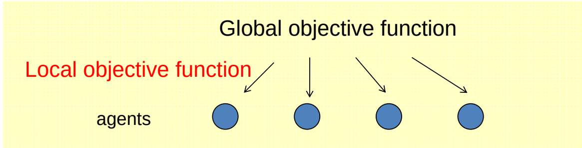
Application of Potential Game for Power Control in Wireless Networks and Network Formation

Goto Tatsuhiko
FL10_04_01
19th,April,2010

Review (Game theoretic approach)



A_i :Action set $\quad A = \prod_{p_i \in p} A_i$:set of joint action

$$a_{-i} = (a_1, a_2, \cdots, a_{i-1}, a_{i+1}, \cdots, a_n)$$

→ joint action $a = (a_i, a_{-i})$

Local objective function $U_i : A \to R$

Control design
1. Designing the player objective function
2. Learning dynamics (repeated game)
   (ex)single stage memory dynamics

Review (Potential game)

Global planner $\phi : A \to \Re$ (potential function)

↓ aligned

Make player's objective function U_i

$$U_i(a_i'', a_{-i}) - U_i(a_i', a_{-i})$$
$$= \phi_i(a_i'', a_{-i}) - \phi_i(a_i', a_{-i})$$

→ Changing in the player's objective function = Changing in the potential function

Every agent select an action to maximize their objective function

Outline

Near-Optimal Power Control in Wireless Networks: A Potential Game Approach

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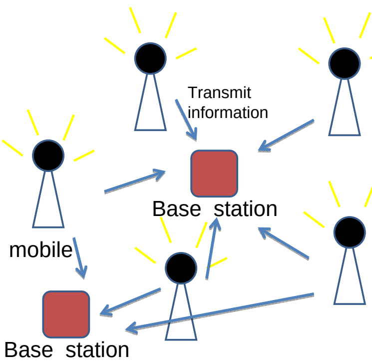
- Background
- Model
- Modified utilities
- Near optimal dynamics
- Convergence analysis
- Simulation Result

background

CDMA system



Mobiles' rates

$$SINR_m(P) = \frac{h_{mm}P_m}{N_0 + \sum_{k \neq m} h_{km}P_k}$$

The signals of other users are noise

rate $r_m(P) = \log(1 + \gamma\, SINR_m(P))$

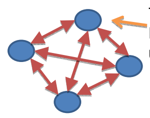
cost $c_m p_m$

net utility $u_m(p) = r_m(p) - \zeta_m c_m p_m$

Maximize net utility

approach

Situation : Self interested
Interfere with each other



Object : Design of power control

Tradeoff
rate ↔ cost
$u_m(p) = r_m(p) - \zeta_m c_m p_m$

→ Centralized optimization problem with full information
→ difficult for complex

↓ General distributed power control

Potential game approach

Original game g → Approximation game $\tilde{g}$

Best response dynamics → Converge within neighborhood of the optimal operating point

Set optimal price c^* → Converge to the optimal operating point

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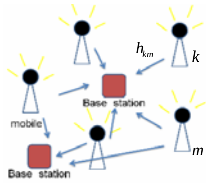
model



mobiles $M=\{1,\cdots,M\}$

Power allocation $P=\{p_1,\cdots,p_M\}$

SINR

$$SINR_m(P)=\frac{h_{mm}P_m}{N_0+\sum_{k\neq m}h_{km}P_k}$$

h_{km} :gain between user k and transmitter m's base station

rate

$$r_m(P)=\log(1+\gamma\, SINR_m(P))$$

cost $c_m p_m$

p_m : transmission power

$$0\le p_{\min m}\le p_m\le \bar{p}_m$$

Objective function (net utility)

$$u_m(p)=r_m(p)-\zeta_m c_m p_m$$

User specific rate vs money Tradeoff coefficient

Power game (definition)

Power game

$$g=<M,\{u_m\}_{m\in M},\{p_m\}_{m\in M}>$$

m's objective function　m's action set

Self interested

$$\max_{\tilde{p}_m\in P_m} u_m(\tilde{p}_m,p_{-m})$$

Nash equilibrium (NE)

$$u_m(p)\ge u_m(\tilde{p}_m,p_{-m})\quad \forall \tilde{p}_m\in p_m,\forall m\in M$$

