Passivity-based Pose Synchronization with Visual Motion Observer

-Analysis of Convergence and Performance-

Tatsuya Ibuki

FL09-15-1

20th, November, 2009

Introduction

Cooperative Control

A distributed control strategy that achieves specified tasks in multi-agent system

Previous Works

Pose(Attitude) Synchronization: Igarashi *et al.*

[1] "Passivity-based Output Synchronization and Flocking Algorithm in SE(3)," Proc. Of the 47th IEEE Conference on Decision and Control, pp. 1024–1029, 2008.

- No Discussion of How to Get Necessary Information

Visual Motion Observer: Fujita *et al.*

[2] "Passivity-based Dynamic Visual Feedback Control for Three Dimensional Target Tracking: Stability and L2-gain Performance Analysis," *IEEE Trans. on Control System Technology*, Vol. 15, No. 1, pp. 40-52, 2007.

- A Nonlinear Observer for Estimating Relative Pose in 3D

Pose Synchronization with VMO: Kobayashi *et al.*

[3] "Visual Motion Observer-based Pose Synchronization: A Passivity Approach," *Proc. of the 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference* (to appear)

- No Proof for Closed Loop System

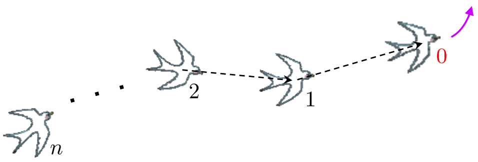
➡ Analysis of Convergence and Control Performance

Problem Setting

Pose Synchronization (Leader Following Type)

Leader doesn't move: All rigid bodies convergent to the leader's pose

Leader moves: Total error of poses is restricted (Control Performance)



Assumption

- Chain Type Information Structure
- Bidirectional Information Structure (Undirected Graph)
- There is leader 0

(A1) Each rigid body's rotation matrix is positive definite

(A2) Estimation error related to orientation is sufficiently small

Outline

Rigid Body Motion

Rigid Body Motion

$$\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi} \quad (1)$$

$$g_{wi} = \begin{bmatrix} e^{\hat{\xi}\theta_{wi}} & p_{wi} \\ 0 & 1 \end{bmatrix} \in \mathcal{R}^{4\times 4}$$

$$V^b_{wi} = \begin{bmatrix} v^b_{wi} \\ \omega^b_{wi} \end{bmatrix} \text{ : Input} \qquad \hat{V}^b_{wi} = \begin{bmatrix} \hat{\omega}^b_{wi} & v^b_{wi} \\ 0 & 0 \end{bmatrix}$$

$p_{wi} \in \mathcal{R}^3$: Position

$e^{\hat{\xi}\theta_{wi}} \in SO(3)$: Orientation

$v^b_{wi} \in \mathcal{R}^3$: Body Linear Velocity

$\omega^b_{wi} \in \mathcal{R}^3$: Body Angular Velocity

"$\wedge$" (wedge) : $\mathcal{R}^3 \to so(3)$

$$\hat{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}^\wedge = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

"$\vee$" (vee) : $so(3) \to \mathcal{R}^3$

Relative Rigid Body Motion

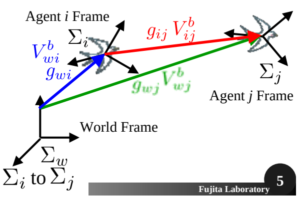
$$V^b_{ij} = V^b_{wj} - \mathrm{Ad}_{(g_{ij}^{-1})} V^b_{wi} \quad (2)$$

Adjoint Transformation

$$\mathrm{Ad}_{(g_{ij})} = \begin{bmatrix} e^{\hat{\xi}\theta_{ij}} & \hat{p}_{ij} e^{\hat{\xi}\theta_{ij}} \\ 0 & e^{\hat{\xi}\theta_{ij}} \end{bmatrix} \in \mathcal{R}^{6\times 6}$$

➡ $(g_{ij}^{-1}\hat{V}^b_{wi} g_{ij})^\vee = \mathrm{Ad}_{(g_{ij}^{-1})} V^b_{wi}$

