.Easier in Programming

Simulation On Developing Progress!!

Conclusion & Future Works

Based on previous work and those works above, we can conclude that :

1. **Each agent is able to meet its duty to cover the targeted area.**
2. **A new control input based on $\frac{\partial F}{\partial E_{S_i}}$ and $\left(\frac{E_S}{E_{max}}\right)$ are developed to guarantee the agents' lifetime and effective motion.**
3. **Small Decreasing of Energy give a better performance.**
4. **Communication Cost has been considered and represented in Energy Model.**
5. **Each Agent can avoid any obstacles.**

Future Works

1. ~~**Designing the optimum pattern of energy station to recharge the energy of agents in order to guarantee the agents lifetime (No need)**~~
2. **Generalizing system to 3 D (Still has a problem with step-size selection and heavy time consuming)**
3. **Optimization in uneven (not flat) regions (need more literature survey)**
4. **Considering the vehicle model (on simulation development)**