Coverage Control for Mobile Networks with Limited-Range Anisotropic Sensors in Limited-Energy Condition

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Outline

- ◆Introduction
- ◆Literature Survey
- ◆Basic Theory
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- ◆Proposed Idea
- ◆Developing Simulations
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Introduction

Background

Coordinated Networked Control; Practically, this research field will be much needed in human life.

The deployment of autonomous vehicles could perform tasks such as :

1. Search and Recovery Operations.
2. Manipulation in hazardous environments.
3. Surveillance
4. Environmental Monitoring.

Goal

To drive the sensors/agents to the position such that a given region is optimally covered by the sensors

Literature Survey

A. Gusrialdi et.al (2008) : "Coverage Control for Mobile Networks with Limited-Range Anisotropic Sensors"

•Coverage control problem solved by limited-range anisotropic sensor
•Using Probabilistic approach

A. Kwok et.al (2007) : "Energy-balancing cooperative strategies for sensor deployment"

•Coverage algorithm for mobile sensor networks with various energy initialization and different control laws.
•Using Weighted Metrics to the corresponding generalized Voronoi.
•Locational Optimization Problems are solved by cost function

S. Martinez et.al (2007) : "Motion Coordination with Distributed Information"

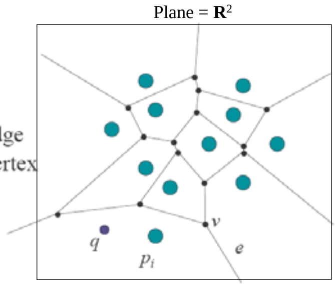
•Surveying methods to model spatially distributed problems, encode various coordination tasks
•Through appropriate cost functions, analyze stability and convergence properties, and design motion-coordination schemes.

Basic Theory - Original Voronoi Diagram

The Properties

Plane = $\mathbf{R}^2$

p_i : site points
q : free point
e : Voronoi edge
v : Voronoi vertex



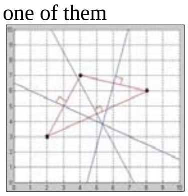
A *partition* of $\mathbb{R}^N$ is a collection of n polytopes $\mathcal{A} = \{A_1, \ldots, A_n\}$ with disjoint interiors whose union is $\mathbb{R}^N$.

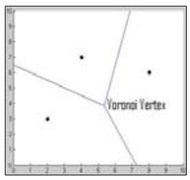
Basic Theory - Original Voronoi Diagram

The Methods :

There are a lot of methods to develop an original Voronoi Diagram.

This is one of them





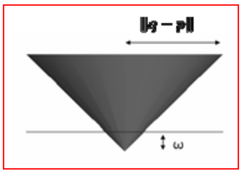
1. Pull the a line between two adjacent sites.
2. Then, pull the perpendicular line of line (1) and through the middle of the line (1). *See Left Picture.
3. Erase a line which is in the area of a sites. *See Right Picture

Basic Theory-Weighted Metrics

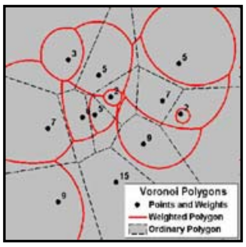
- The area of a site which is bordered by Voronoi edge can be transformed or change by modifying the function of the sensing radius of a site.
- The sensing radius can be modified by using the weighted metrics,

The following weighted metrics yield non-equivalent Voronoi Partitions.

1. Multiplicatively Weighted
   $d_w(q,p) = \frac{1}{w}\|q-p\|$
2. Additively Weighted
   $d_w(q,p) = \|q-p\| - w$
3. Power Metrics
   $d_w(q,p) = \|q-p\|^2 - w$

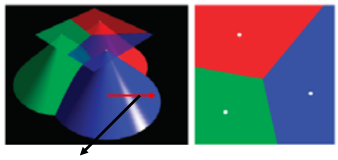


Basic Theory - Original & Generalized Voronoi Diagram



- Original Voronoi Diagram :

$V(p_i) = \{q \in \mathbb{R}^N \,|\, \|q - p_i\| \le \|q - p_j\|, \forall j \ne i\}.$
$\pi(P) = \{V(p_1), \ldots, V(p_n)\}, where\ for\ i \in \{1, \ldots, n\}.$



This radius is changed..