ε – Nash equilibrium

$$u_m(p)\ge u_m(q_m,p_{-m})-\varepsilon\quad \forall q_m\in p_m,\forall m\in M$$

Central planner wishes to impose some performance objective

System Utility

$$\max_{p\in P} U_0(p)$$

(ex) $U_0(p)=\sum_m r_m(p)$ Sum rate objective

Optimal solution p^* (desired operating point)

Modified utilites

modified Utility

$$\tilde{u}_m(p)=\tilde{r}_m(p)-\zeta_m c_m p_m$$

$$\tilde{r}_m(P)=\log(\gamma\, SINR_m(P))$$

$$r_m(P)=\log(1+\gamma\, SINR_m(P))$$

$$SINR_m(P)=\frac{h_{mm}P_m}{N_0+\sum_{k\neq m}h_{km}P_k}$$

Approximation is good

Spreading gain $\gamma>>1$

or

$h_{mm}>>h_{mk}$

Can make potential function

$$\phi(p)=\sum_m \log(p_m)-\zeta_m c_m p_m$$

$$(\phi(p_m,p_{-m})-\phi(q_m,p_{-m})=\tilde{u}_m(p_m,p_{-m})-\tilde{u}_m(q_m,p_{-m}))$$

Strictly concave → unique NE

Potential game

$$\tilde{g}=<M,\{\tilde{u}_m\}_{m\in M},\{p_m\}_{m\in M}>$$

Assigning prices

Assigning prices c^* to coincide with NE of $\tilde{g}$ and p^*

$$\tilde{u}_m(p)=\tilde{r}_m(p)-\zeta_m c_m p_m$$

[Theorem]

Let p^* be the desired operating point .Then the prices c^* are given by

$$c_m^*=(\zeta_m p_m^*)^{-1}\qquad m\in M$$

(proof) $\phi(p)$ Strictly concave → unique NE

→ Maxima of $\phi(p)$ is NE

$$\phi(p)=\sum_m \log(p_m)-\zeta_m c_m^* p_m$$

$$c_m^*=(\zeta_m p_m^*)^{-1}$$

$$\frac{\partial\phi}{\partial p_m}=\frac{1}{p_m}-\frac{1}{p_m^*}\ \Rightarrow\ p=p^*\rightarrow\frac{\partial\phi}{\partial p_m}=0$$

⇒ p^* Global maximum of the potential

Near optimal dynamics

p^* is not NE of the game g with c^*

→ Converge neighbor of p^* ?

Best Response dynamics

$$p_m\leftarrow p_m+\alpha(\beta_m(p_{-m})-p_m)$$

Best Response $\beta_m(p_{-m})=\arg\max_{p_m\in P_m} u_m(p_m,p_{-m})$

most good action for user m

$$\dot{p}_m=\beta_m(p_{-m})-p_m$$

α is small

$\tilde{g}$ with $c=c^*$ → (BR) Converge to p^* (Lyapunov analysis)

How about g ?

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- Background
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Convergence analysis

Best Response of $\tilde{g}$ $\quad \tilde{\beta}_m(p_{-m}) = \arg\max_{p_m \in P_m} \tilde{u}_m(p_m, p_{-m})$

$= \arg\max_{p_m \in P_m} \phi(p_m, p_{-m})$ (From PG)

ε -equilibria of $\tilde{g}$ $\quad \tilde{I}_\varepsilon = \{ p \mid \tilde{u}_m(p_m, p_{-m}) \ge \tilde{u}_m(q_m, p_{-m}) - \varepsilon \}$

[Lemma]

The BR in g converge to $\tilde{I}_\varepsilon$ $\quad \varepsilon \le \frac{1}{\gamma} \sum_{m \in M} \frac{1}{SINR_{\min m}}$

$SINR_{\min m}(P) = \frac{h_{mm} P_{\min m}}{N_0 + \sum_{k \ne m} h_{km} P_{\max k}}$