$\mathrm{Ad}_{(g_{ij}^{-1})}$: Coordinate Transformation from Σ_i to Σ_j



Passivity of Rigid Body Motion

Rigid Body Motion

$$\dot{g}_{wi} = g_{wi}\hat{V}^b_{wi} \quad (1)$$

$$e_{cij} = \begin{bmatrix} p_{ij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix} \qquad e_{wi} = \begin{bmatrix} e^{-\hat{\xi}\theta_{wi}} & 0 \\ 0 & e^{-\hat{\xi}\theta_{wi}} \end{bmatrix} e_{cwi}$$

Lemma 1 (Passivity)

The rigid body motion (1) satisfies

$$\int_0^T (V^b_{wi})^T e_{wi} dt \geq -\beta_i, \ \forall T > 0$$

where β_i is a positive scalar.

$$\mathrm{sk}(e^{\hat{\xi}\theta_{ij}}) = \frac{1}{2}(e^{\hat{\xi}\theta_{ij}} - e^{-\hat{\xi}\theta_{ij}})$$

$\mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee = 0$ ➡ $e^{\hat{\xi}\theta_{ij}} = I_3$

Storage Function

$$E(g_{ij}) := \frac{1}{2}\|p_{ij}\|^2 + \phi(e^{\hat{\xi}\theta_{ij}}) \geq 0$$

$$\phi(e^{\hat{\xi}\theta_{ij}}) = \frac{1}{2}\mathrm{tr}(I_3 - e^{\hat{\xi}\theta_{ij}})$$

Relative Rigid Body Motion (RRBM)

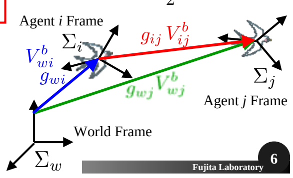
$$V^b_{ij} = V^b_{wj} - \mathrm{Ad}_{(g_{ij}^{-1})} V^b_{wi} \quad (2)$$

Lemma 2 (Passivity)

If rigid body j is static $(V^b_{wj} = 0)$, then RRBM (2) satisfies

$$\int_0^T (V^b_{wi})^T (-e_{cij}) dt \geq -\beta_{ij}, \ \forall T > 0$$

where β_{ij} is a positive scalar.



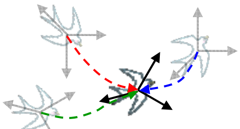
Pose Synchronization

Pose Synchronization

$$\lim_{t\to\infty} E(g_{ij}) = 0 \ \forall i,j \in \{1,\cdots,n\} \quad (3)$$

$E(g_{ij}) = 0$ ➡ $g_{wi}^{-1} g_{wj} = I_4$ ➡ $g_{wi} = g_{wj}$

Relative Pose $g_{wi}^{-1} g_{wj}$



Control Input for Pose Synchronization

$$V_{wi}^b = -K_{ci}(-e_{cij}) = K_{ci} \sum_{j\in\mathcal{N}_i} \begin{bmatrix} p_{ij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee \end{bmatrix} \quad (4)$$

$K_i = \begin{bmatrix} k_{pi}I_3 & 0 \\ 0 & k_{ai}I_3 \end{bmatrix}$ $k_{pi} > 0$, $k_{ai} > 0$

$\mathcal{N}_i$: i's neighbors

Lemma 3
Consider the n rigid bodies represented by (1). Then, under the assumption A1, the velocity input (4) achieves pose synchronization in the sense of (3).