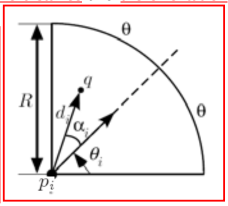
- Generalized Voronoi Diagram :

$V(p_i) = \{q \in \mathbb{R}^N \,|\, d_w(q,p_i) \le d_w(q,p_j), \forall j \ne i\}.$
$d_w(q,p) = \|q - p\|^2 - w$ => Power Metrics (example)

Basic Theory - Sensor Model Formulation

- Most of the sensors such as camera, radar etc have anisotropic characteristic,
- We consider the coverage problem with an anisotropic sensor model.
- The performance of this model depends on the distance and the orientation to the target.

✔ Let Q be a polyhedron in $\mathcal{R}^2$
✔ Let R be the maximum sensing radius of an agent
✔ Let $p = (p_1 \ldots \ldots p_N)$be the location of the agents moving in the region
✔ Let $\theta = (\theta_1, \ldots \ldots, \theta_N)$be the orientation of the agents moving in the region



The kinematic model of the agents are given by :

$p_i(k+1) = p_i(k) + u_i(k)$
$\theta_i(k+1) = \theta_i(k) + v_i(k)$

k is the iteration index, $u_i(k)$ and $v_i(k)$ are the control input of sensor i

Basic Theory - Sensor Model Formulation

The sensory domain :

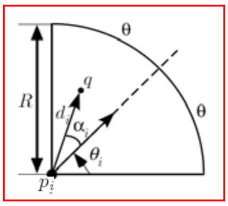
$Q_i = \{q \in Q : d_i \le R \wedge |\alpha_i| \le \theta\}$

where

$d_i = \|q - p_i\| \ge 0$

$\alpha_i = \cos^{-1}\left(\frac{(q - p_i)(\cos\theta_i, \sin\theta_i)}{\|q - p_i\|}\right)$

$\theta \in \left(0, \frac{\pi}{2}\right]$



Assumption :

R => All sensors have constant and identical sensing radius

R is maximum sensing radius

$P_i(q) = 0, \frac{\partial P_i(q)}{\partial d_i(q)} = 0, \frac{\partial P_i(q)}{\partial \alpha_i(q)} = 0\ if\ q \notin Q_i$

=> The sensor can sense only in its region

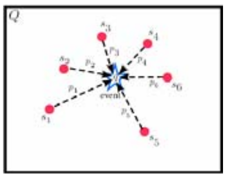
$$P_i(q) = \begin{cases} \frac{(d_i - R)^2(\alpha_i - \theta)^2}{R^2\theta^2} & if\ q \in Q_i \\ 0 & otherwise \end{cases}$$

=> Sensor model used in simulation

Basic Theory - Optimization Problem

$$P(q,p,\theta) = 1 - \prod_{i=1}^{N}[1 - P_i(q)]$$

⇒Joint Probability that this event (q) is detected by the others agents.



The optimal coverage problem can be formulated as an optimization problem of maximizing the objective function defined by

$$F(p,\theta) = \int_Q \phi(q).P(q,p,\theta)dq$$

where : $\phi(q)$ = density function

Basic Theory - Gradient Flows Approach

$F(p,\theta) = \int_Q \phi(q).P(q,p,\theta).dq$ ....(1.

Optimization Problem Formula

$\frac{\partial F}{\partial p_i} = \int_Q \phi(q).\frac{\partial P(q,p,\theta)}{\partial p_i}dq$

$\frac{\partial F}{\partial p_i} = \int_Q \phi(q).\prod_{k\in N_i}[1 - P_k(q)].\left(\frac{\partial P_i(q)}{\partial d_i}.\frac{\partial d_i}{\partial p_i} + \frac{\partial P_i(q)}{\partial \alpha_i}.\frac{\partial \alpha_i}{\partial p_i}\right).dq$ ....(2.

Small changes of agents position will affect the optimization value.

$\frac{\partial F}{\partial \theta_i} = \int_Q \phi(q).\frac{\partial P(q,p,\theta)}{\partial \theta_i}dq$

Small changes of agents orientation will affect the optimization value.