(proof) $\bar{\phi}$: maximum value of ϕ

$V = \bar{\phi} - \phi \ge 0$: Lyapunov function

proof

$$-\dot{V} = \sum_{m \in M} \frac{\partial \phi}{\partial p_m}\left(\tilde{\beta}_m(p_{-m}) - p_m\right) + \sum_{m \in M} \frac{\partial \phi}{\partial p_m}\left(\beta_m(p_{-m}) - \tilde{\beta}_m(p_{-m})\right)$$

$$\sum_{m \in M} \frac{\partial \phi}{\partial p_m}\left(\tilde{\beta}_m(p_{-m}) - p_m\right) \ge \tilde{u}_m\left(\tilde{\beta}_m(p_{-m}), p_{-m}\right) - \tilde{u}_m(p_m, p_{-m})$$

$$\left| \frac{\partial \phi}{\partial p_m}\left(\beta_m(p_{-m}) - \tilde{\beta}_m(p_{-m})\right) \right| \le \left(\frac{1}{p_{\min m}} - \frac{1}{\overline{p}_m} \right) \frac{\xi_m}{\gamma}$$

$$\left(\xi_m = \frac{N_0}{h_{mm}} + \sum_{k \ne m} \frac{h_{km}}{h_{mm}} \overline{p}_k \right)$$

$$-\dot{V} \ge \sum_{m \in M}\left(\tilde{u}_m\left(\tilde{\beta}_m(p_{-m}), p_{-m}\right) - \tilde{u}_m(p_m, p_{-m})\right) - \sum_{m \in M}\left(\frac{1}{p_{\min m}} - \frac{1}{\overline{p}_m} \right) \frac{\xi_m}{\gamma}$$

$$\sum_{m \in M}\left(\tilde{u}_m\left(\tilde{\beta}_m(p_{-m}), p_{-m}\right) - \tilde{u}_m(p_m, p_{-m})\right) \ge \sum_{m \in M} \frac{\xi_m}{\gamma p_{\min m}}$$

$$\dot{V} \le -\sum_{m \in M} \frac{\xi_m}{\gamma \overline{p}_m}$$

proof

$$\sum_{m \in M}\left(\tilde{u}_m\left(\tilde{\beta}_m(p_{-m}), p_{-m}\right) - \tilde{u}_m(p_m, p_{-m})\right) \ge \sum_{m \in M} \frac{\xi_m}{\gamma p_{\min m}}$$

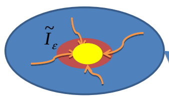
$$\dot{V} \le -\sum_{m \in M} \frac{\xi_m}{\gamma \overline{p}_m}$$

Converge to this set from Lyapunov method

$$\tilde{u}_m\left(\tilde{\beta}_m(p_{-m}), p_{-m}\right) - \tilde{u}_m(p_m, p_{-m}) \le \sum_{m \in M} \frac{\xi_m}{\gamma p_{\min m}}$$

$$\varepsilon \le \frac{1}{\gamma} \sum_{m \in M} \frac{1}{SINR_{\min m}}$$

$$\left(\tilde{I}_\varepsilon = \{ p \mid \tilde{u}_m(p_m, p_{-m}) \ge \tilde{u}_m(q_m, p_{-m}) - \varepsilon \} \right)$$



$$\dot{V} \le -\sum_{m \in M} \frac{\xi_m}{\gamma \overline{p}_m}$$

How far ?

How far the set of ε- equilibria of $\tilde{g}$ from p^*?

[Theorem] $\quad \left| \tilde{p}_m - p_m^* \right| \le \overline{P}_m \sqrt{2\varepsilon} \qquad \tilde{p} \in \tilde{I}_\varepsilon$

ε -equilibria of $\tilde{g}$ — Global maximum of the potential

$\tilde{I}_\varepsilon = \{ p \mid \tilde{u}_m(p_m, p_{-m}) \ge \tilde{u}_m(q_m, p_{-m}) - \varepsilon \}$

(proof)

$$\phi\left(p_m^*, \tilde{p}_{-m}\right) - \phi\left(\tilde{p}_m, \tilde{p}_{-m}\right) \le \varepsilon$$

$$\rightarrow \left(\log(p_m^*) - \lambda_m p_m^*\right) - \left(\log(\tilde{p}_m) - \lambda_m \tilde{p}_m\right) \le \varepsilon \qquad \left(\phi(p) = \sum_m \log(p_m) - \zeta_m c_m^* p_m \right)$$

$$\rightarrow f_m(p_m^*) - f_m(\tilde{p}_m) \le \varepsilon \qquad \left(f_m = \log(p_m) - \lambda_m p_m \right)$$