Lyapunov Function Candidate

$$E_c' = \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i} \left(\frac{1}{2k_{pi}} \|p_{wi}\|^2 + \frac{1}{k_{ai}} \phi(e^{\hat{\xi}\theta_{wi}}) \right) \ ➡ \ E(g_{ij}) = \frac{1}{2}\|p_{ij}\|^2 + \phi(e^{\hat{\xi}\theta_{ij}})$$

Estimation Error System

Actual Pose g_{ij} : *not measurable*

Relative Rigid Body Motion

$$V_{ij}^b = V_{wj}^b - \mathrm{Ad}_{(g_{ij}^{-1})} V_{wi}^b$$

➡ Estimated Pose $\bar{g}_{ij}$

Relative Rigid Body Motion Model

➡ $\bar{V}_{ij}^b = V_{wj}^b - \mathrm{Ad}_{(\bar{g}_{ij}^{-1})} u_{eij}$

u_{eij} : Input for Estimation Error

Estimation Error

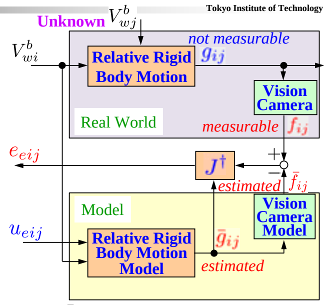
$$g_{eij} = \bar{g}_{ij}^{-1} g_{ij}$$

$g_{eij} = I_4$ ➡ $\bar{g}_{ij} = g_{ij}$

Estimation Error Vector

$$e_{eij} = \begin{bmatrix} p_{eij} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{eij}})^\vee \end{bmatrix} \begin{matrix} \text{(Position)} \\ \text{(Orientation)} \end{matrix}$$

$e_{eij} = 0$ ➡ $g_{eij} = I_4$



$f_{ij}, \bar{f}_{ij}$: Feature Point (Estimated)

J : Image Jacobian

Estimation Error System

$$V_{eij}^b = -\mathrm{Ad}_{(g_{eij}^{-1})} u_{eij} + V_{wj}^b \quad (5)$$

Visual Motion Observer

Lemma 4 (Passivity)
If agent j is static $(V_{wj}^b = 0)$, then the estimation error system (5) satisfies

$$\int_0^T u_{eij}^T(-e_{eij})dt \geq -\beta_{eij}, \ \forall T > 0$$

where β_{eij} is a positive scalar.

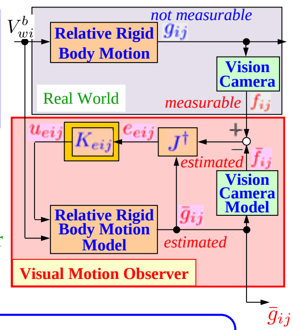
Storage Function

$$E(g_{eij}) := \frac{1}{2}\|p_{eij}\|^2 + \phi(e^{\hat{\xi}\theta_{eij}}) \geq 0$$

Control Law for Visual Motion Observer

$$u_{eij} = -K_{eij}(-e_{eij}) = K_{eij}e_{eij} \quad (6)$$

$K_{eij} = k_{eij}I_6, \ k_{eij} > 0$



Lemma 5 (Visual Motion Observer)
If agent j is static $(V_{wj}^b = 0)$, then the equilibrium point $e_{eij} = 0$ for the closed-loop system (5) and (6) is asymptotically stable.

Outline

- Introduction
- Previous Work
- Pose Synchronization with VMO: 2 Rigid Bodies
  - Convergence
  - Control Performance
- Pose Synchronization with VMO: 3 Rigid Bodies
- Pose Synchronization with VMO: n Rigid Bodies
- Future Works

Pose Synchronization with VMO (2 Rigid Bodies)

Pose Synchronization + Visual Motion Observer

$$\begin{bmatrix} V_{10}^b \\ V_{e10}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(g_{10}^{-1})} & 0 \\ 0 & -\mathrm{Ad}_{(g_{e10}^{-1})} \end{bmatrix} \begin{bmatrix} V_{w1}^b \\ u_{e10} \end{bmatrix} + \begin{bmatrix} V_{w0}^b \\ V_{w0}^b \end{bmatrix} \quad (7)$$

Relative Rigid Body Motion + Estimation Error System

Input $u_1 := \begin{bmatrix} V_{w1}^b \\ u_{e10} \end{bmatrix}$ Velocity Input / Observer's Input

Error $e_1 := \begin{bmatrix} e_{c10} \\ e_{e10} \end{bmatrix}$ Control Error / Estimation Error