$\frac{\partial F}{\partial \theta_i} = \int_Q \phi(q).\prod_{k\in N_i}[1 - P_k(q)].\left(\frac{\partial P_i(q)}{\partial \alpha_i}.\frac{\partial \alpha_i}{\partial \theta_i}\right).dq$ ....(3

So that, the result of equations (2 and (3 respectively are :

$$\frac{\partial F}{\partial p_i} = \int_Q \phi(q).\prod_{k\in N}[1 - P_k(q)].\left(\frac{2.(d_i - R).(\alpha_i - \theta)^2}{R^2\theta^2}.\frac{p_i - q}{m} + \frac{2.(\alpha_i - \theta).(d_i - R)^2}{R^2\theta^2}.\frac{z}{m^2\sqrt{m^2 - l^2}}((q - p_i)_y, -(q - p_i)_x.)\right).dq$$

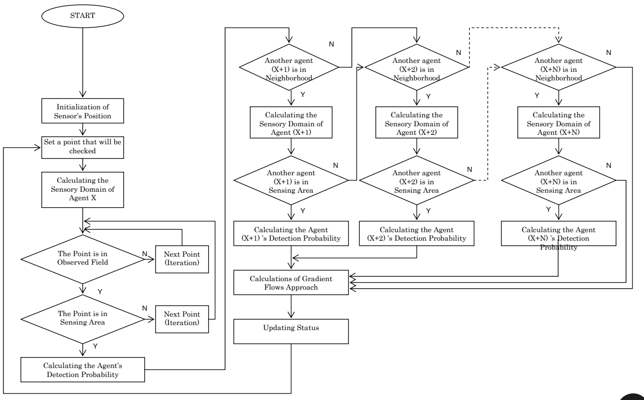
$\therefore m = \|q - p_i\| = \sqrt{(q - p_i)_x{}^2 + (q - p_i)_y{}^2}$

$$\frac{\partial F}{\partial \theta_i} = \int_Q \phi(q).\prod_{k\in N}[1 - P_k(q)].\left(-\frac{2.(d_i - R).(\alpha_i - \theta)^2}{R^2\theta^2}.\frac{z}{\sqrt{m^2 - l^2}}\right).dq$$

$\therefore z = (q - p_i).(-\sin\theta_i, \cos\theta_i)$

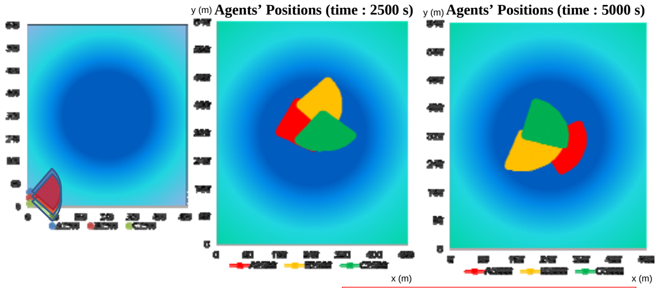
$l = (q - p_i).(\cos\theta_i, \sin\theta_i)$

Simulations - Original Algorithm



Simulation of Previous Work - Result I (1)

Assumptions : Infinite Energy Supplies



$\phi(q) = 40 - 0.1\|q - A_{max}\|$

where $A_{max} = (240,320)$

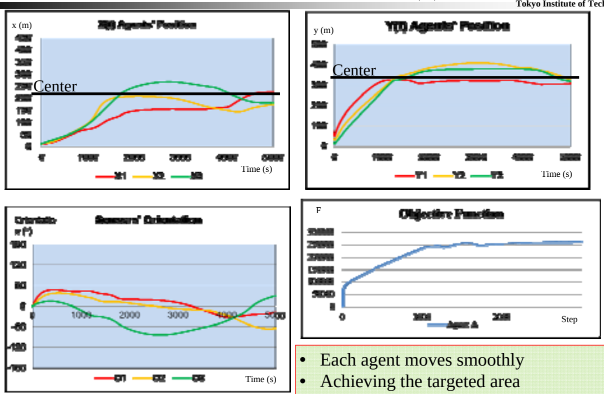
Initial Condition: Agent (X, Y, Orientation)

A (10,50,0°) ; B (10,30, 0°) ; C (10,10, 0°)

Goal

To drive the sensors/agents to the position such that a given region is optimally covered by the sensors

Simulation - Result I (2)



- Each agent moves smoothly
- Achieving the targeted area
- Objective function is maximized

Proposed Idea (1)

- Consider the limited energy consumption of each agent.
- Using the concept of weighted metrics,
- Calculate the relationship between the energy consumption and sensing performance.