proof

$$f_m = \log(p_m) - \lambda_m p_m$$

$$\rightarrow f_m(\tilde{p}_m) = f_m(p_m^*) + (\tilde{p}_m - p_m^*)\frac{\partial f_m(p_m^*)}{\partial p_m} + \frac{1}{2}(p_m^* - \tilde{p}_m)^2 \frac{\partial^2 f_m(p_m^* + \alpha(\tilde{p}_m - p_m^*))}{\partial p_m^2}$$

p^* Is desired operating point

$$\rightarrow \frac{\partial f_m(p_m^*)}{\partial p_m} = \frac{\partial \phi(p_m^*)}{\partial p_m} = 0$$

$$\rightarrow f_m(p_m^*) - f_m(\tilde{p}_m) = \frac{1}{2}(p_m^* - \tilde{p}_m)^2 \frac{1}{(p_m^* + \alpha(\tilde{p}_m - p_m^*))^2}$$

$$\rightarrow 2\left(p_m^* + \alpha(\tilde{p}_m - p_m^*)\right)^2\left(f_m(p_m^*) - f_m(\tilde{p}_m)\right) = (p_m^* - \tilde{p}_m)^2$$

$$\rightarrow 2\varepsilon \overline{P}_m^2 \ge \left(p_m^* - \tilde{p}_m\right)^2 \qquad \left(f_m(p_m^*) - f_m(\tilde{p}_m) \le \varepsilon, 0 < p_m^*, \tilde{p}_m \le \overline{p}_m \right)$$

$$\rightarrow \left| \tilde{p}_m - p_m^* \right| \le \overline{P}_m \sqrt{2\varepsilon}$$

Near optimal performance

Near optimal performance in terms of system utility

Performance loss decrease with small ε
increase with large L, L_m

(ex) $U_0(p) = \sum_m r_m(p)$
Sum rate objective

[Theorem] Let $\varepsilon \le \frac{1}{\gamma}\sum_{m\in M}\frac{1}{SINR_{\min m}}$ then

(1) U_0 is Lipschitz continuous function ,with L . Then

$$|U_0(p^*) - U_0(\tilde{p})| \le \sqrt{2\varepsilon}L\sqrt{\sum_{m\in M} \bar{P}_m^2}$$

(2) Assume that U_0 is a continuous differentiable function such that

$\left|\frac{\partial U_0}{\partial p_m}\right| \le L_m$ Then $|U_0(p^*) - U_0(\tilde{p})| \le \sqrt{2\varepsilon}\sum_{m\in M}\bar{P}_m L_m$

Difference between p^* and $\tilde{p}$ ➡ Difference between $U_0(p^*)$ and $U_0(\tilde{p})$

Outline

- Background
- Model
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- Simulation Result

Simulation result

・Three users

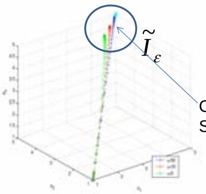
・desired operating point $p^* = [5,5,5]$

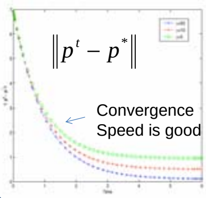
$r_m(P) = \log(1+\gamma SINR_m(P))$ $\gamma \in \{5,10,15\}$

$SINR_m(P) = \frac{h_{mm}P_m}{N_0 + \sum_{k\ne m} h_{km}P_k}$

$N_0 = 1$ $h_{km} \in [0,2]$ $h_{mm} \in [2,4]$

Do the BR dynamics with c^*





➡ Big γ is good $\varepsilon \le \frac{1}{\gamma}\sum_{m\in M}\frac{1}{SINR_{\min m}}$

➡ Monotonously decreasing

Simulation result (system utility)

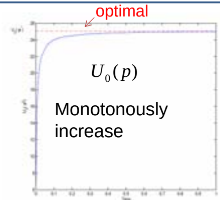
Sum rate objective

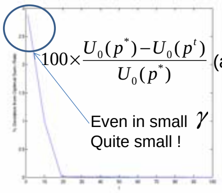
$U_0(p) = \sum_m r_m(p)$ ➡ $\max_{p\in P} U_0(p)$