$e_{e10} = 0$ ➡ $\bar{g}_{10} \to g_{10}$

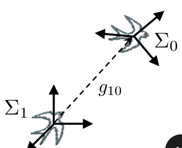
$e_{c10} = 0$ ➡ $g_{w1} \to g_{w0}$

$\left[\mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee = 0, |\theta_{ij}| < \pi \right.$ ➡ $\left. e^{\hat{\xi}\theta_{wi}} = e^{\hat{\xi}\theta_{wj}} \right]$ A2

Lemma 6 (Passivity)
If the leader is static $(V_{w0}^b = 0)$, then system (7) satisfies

$$\int_0^T u_1^T(-e_1)dt \geq -\beta_1$$

where β_1 is a positive scalar.



Proof of Passivity (2 Rigid Bodies)

Proof:

Storage Function

$$E_1 := E(g_{10}) + E(g_{e10})$$

Sum of Storage Function

$$E(g_{ij}) = \frac{1}{2}\|p_{ij}\|^2 + \phi(e^{\hat{\xi}\theta_{ij}})$$

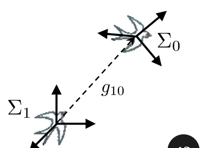
$$\phi(e^{\hat{\xi}\theta_{ij}}) = \frac{1}{2}\mathrm{tr}(I_3 - e^{\hat{\xi}\theta_{ij}})$$

$$\dot{\phi}(e^{\hat{\xi}\theta_{ij}}) = (\mathrm{sk}(e^{\hat{\xi}\theta_{ij}})^\vee)^T (e^{-\hat{\xi}\theta_{ij}} \dot{e}^{\hat{\xi}\theta_{ij}})^\vee$$

Time derivative:

$$\begin{aligned} \dot{E}_1 &= \frac{d}{dt}\left(\frac{1}{2}\|p_{ij}\|^2 + \phi(e^{\hat{\xi}\theta_{ij}}) + \frac{1}{2}\|p_{eij}\|^2 + \phi(e^{\hat{\xi}\theta_{eij}}) \right) \\ &= \dot{p}_{10}^T p_{10} + (\mathrm{sk}(e^{\hat{\xi}\theta_{10}})^\vee)^T (e^{-\hat{\xi}\theta_{10}} \dot{e}^{\hat{\xi}\theta_{10}})^\vee \\ &\quad + \dot{p}_{e10}^T p_{e10} + (\mathrm{sk}(e^{\hat{\xi}\theta_{e10}})^\vee)^T (e^{-\hat{\xi}\theta_{e10}} \dot{e}^{\hat{\xi}\theta_{e10}})^\vee \\ &= (V_{w1}^b)^T(-e_{c10}) + u_{e10}^T(-e_{e10}) \\ &= -\left[(V_{w1}^b)^T \ \ u_{e10}^T \right] \begin{bmatrix} e_{c10} \\ e_{e10} \end{bmatrix} \\ &= u_1^T(-e_1) \end{aligned}$$

□



Pose Synchronization with VMO (2 Rigid Bodies)

Control Input for Pose Synchronization with VMO : Passivity Approach

$$u_1 = -K_1(-e_1) = K_1 e_1 \quad (8) \qquad K_1 = \begin{bmatrix} k_{c1} I_6 & 0 \\ 0 & k_{e1} I_6 \end{bmatrix} \; k_{c1}, k_{e1} > 0$$

$e_{c10} : g_{10} = \bar{g}_{10} g_{e10}$

Include Estimation Error $\quad e_1 = \begin{bmatrix} e_{c10} \\ e_{e10} \end{bmatrix} \quad e_{c10} = \begin{bmatrix} p_{10} \\ \mathrm{sk}(e^{\hat{\xi}\theta_{10}})^{\vee} \end{bmatrix}$

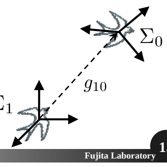
➡ $k_{e1} > k_{c1}$ is prefer

Theorem 1
If the leader is static $(V^b_{w0} = 0)$, then the equilibrium point $e_1 = 0$ for the closed-loop system (7) and (8) is asymptotic stable.