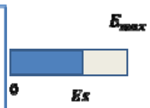
$$d_w(q,p) = \frac{1}{w}\|p - q\| \rightarrow \textit{Multiplicatively Weighted}$$

*Since energy related to the sensors' performance, then the maximum value of radius of sensory domain will be influenced by the energy supplies $R(Es)$.

The following equations will describe the idea :

$$R(Es) = \frac{Es}{E_{max}} R_{max}$$

Where
R_{max} is max sensing radius
R is sensing radius
Es is Energy Contained
E_{max} is Maximum Energy



Proposed Idea (2)

which will give an effect on the following sensor model :

$$P_i(q) = \frac{(d_i - R(Es))^2(\alpha_i - \theta)^2}{R(Es)^2\theta^2} \quad if\ q \in Q_i$$

Assume :
- The energy will be decreasing proportional to the trajectory

Before:

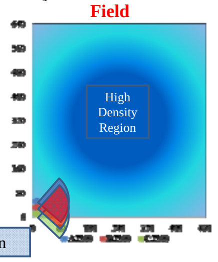
$$P_i(q) = \begin{cases} \frac{(d_i - R)^2(\alpha_i - \theta)^2}{R^2\theta^2} & if\ q \in Q_i \\ 0 & otherwise \end{cases}$$

$$E(t) = -k_e \frac{\|dp_i(t)\|}{dt}$$

Effect :
- The sensing radius will be proportional to the energy contained

$$R(Es) = \frac{Es}{E_{max}} R_{max}$$

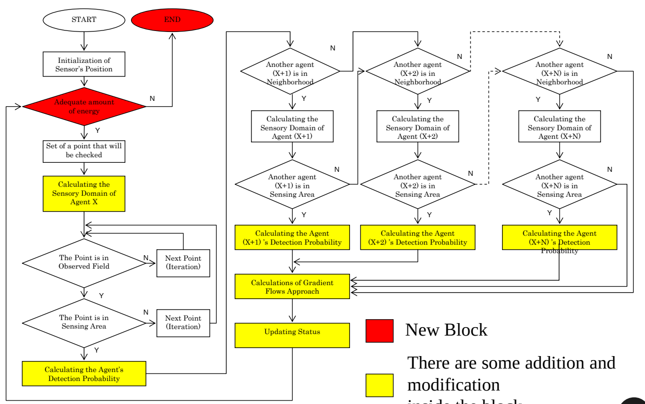
Field



FROM NOW
Assumptions : Finite Energy Supplies

Initial Condition

Developing Simulation - Algorithm

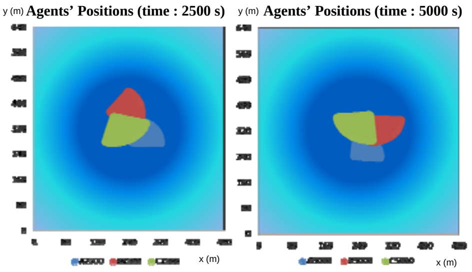


New Block

There are some addition and modification inside the block

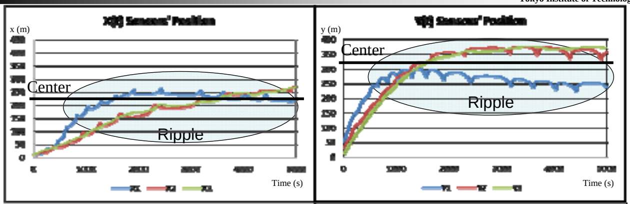
Developing Simulation - Result I (1)

Initial Condition :
Agent
(X, Y, Orient, Energy)



Analysis :
Sensors are still covering the targeted area very well although their sensing radius are decreasing