[Theorem]
U_0 is a continuous differentiable function
$\left|\frac{\partial U_0}{\partial p_m}\right| \le \frac{M-1}{p_{\min m}}$ ➡

$$|U_0(p^*) - U_0(\tilde{p})| \le \sqrt{2\varepsilon}(M-1)\sum_{m\in M}\frac{\bar{P}_m}{P_{\min m}}$$

$$\left(|U_0(p^*) - U_0(\tilde{p})| \le \sqrt{2\varepsilon}\sum_{m\in M}\bar{P}_m L_m\right)$$





General type

u_m → ϕ → $\hat{u}_m$

Find the most close potential function of the game by u_m

$$d^2(g) = \min_{\phi\in C_0}\left\|\delta_0\phi - \sum_{m\in M} D_m u^m\right\|_2^2$$

Find the most close objective function of ϕ to u_m

$$\hat{u}^m = \operatorname{argmin}_{\bar{u}^m}\left\|u^m - \bar{u}^m\right\|_2^2$$

$$D_m\bar{u}_m = D_m\phi$$

D_m : Difference operator

$(D_m\phi)(p,q) = W^n(p,q)(\phi(q) - \phi(p))$ $W^n(p,q) = 1$ $p \ne q$ $W^n(p,q) = 0$ $p = q$

Potential game
$D_m u_m = D_m\phi$

→ $\sum_{m\in M} D_m u_m = \sum_{m\in M} D_m\phi = \delta_0\phi$

combinatorial gradient operator $\delta_0 = \sum_{m\in M} D_m$

Theorem of Projection

[Theorem] Optimal projection

$$\phi = \left(\sum_{m\in\mathcal{M}}\Pi_m\right)^\dagger \sum_{m\in\mathcal{M}}\Pi_m u^m$$

$$\hat{u}^m = (I - \Pi_m)u^m + \Pi_m\left(\sum_{k\in\mathcal{M}}\Pi_k\right)^\dagger \sum_{k\in\mathcal{M}}\Pi_k u_k. \quad \Pi_m = D_m^* D_m$$

[Theorem]

Any equilibrium of $\tilde{g}$ is an ε–equilibrium of g .

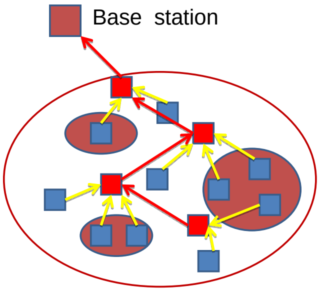
$\varepsilon \le \sqrt{2d(g)}$ $d^2(g) = \min_{\phi\in C_0}\left\|\delta_0\phi - \sum_{m\in M} D_m u^m\right\|_2^2$

g : game

$\tilde{g}$: projection of the game g

$u_m(p) \ge u_m(q_m, p_{-m}) - \varepsilon$

Coverage considering wireless network



Searching agent

Information transmission agent

Information from (blue) to (red)

Information from (red) to (red)

Object : optimal coverage and
make small transmission cost of information
→ Several Information transmission agent needed

Agent can change from (blue) to (red) and from (red) to (blue)

Outline

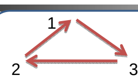
Distributed Dynamic Reinforcement of Efficient Outcomes in Multiagent Coordination and Network Formation*

Georgios C. Chasparis[†] and Jeff S. Shamma[‡]

November 7, 2009

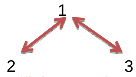
- Background
- Reinforcement learning
- Asymptotic stability analysis
- Dynamic Reinforcement
- Asymptotic stability of RADR
- Simulation Result

Distributed network formation



→ Efficient network

Want to make it!



Non-efficient nash network

Reward

$$R_i(\alpha)=\sum_{s\in I, s\neq i}\chi_\alpha(i\to s)-\kappa_0|\alpha_i|$$

connection reward cost

$\chi_\alpha(i\leftarrow s)=1$: g_α contain the path $(i\leftarrow s)$

$\chi_\alpha(i\leftarrow s)=0$: otherwise

Actions of agent

(ex) $A_1=\{\{1\},\{2\},\{3\},\{2,3\}\}$

Nash network

α^* Nash network ⟺ $R_i(\alpha_i^*,\alpha_{-i}^*)\geq R_i(\alpha_i',\alpha_{-i}^*)\quad \alpha_i'\in A_i\setminus\alpha_i^*$

wheel network

Connected network is uniquely defined by a path $(i\leftarrow i)$

→ Every agent realizes its maximum possible utility

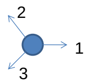
Reinforcement learning

Learning algorithm

$$x_i(k+1)=x_i(k)+\varepsilon(k)R_i(\alpha(k))[\alpha_i(k)-x_i(k)]$$

Propensities of action — reward — Action at time k

(ex)