Proof:

$$\begin{aligned} \dot{E}_1 &= u_1^T(-e_1) \\ &= (-K_1(-e_1))^T(-e_1) \\ &= -e_1^T K_1 e_1 \\ &< 0 \; (\because K_1 > 0) \end{aligned}$$

□



Control Performance (2 Rigid Bodies)

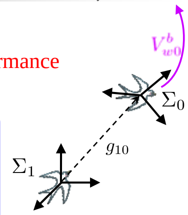
Velocity of Leader V^b_{w0} : Disturbance

$d_0 := \begin{bmatrix} V^b_{w0} \\ V^b_{w0} \end{bmatrix}$ ➡ $e_1 = \begin{bmatrix} e_{c10} \\ e_{e10} \end{bmatrix}$ L_2 Gain Performance

Theorem 2 Given a positive scalar γ_1, assume

$$k_{c1} > \frac{\gamma_1^2 + 1}{2\gamma_1^2}, \quad k_{e1} > \frac{\gamma_1^2 + 1}{2\gamma_1^2}.$$

Then the closed-loop system (7) and (8) has L_2-gain $\leq \gamma_1$.



Ex.) $k_{c1} = 10, k_{e1} = 20$ ➡ $\gamma_1 > 0.229$

Finite-gain L_2 Stable $\quad u$: Input $\quad H$: Map to Output

$^{\exists}\gamma, \beta > 0$ such that

$$\|(Hu)_\tau\|_{\mathcal{L}_2} \leq \gamma \|u_\tau\|_{\mathcal{L}_2} + \beta \quad ^{\forall}\tau \in [0, \infty)$$

$$\|u_\tau\|_{\mathcal{L}_2} := \sqrt{\int_0^\tau \|u(t)\|_2 dt} < \infty$$

H. K. Khalil, *Nonlinear Systems*, Third Edition, Prentice Hall, 2002.

Proof of L_2 Gain Performance (2 Rigid Bodies)

Proof:

$$E(g_{ij}) = \frac{1}{2}\|p_{ij}\|^2 + \phi(e^{\hat{\xi}\theta_{ij}})$$

Storage Function

$$E_1 := E(g_{10}) + E(g_{e10})$$

Time derivative:

$$\begin{aligned} \dot{E}_1 &= e_{c10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{10}})} V^b_{10} + e_{e10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e10}})} V^b_{e10} \\ &= (V^b_{w1})^T(-e_{c10}) + e_{c10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{10}})} V^b_{w0} + u_{e10}^T(-e_{e10}) + e_{e10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e10}})} V^b_{w0} \\ &= u_1^T(-e_1) + e_1^T \begin{bmatrix} \mathrm{Ad}_{(e^{\hat{\xi}\theta_{10}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e10}})} \end{bmatrix} d_0 \\ &= u_1^T(-e_1) - \frac{1}{2}\gamma_1^2 \left\| d_0 - \frac{1}{\gamma_1^2} \begin{bmatrix} \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{10}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e10}})} \end{bmatrix} e_1 \right\|^2 \\ &\quad + \frac{1}{2\gamma_1^2} \left\| \begin{bmatrix} \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{10}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e10}})} \end{bmatrix} e_1 \right\|^2 + \frac{1}{2}\gamma_1^2 \|d_0\|^2 \\ &\leq -e_1^T K_1 e_1 + \frac{1}{2\gamma_1^2}\|e_1\|^2 + \frac{1}{2}\gamma_1^2 \|d_0\|^2 \quad \left(\because \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ij}})} : \text{ Unitary}\right) \end{aligned}$$

Proof of L_2 Gain Performance (2 Rigid Bodies)