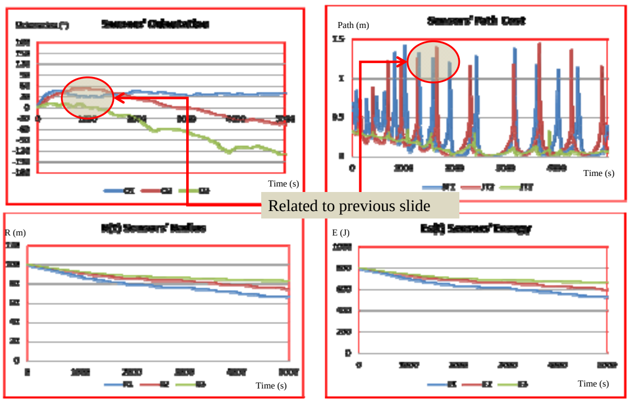
Developing Simulation - Result I (2)



Analysis :
•Sensors are moving ineffectively. Those movement will cost more energy.
•Objective Function is close to maximized

Possible Root Causes :
1.There is a problem in the algorithm
2.Gradient Flows should consider the time or energy

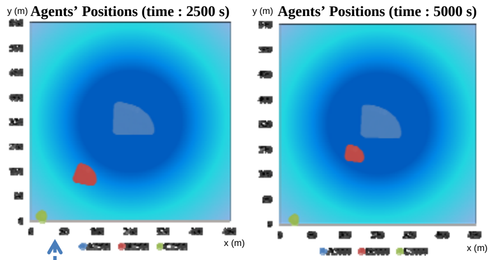
Developing Simulation - Result I (3)



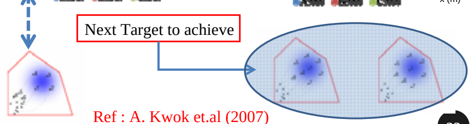
Developing Simulation - Result II (1)

Assumptions : Finite Energy Supplies and Different Initial Energy

Initial Condition :
Agent (X, Y, Orient, Energy)

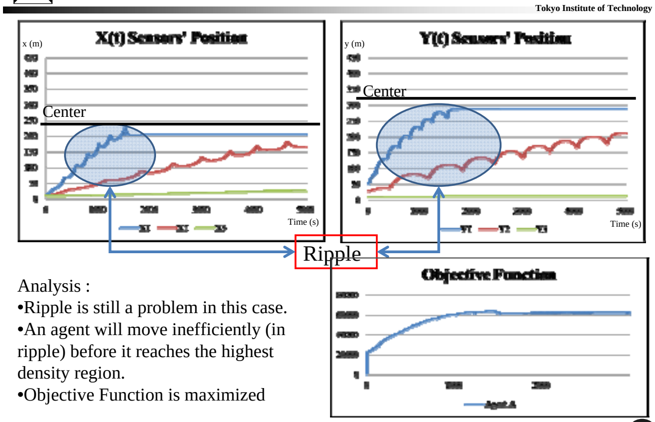


Analysis :
Low Energy Agents are almost impossible to reach the high density region. So that, the agents lifetime are not guaranteed



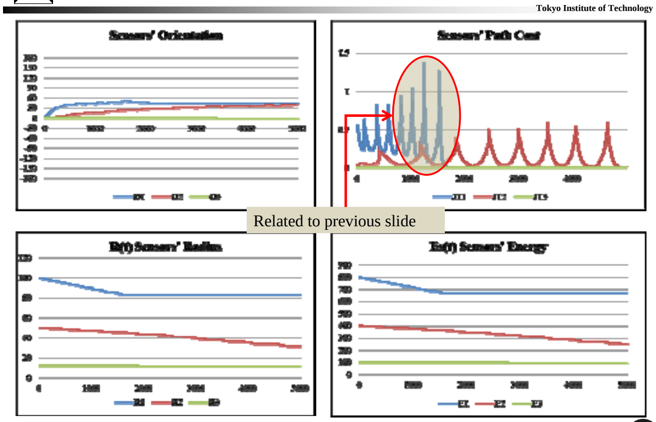
Ref : A. Kwok et.al (2007)

Developing Simulation - Result II (1)



Analysis :
•Ripple is still a problem in this case.
•An agent will move inefficiently (in ripple) before it reaches the highest density region.
•Objective Function is maximized

Developing Simulation - Result II (3)



Conclusion - Summary

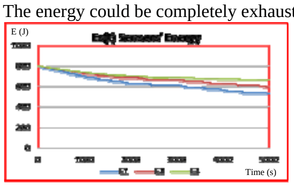
Based on those specifications (previous work), each agent is able to work together and cover the targeted area at its optimum.