$x_i(k)=\begin{bmatrix}0.2\\0.3\\0.5\end{bmatrix}$ → Do action 1 and get reward 1.5 → $x_i(k+1)=\begin{bmatrix}0.2\\0.3\\0.5\end{bmatrix}+\frac{1.5}{k+1}\begin{bmatrix}0.8\\-0.3\\-0.5\end{bmatrix}$

Probability of action selection

$$\sigma_i(k)=(1-\lambda)x_i(k)+\lambda\frac{1}{|A_i|}\mathbf{1}$$

Random selection

$\varepsilon(k)=\frac{1}{k+1}$
Propensities are proportional to the cumulative reward

→ $x_i(k)$ evolves over the probability simplex $\Delta(|A_i|)\quad(\Delta(m)=\{v\in\Re^m\mid v\geq 0, 1^T v=1\})$

Asymptotic stability analysis

$$x_i(k+1)=x_i(k)+\varepsilon(k)R_i(\alpha(k))[\alpha_i(k)-x_i(k)]$$

$$\bar{g}_i(x(k))=\bar{r}_i(x(k))-\bar{R}_i(x(k))x_i(k)$$

$\bar{r}_i(x(k))=E[R_i(\alpha(k))\alpha_i(k)\mid x(k)]\quad \bar{R}_i(x(k))=E[R_i(\alpha(k))\mid x(k)]$

$$\xi_i(k)=R_i(\alpha(k))[\alpha_i(k)-x_i(k)]-\bar{g}_i(x(k))$$

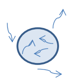
$$x_i(k+1)=x_i(k)+\varepsilon(k)[\bar{g}_i(x(k))+\xi_i(k)]$$

deterministic — noise

[proposition3.1] $(\lambda>0)$

$$x(k+1)=x(k)+\varepsilon(k)[\bar{g}(x(k))+\xi(k)]\quad x(k)=\{x_1(k),\cdots,x_n(k)\}$$

$x(k)$ convergence to an invariant set of the $\dot{x}=\bar{g}(x)$
$A\subset\Delta$ be a locally Asymptotically stable set for $\dot{x}=\bar{g}(x)$ → $\Pr[\lim_{k\to\infty}x(k)\in A]>0$



Asymptotic stability analysis

$\dot{x}=\bar{g}(x)$ → Stationary point $S=\{x\in X:\bar{g}(x)=\bar{r}(x)-\bar{R}(x)x=0\}$

[proposition3.2] $(\lambda>0)$

x^* linearly unstable stationary point of $\dot{x}=\bar{g}(x)$

→ $\Pr[\lim_{k\to\infty}x(k)=x^*]=0$

Expected reward $\bar{v}_i(j,x^*)=E[R_i(\alpha)\mid\alpha_i=j,x_{-i}=x_{-i}^*]\quad x^*\in S$

[proposition3.3] (Stationary point)

x^* stationary point of $\dot{x}=\bar{g}(x)$ ⟷ $\bar{v}_i(j,x^*)=c_i\quad j\in A_i$
Expected reward not change by one agent

proposition

[proposition3.4] (pure strategies) $(\lambda=0)$

pure strategies profile → stationary point of $\dot{x}=\bar{g}(x)$

[proposition3.5] (sensitivity of pure strategies) $(\lambda>0)$

x^* pure strategies profile and strict NE → Exist a differentiable function $\upsilon^*(\lambda)$,such $\tilde{x}=x^*+\upsilon^*(\lambda)$ stationary point of $\dot{x}=\bar{g}(x)$ $(\lim_{\lambda\to 0}\upsilon^*(\lambda)=\upsilon^*(0)=0)$