Proof:

$$\begin{aligned} \dot{E}_1 &\leq -e_1^T K_1 e_1 + \frac{1}{2\gamma_1^2}\|e_1\|^2 + \frac{1}{2}\gamma_1^2 \|d_0\|^2 \\ &= -e_1^T \begin{bmatrix} (k_{c1} - \frac{1}{2\gamma_1^2}) I_6 & 0 \\ 0 & (k_{e1} - \frac{1}{2\gamma_1^2}) I_6 \end{bmatrix} e_1 + \frac{1}{2}\gamma_1^2 \|d_0\|^2 \\ &\leq -\frac{1}{2}\|e_1\|^2 + \frac{1}{2}\gamma_1^2 \|d_0\|^2 \quad \left(\because k_{c1} > \frac{\gamma_1^2 + 1}{2\gamma_1^2}, k_{c1} > \frac{\gamma_1^2 + 1}{2\gamma_1^2}\right) \end{aligned}$$

Integrating from 0 to T,

$$E_1(T) - E_1(0) \leq \frac{1}{2}\gamma_1^2 \int_0^T \|d_0\|^2 dt - \frac{1}{2}\int_0^T \|e_1\|^2 dt$$

➡ $$\int_0^T \|e_1\|^2 dt \leq \gamma_1^2 \int_0^T \|d_0\|^2 dt + 2E_1(0)$$

Thus,

$$\|e_1\|_{\mathcal{L}_2} \leq \gamma_1 \|d_0\|_{\mathcal{L}_2} + \sqrt{2E_1(0)}$$

$$(\alpha + \beta \geq \sqrt{\alpha^2 + \beta^2}, \alpha, \beta > 0)$$ □

Outline

- Introduction
- Previous Work
- Pose Synchronization with VMO: 2 Rigid Bodies
- Pose Synchronization with VMO: 3 Rigid Bodies
  - Convergence
  - Control Performance
- Pose Synchronization with VMO: n Rigid Bodies
- Future Works

Pose Synchronization with VMO (3 Rigid Bodies)

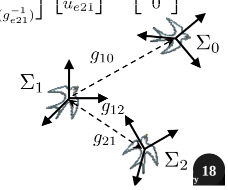
Relative Rigid Body Motion + Estimation Error System

$$\begin{bmatrix} V^b_{10} \\ V^b_{21} \\ V^b_{e10} \\ V^b_{e12} \\ V^b_{e21} \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(g_{10}^{-1})} & 0 & 0 & 0 & 0 \\ 0 & -\mathrm{Ad}_{(g_{21}^{-1})} & 0 & 0 & 0 \\ 0 & 0 & -\mathrm{Ad}_{(g_{e10}^{-1})} & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{Ad}_{(g_{e12}^{-1})} & 0 \\ 0 & 0 & 0 & 0 & -\mathrm{Ad}_{(g_{e21}^{-1})} \end{bmatrix} \begin{bmatrix} V^b_{w1} \\ V^b_{w2} \\ u_{e10} \\ u_{e12} \\ u_{e21} \end{bmatrix} + \begin{bmatrix} V^b_{w0} \\ V^b_{w1} \\ V^b_{w0} \\ V^b_{w2} \\ V^b_{w1} \end{bmatrix}$$

$$= \begin{bmatrix} -\mathrm{Ad}_{(g_{10}^{-1})} & 0 & 0 & 0 & 0 \\ I_6 & -\mathrm{Ad}_{(g_{21}^{-1})} & 0 & 0 & 0 \\ 0 & 0 & -\mathrm{Ad}_{(g_{e10}^{-1})} & 0 & 0 \\ 0 & I_6 & 0 & -\mathrm{Ad}_{(g_{e12}^{-1})} & 0 \\ I_6 & 0 & 0 & 0 & -\mathrm{Ad}_{(g_{e21}^{-1})} \end{bmatrix} \begin{bmatrix} V^b_{w1} \\ V^b_{w2} \\ u_{e10} \\ u_{e12} \\ u_{e21} \end{bmatrix} + \begin{bmatrix} V^b_{w0} \\ 0 \\ V^b_{w0} \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

➡ Each body velocity is included!