With a slightly modified algorithm,
•Each sensor is able to meet its duty to cover the target area,
•It appears the new problems associated with lifetime of each sensor and,
•Motion sensors that are not effective with respect to energy consumption.

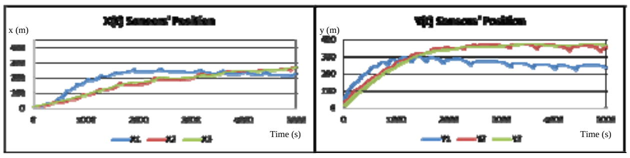
These problems will be the focus for settlement on the future work.

Conclusion - Problem to Tackle

1. The energy could be completely exhausted. (Agents lifetime are not guaranteed)



2. There are quite significant ripples in agents movement.



3. This simulation is only valid in flat region

Proposed Solution and Future Works (1)

1. Reducing any ripples which is occurred on sensors/agents movement (there is no problem in the algorithm) :
   There are at least 2 alternatives :
   a. Designing new sensor model approach related to position and energy contained (currently on progress)

   $$P(p_i, E_n) = \frac{R^2 - d_i^2(E_{max} + 1 - E_n)}{R^2} \rightarrow \dot{E}_n(t) = -\frac{4}{R^2\|E_{max} + 1 - E_n\|^2}\frac{\partial P}{\partial p_i}$$

   b. Designing new control system which is suitable to this new condition
2. Designing the optimum pattern of energy station to recharge the energy of agents in order to guarantee the agents lifetime

3. Generalizing the energy consumption equations with respect to three dimensional region in order to satisfied odd (not flat) regions

4. Considering the vehicle model

Reference

- **S. Martinez**, F. Bullo, J. Cortes, "Motion coordination with distributed information," *IEEE Control Systems Magazine*, Vol. 27, No. 4, pp. 75- 88, 2007.
- **A. Kwok** and S. Martinez, "Energy-balancing cooperative strategies for sensor deployment," *Proc. of IEEE Conference on Decision and Control*, pp. 6136-6141, 2007.
- **A. Gusrialdi,** T. Hatanaka and M. Fujita, "Coverage Control for Mobile Networks with Limited-Range Anisotropic Sensors" *Proc. of the 47th IEEE Conference on Decision and Control*, pp. 4263-4268, 2008
- **K. Laventall** and J. Cortes, "Coverage control by robotic networks with limited-range anisotropic sensory," *Proc. of the American Control Conference*, pp. 2666-2671, 2008.

Appendix (1)

$$\frac{\partial d_i}{\partial p_i} = \frac{\partial\|q-p_i\|}{\partial p_i} = \frac{\partial\|q-p_i\|}{\partial(q-p_i)} \cdot \frac{\partial(q-p_i)}{\partial p_i} \longrightarrow -1$$

$$\frac{\partial d_i}{\partial p_i} = -\frac{\partial\|q-p_i\|}{\partial(q-p_i)}$$

$$\frac{\partial d_i}{\partial p_i} = -\left(\frac{\partial(\sqrt{(q-p_i)_x^2 + (q-p_i)_y^2})}{\partial(q-p_i)_x}, \frac{\partial(\sqrt{(q-p_i)_x^2 + (q-p_i)_y^2})}{\partial(q-p_i)_y}\right)$$

$$\frac{\partial d_i}{\partial p_i} = -\left(\frac{2(q-p_i)_x}{2\sqrt{(q-p_i)_x^2 + (q-p_i)_y^2}}, \frac{2(q-p_i)_x}{2\sqrt{(q-p_i)_x^2 + (q-p_i)_y^2}}\right)$$

$$\frac{\partial d_i}{\partial p_i} = -\left(\frac{2(q-p_i)_x}{2m}, \frac{2(q-p_i)_x}{2m}\right)$$

$$\frac{\partial d_i}{\partial p_i} = -\frac{(q-p_i)}{m} = \frac{(p_i-q)}{m} \longrightarrow \therefore m = \|q-p_i\| = \sqrt{(q-p_i)_x^2 + (q-p_i)_y^2}$$

$$\frac{\partial d_i}{\partial p_i} = \frac{(p_i-q)}{\|q-p_i\|}$$

Appendix (2)