[proposition3.6] (LAS) $(\lambda>0)$

$\tilde{x}$ Locally asymptotically Stable point of $\dot{x}=\bar{g}(x)$ ↔ $\bar{v}_i(j^*,\tilde{x})>\bar{v}_i(s,\tilde{x}) \quad \forall s\in A_i\setminus j^*$, $x_i^*=e_{j^*}$

[proposition3.7] $(\lambda>0)$

$\bar{v}_i(j^*,\tilde{x})>\bar{v}_i(s,\tilde{x}) \quad \forall s\in A_i\setminus j^*$ $\tilde{x}$:strict NE → $\Pr[\lim_{k\to\infty}x(k)=\tilde{x}]>0$ (from p3.6)

$\bar{v}_i(j^*,x^*)<\bar{v}_i(s,x^*) \quad \exists s\in A_i\setminus j^*$ x^*:not NE → $\Pr[\lim_{k\to\infty}x(k)\in B_\delta(x^*)]=0$

Neighbor of x^*

Outline

- Background
- Reinforcement learning
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- Simulation Result

Dynamic Reinforcement

Before depend only on the probability distribution → Also affected by the history of x_i

→ Goal is to investigate the effects on convergence to an Efficient pure equilibrium

Learning algorithm

$$x_i(k+1)=x_i(k)+\varepsilon(k)R_i(\alpha(k))[\alpha_i(k)-x_i(k)]$$

Propensities of action — reward — Action at time k

Probability of action selection

$$\sigma_i(k)=\Pi_\Delta[(1-\lambda)(x_i(k)+u_i(k))+\lambda\frac{1}{|A_i|}1] \qquad \Pi_\Delta[x]=\arg\min_{s\in\Delta(m)}|x-s| \qquad \varepsilon(k)=\frac{1}{k+1}$$

Correspond to history of x_i

$u_i(k)=\gamma_i(\rho_i(k))(x_i(k)-y_i(k))$ $\gamma_i(\rho_i(k))$: RADR parameter

$y_i(k+1)=y_i(k)+\varepsilon(k)(x_i(k)-y_i(k))$ Running average of x_i

$\rho_i(k+1)=\rho_i(k)+\varepsilon(k)(R_i(\alpha(k))-\rho_i(k))$ Running average of $R_i(\alpha(k))$

Asymptotic stability of RADR

$$z(k)=\begin{pmatrix}x(k)\\y(k)\\\rho(k)\end{pmatrix} \xrightarrow{\text{Relevant ODE}} \begin{pmatrix}\dot{x}\\\dot{y}\\\dot{\rho}\end{pmatrix}=\begin{pmatrix}\bar{g}(z)\\x-y\\\bar{R}(z)-\rho\end{pmatrix} \xrightarrow{\text{linearization}} \frac{d}{dt}\begin{pmatrix}\delta x(t)\\\delta y(t)\\\delta\rho(t)\end{pmatrix}=\tilde{A}^{\lambda,\gamma}\begin{pmatrix}\delta x(t)\\\delta y(t)\\\delta\rho(t)\end{pmatrix}$$

α is depend on y,ρ

$x_i(t)=\tilde{x}_i+N\delta x_i(t)$, $y_i(t)=\tilde{y}_i+N\delta y_i(t)$, $\delta\rho(t)=\rho(t)-\tilde{\rho}$

[Theorem 4.1] (LAS of RADR)

equilibrium $\tilde{z}=(\tilde{x},\tilde{y},\tilde{\rho})$ is LAS point of linearization ↔ RADR parameter $0\le\gamma_i(\tilde{\rho}_i(k))\le\frac{\bar{v}_i(j^*,\tilde{x})+1}{\bar{v}_i(s,\tilde{x})}-1 \quad \forall s\ne j^*$

[Theorem 4.2] (RADR convergence)

$0\le\gamma_i(\tilde{\rho}_i(k))\le\frac{\bar{v}_i(j^*,\tilde{x})+1}{\bar{v}_i(s,\tilde{x})}-1 \quad \forall s\ne j^*,\forall i$ → $\Pr[\lim_{k\to\infty}x(k)=\tilde{x}]>0$