Input $u_2 := \begin{bmatrix} V^b_{w1} \\ V^b_{w2} \\ u_{e10} \\ u_{e12} \\ u_{e21} \end{bmatrix}$ $\quad$ Error $e_2 := \begin{bmatrix} e_{c10} \\ e_{c21} \\ e_{e10} \\ e_{e12} \\ e_{e21} \end{bmatrix}$



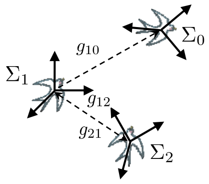
Passivity (3 Rigid Bodies)

Lemma 7 (Passivity)

If the leader is static $(V_{w0}^b = 0)$, then system (9) satisfies

$$\int_0^T u_2^T \nu_2 dt \geq -\beta_2$$

where $\nu_2 = -N_2 e_2$ and β_2 is a positive scalar.



$$N_2 = \begin{bmatrix} I_6 & -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{21}})} & 0 & 0 & -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e21}})} \\ 0 & I_6 & 0 & -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e12}})} & 0 \\ 0 & 0 & I_6 & 0 & 0 \\ 0 & 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & 0 & I_6 \end{bmatrix} \in \mathcal{R}^{30\times 30}$$

N_2 : Regular Matrix (Upper Triangular)

Proof:

$$E(g_{ij}) = \frac{1}{2}\|p_{ij}\|^2 + \phi(e^{\hat{\xi}\theta_{ij}})$$

Storage Function

$$E_2 := E(g_{10}) + E(g_{21}) + E(g_{e10}) + E(g_{e12}) + E(g_{e21})$$

Proof of Passivity (3 Rigid Bodies)

Proof:

Time derivative:

$$\begin{aligned} \dot{E}_2 &= \dot{E}(g_{10}) + \dot{E}(g_{21}) + \dot{E}(g_{e10}) + \dot{E}(g_{e12}) + \dot{E}(g_{e21}) \\ &= e_{c10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{10}})} V_{10}^b + e_{c21}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{21}})} V_{21}^b + e_{e10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e10}})} V_{e10}^b \\ &\quad + e_{e12}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e12}})} V_{e12}^b + e_{e21}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e21}})} V_{e21}^b \\ &= -e_{c10}^T V_{w1}^b - e_{c21}^T V_{w2}^b - e_{e10}^T u_{e10} - e_{e12}^T u_{e12} - e_{e21}^T u_{e21} \\ &\quad + e_{c21}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{21}})} V_{w1}^b + e_{e12}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e12}})} V_{w2}^b + e_{e21}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e21}})} V_{w1}^b \\ &= -\begin{bmatrix} e_{c10}^T & e_{c21}^T & e_{e10}^T & e_{e12}^T & e_{e21}^T \end{bmatrix} \begin{bmatrix} I_6 & 0 & 0 & 0 & 0 \\ -\mathrm{Ad}_{(e^{\hat{\xi}\theta_{21}})} & I_6 & 0 & 0 & 0 \\ 0 & 0 & I_6 & 0 & 0 \\ 0 & -\mathrm{Ad}_{(e^{\hat{\xi}\theta_{e12}})} & 0 & I_6 & 0 \\ -\mathrm{Ad}_{(e^{\hat{\xi}\theta_{e21}})} & 0 & 0 & 0 & I_6 \end{bmatrix} \begin{bmatrix} V_{w1}^b \\ V_{w2}^b \\ u_{e10} \\ u_{e12} \\ u_{e21} \end{bmatrix} \\ &= -e_2^T N_2^T u_2 \\ &= \nu_2^T u_2 = u_2^T \nu_2 \end{aligned}$$

□

Pose Synchronization with VMO (3 Rigid Bodies)

Control Input for Pose Synchronization with VMO : Passivity Approach

$$u_2 = -K_2\nu_2 = K_2 N_2 e_2 \quad (10)$$

$$K_2 = \mathrm{diag}\{k_{c1}I_6, k_{c2}I_6, k_{e1}I_6, k_{e1}I_6, k_{e2}I_6\} \quad k_{ci}, k_{ei} > 0,\ i \in \{1,2\}$$

$$\begin{bmatrix} V_{w1}^b \\ V_{w2}^b \end{bmatrix} = \begin{bmatrix} e_{c10} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{21}})} e_{c21} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e21}})} e_{e21} \\ e_{c21} - \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e12}})} e_{e12} \end{bmatrix} = \begin{bmatrix} e_{c10} + e_{c12} + e_{e12} \\ e_{c21} + e_{e21} \end{bmatrix}$$