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{\partial\left(\cos^{-1}\left(\frac{(q-p_i).(\cos\theta_i, \sin\theta_i)}{\|q-p_i\|}\right)\right)}{\partial p_i} \longrightarrow \frac{\partial(\cos^{-1} x)}{\partial p_i} = -\frac{1}{\sqrt{1-x^2}} \longleftarrow \therefore (|x| < 1)$$

$$\frac{\partial \alpha_i}{\partial p_i} = -\frac{1}{\sqrt{1 - \frac{((q-p_i).(\cos\theta_i, \sin\theta_i))^2}{\|q-p_i\|^2}}} \cdot \frac{\partial\left(\frac{(q-p_i).(\cos\theta_i, \sin\theta_i)}{\|q-p_i\|}\right)}{\partial p_i}$$

$$\frac{\partial \alpha_i}{\partial p_i} = -\frac{1}{\sqrt{1 - \frac{l^2}{m^2}}} \cdot \frac{\partial\left(\frac{(q-p_i)_x \cos\theta_i + (q-p_i)_y .\sin\theta_i}{\sqrt{(q-p_i)_x^2 + (q-p_i)_y^2}}\right)}{\partial(q-p_i)} \cdot \frac{\partial(q-p_i)}{\partial p_i} \longrightarrow l = (q-p_i).(\cos\theta_i, \sin\theta_i)$$

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{m}{\sqrt{m^2 - l^2}} \cdot \left(\frac{\partial\left(\frac{(q-p_i)_x \cos\theta_i + (q-p_i)_y .\sin\theta_i}{\sqrt{(q-p_i)_x^2 + (q-p_i)_y^2}}\right)}{\partial(q-p_i)_x}, \frac{\partial\left(\frac{(q-p_i)_x \cos\theta_i + (q-p_i)_y .\sin\theta_i}{\sqrt{(q-p_i)_x^2 + (q-p_i)_y^2}}\right)}{\partial(q-p_i)_y}\right)$$

Appendix (3)

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{m}{\sqrt{m^2 - l^2}} \left(\frac{\cos\theta_i . m - (q - p_i).(\cos\theta_i, \sin\theta_i).\frac{(q-p_i)_x}{m}}{m^2}, \frac{\sin\theta_i . m - (q - p_i).(\cos\theta_i, \sin\theta_i).\frac{(q-p_i)_y}{m}}{m^2} \right)$$

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{1}{m^2\sqrt{m^2 - l^2}} \left(\cos\theta_i . m^2 - (q - p_i).(\cos\theta_i, \sin\theta_i).(q-p_i)_x, \sin\theta_i . m^2 - (q - p_i).(\cos\theta_i, \sin\theta_i).(q-p_i)_y \right)$$

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{1}{m^2\sqrt{m^2 - l^2}} \begin{pmatrix} (q-p_i)_x{}^2.\cos\theta_i + (q-p_i)_y{}^2.\cos\theta_i - (q-p_i)_x{}^2.\cos\theta_i - (q-p_i)_x.(q-p_i)_y \sin\theta_i, \\ (q-p_i)_x{}^2.\sin\theta_i + (q-p_i)_y{}^2.\sin\theta_i - (q-p_i)_y{}^2.\sin\theta_i - (q-p_i)_x.(q-p_i)_y \cos\theta_i \end{pmatrix}$$

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{1}{m^2\sqrt{m^2 - l^2}} \begin{pmatrix} (q-p_i)_y{}^2.\cos\theta_i - (q-p_i)_x.(q-p_i)_y \sin\theta_i, \\ (q-p_i)_x{}^2.\sin\theta_i - (q-p_i)_x.(q-p_i)_y \cos\theta_i \end{pmatrix}$$