$\gamma_i(\tilde{\rho}_i(k))\ge\frac{\bar{v}_i(j^*,\tilde{x})+1}{\bar{v}_i(s,\tilde{x})}-1 \quad \exists s\ne j^*,\exists i$ → $\Pr[\lim_{k\to\infty}x(k)=\tilde{x}]=0$

example

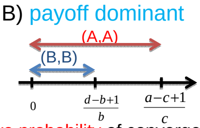
	2.A	2.B
1.A	a,a	b,c
1.B	c,b	d,d

Symmetric coordination game

(case1) $a>c,d>b,(a-c)>(d-b),a<d$

→ (A,A) risk dominant
(B,B) payoff dominant

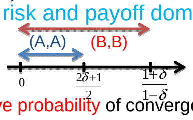
$0\le\gamma_i(\tilde{\rho}_i(k))\le\frac{\bar{v}_i(j^*,\tilde{x})+1}{\bar{v}_i(s,\tilde{x})}-1 \quad \forall s\ne j^*$



→ $\frac{d-b+1}{b}<\gamma<\frac{a-c+1}{c}$ Positive probability of convergence to (A,A)
Zero probability of convergence to (B,B)

(case2) $a=2+2\delta,b=1-\delta,c=2,d=1$

→ (A,A) risk and payoff dominant



$\frac{2\delta+1}{2}<\gamma<\frac{\delta+1}{1-\delta}$ → Positive probability of convergence to (B,B)
Zero probability of convergence to (A,A)

Example 2

	2.A	2.B
1.A	a_1,a_2	b,c
1.B	c,b	d_1,d_2

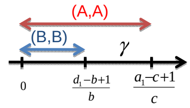
Asymmetric coordination game

$a_1>a_2>0,d_1=a_2,d_2=a_1,b=c>0,a_1>c,d_1>b$

→ (A,A) Desirable for agent 1 $(a_1<a_2)$
(B,B) Desirable for agent 2 $(a_1>a_2)$

Only agent 1

$\frac{d_1-b+1}{b}<\gamma<\frac{a_1-c+1}{c}$ →



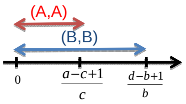
Positive probability of convergence to (A,A)
Zero probability of convergence to (B,B)

→ The agent that applies RADR destabilizes the less Desirable equilibrium in favor of the desirable one

Payoff dependent γ

	2.A	2.B
1.A	a,a	b,c
1.B	c,b	d,d

Symmetric coordination game



Risk dominant but not Pay off dominant can't destabilized with constant γ

$$a>c>0, d>b>0, (a-c)<(d-b), a>d$$

(A,A) payoff dominant
(B,B) risk dominant
can't be destabilized

$$\gamma_i(\rho_i)=\frac{\gamma_0}{\rho_i^k}$$ RADR parameter

$$\kappa>\frac{\log\left(\frac{a-c+1}{d-b+1}\frac{b}{c}\right)}{\log\left(\frac{d}{a}\right)} \qquad d^{\kappa}\frac{d-b+1}{b}<\gamma_0<a^{\kappa}\frac{a-c+1}{c}$$

Positive probability of convergence to (A,A)

Zero probability of convergence to (B,B)

The risk dominant equilibrium is no longer stable in favor of the payoff dominant equilibrium

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simulation

3 agent

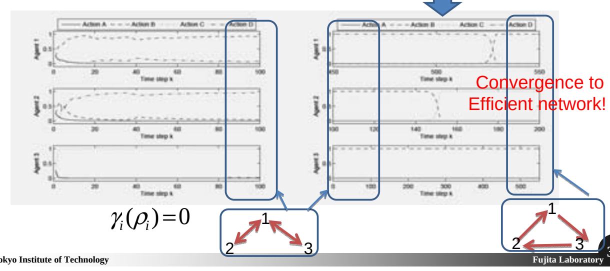
Actions of agent

$$A_i=\{A,B,C,D\}$$
$$A_1=\{\{1\},\{2\},\{3\},\{2,3\}\}$$
$$A_2=\{\{1\},\{2\},\{3\},\{1,3\}\}$$
$$A_3=\{\{1\},\{2\},\{3\},\{2,1\}\}$$

$$\gamma_i(\rho_i)=\frac{\gamma_0}{\rho_i^k}$$ RADR parameter

$$\kappa_0=\frac{1}{2} \qquad \gamma_i\in(2/3, 3/2)$$

Non-efficient Nash network is unstable



$$\gamma_i(\rho_i)=0$$