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{1}{m^2\sqrt{m^2 - l^2}} \begin{pmatrix} (q-p_i)_y \left((q-p_i)_y.\cos\theta_i - (q-p_i)_x.\sin\theta_i\right), \\ (q-p_i)_x.\left((q-p_i)_x.\sin\theta_i - (q-p_i)_y \cos\theta_i\right). \end{pmatrix}$$

Appendix (4)

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{1}{m^2\sqrt{m^2 - l^2}} \begin{pmatrix} (q-p_i)_y \left((q-p_i).(-\sin\theta_i, \cos\theta_i)\right), \\ -(q-p_i)_x.\left((q-p_i).(-\sin\theta_i, \cos\theta_i)\right). \end{pmatrix}$$

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{1}{m^2\sqrt{m^2 - l^2}} \begin{pmatrix} (q-p_i)_y z, \\ -(q-p_i)_x.z. \end{pmatrix}$$

$$\frac{\partial \alpha_i}{\partial p_i} = \frac{z}{m^2\sqrt{m^2 - l^2}} \left((q-p_i)_y, -(q-p_i)_x.\right)$$

$$\therefore z = (q - p_i).(-\sin\theta_i, \cos\theta_i)$$

$$\frac{\partial P_i(q)}{\partial d_i} = \frac{\partial\left(\frac{(d_i - R)^2.(\alpha_i - \theta)^2}{R^2\theta^2}\right)}{\partial d_i}$$

$$\frac{\partial P_i(q)}{\partial d_i} = \frac{2.(d_i - R).(\alpha_i - \theta)^2}{R^2\theta^2}$$

$$\frac{\partial P_i(q)}{\partial \alpha_i} = \frac{\partial\left(\frac{(d_i - R)^2.(\alpha_i - \theta)^2}{R^2\theta^2}\right)}{\partial \alpha_i}$$

$$\frac{\partial P_i(q)}{\partial \alpha_i} = \frac{2.(\alpha_i - \theta).(d_i - R)^2}{R^2\theta^2}$$

Appendix (5)

$$\frac{\partial \alpha_i}{\partial \theta_i} = \frac{\partial\left(\cos^{-1}\left(\frac{(q-p_i).(\cos\theta_i, \sin\theta_i)}{\|q-p_i\|}\right)\right)}{\partial \theta_i}$$

$$\frac{\partial \alpha_i}{\partial \theta_i} = -\frac{1}{\sqrt{1 - \frac{((q-p_i).(\cos\theta_i, \sin\theta_i))^2}{\|q-p_i\|^2}}} . \frac{\partial\left(\frac{(q-p_i).(\cos\theta_i, \sin\theta_i)}{\|q-p_i\|}\right)}{\partial \theta_i}$$

$$\frac{\partial \alpha_i}{\partial \theta_i} = -\frac{1}{\sqrt{1 - \frac{l^2}{m^2}}} . \frac{\partial\left(\frac{(q-p_i)_x \cos\theta_i + (q-p_i)_y.\sin\theta_i}{\sqrt{(q-p_i)_x{}^2 + (q-p_i)_y{}^2}}\right)}{\partial \theta_i}$$

$$\frac{\partial \alpha_i}{\partial \theta_i} = -\frac{m}{\sqrt{m^2 - l^2}} . \frac{\partial\left(\frac{(q-p_i)_x \cos\theta_i + (q-p_i)_y.\sin\theta_i}{m}\right)}{\partial \theta_i}$$

Appendix (6)

$$\frac{\partial \alpha_i}{\partial \theta_i} = -\frac{1}{\sqrt{m^2 - l^2}} . \frac{\partial\left((q-p_i)_x \cos\theta_i + (q-p_i)_y.\sin\theta_i\right)}{\partial \theta_i}$$

$$\frac{\partial \alpha_i}{\partial \theta_i} = -\frac{1}{\sqrt{m^2 - l^2}} (q - p_i).(-\sin\theta_i, \cos\theta_i)$$

$$\frac{\partial \alpha_i}{\partial \theta_i} = -\frac{z}{\sqrt{m^2 - l^2}}$$