Estimation Error

Theorem 3

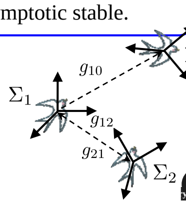
If the leader is static $(V_{w0}^b = 0)$, then the equilibrium point $e_2 = 0$ for the closed-loop system (9) and (10) is asymptotic stable.

Proof:

$$\begin{aligned} \dot{E}_2 &= u_2^T \nu_2 \\ &= (-K_2\nu_2)^T \nu_2 \\ &= -\nu_2 K_2 \nu_2 \\ &= -e_2^T N_2^T K_2 N_2 e_2 < 0 \end{aligned}$$

□

$\left(K_2 > 0, N_2 : \text{Regular} \Rightarrow N_2^T K_2 N_2 > 0\right)$

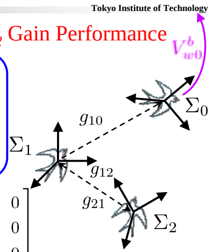


Control Performance (3 Rigid Bodies)

Velocity of Leader V_{w0}^b : Disturbance d_0 ➡ e_2 L_2 Gain Performance

Theorem 4

Given a positive scalar γ_2, assume $P_2 > 0$ (defined below). Then the closed-loop system (9) and (10) has L_2-gain $\leq \gamma_2$



$$P_2 := N_2^T K_2 N_2 - \frac{1}{2}\begin{bmatrix} \left(1+\frac{1}{\gamma_2^2}\right) I_6 & 0 & 0 & 0 & 0 \\ 0 & I_6 & 0 & 0 & 0 \\ 0 & 0 & \left(1+\frac{1}{\gamma_2^2}\right) I_6 & 0 & 0 \\ 0 & 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & 0 & I_6 \end{bmatrix}$$

Finite-gain L_2 Stable $\quad u$: Input $\quad H$: Map to Output

$^\exists \gamma, \beta > 0$ such that

$$\|(Hu)_\tau\|_{\mathcal{L}_2} \leq \gamma \|u_\tau\|_{\mathcal{L}_2} + \beta \quad {}^\forall \tau \in [0, \infty)$$

$$\|u_\tau\|_{\mathcal{L}_2} := \sqrt{\int_0^\tau \|u(t)\|_2 dt} < \infty$$

H. K. Khalil, *Nonlinear Systems*, Third Edition, Prentice Hall, 2002.

Proof of L_2 Gain Performance (3 Rigid Bodies)

Proof:

$$E(g_{ij}) = \frac{1}{2}\|p_{ij}\|^2 + \phi(e^{\hat{\xi}\theta_{ij}})$$

Storage Function

$$E_2 := E(g_{10}) + E(g_{21}) + E(g_{e10}) + E(g_{e12}) + E(g_{e21})$$

Time derivative:

$$\begin{aligned} \dot{E}_2 &= e_{c10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{10}})} V_{10}^b + e_{c10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e10}})} V_{e10}^b + e_{c21}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{21}})} V_{21}^b + e_{c21}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e21}})} V_{e21}^b \\ &= u_2^T \nu_2 + e_{c10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{10}})} V_{w0}^b + e_{e10}^T \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e10}})} V_{w0}^b \\ &= u_2^T \nu_2 + e_1^T \begin{bmatrix} \mathrm{Ad}_{(e^{\hat{\xi}\theta_{10}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{\hat{\xi}\theta_{e10}})} \end{bmatrix} d_0 \\ &= u_2^T \nu_2 - \frac{1}{2}\gamma_2^2 \left\| d_0 - \frac{1}{\gamma_2^2} \begin{bmatrix} \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{10}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e10}})} \end{bmatrix} e_1 \right\|^2 \\ &\quad + \frac{1}{2\gamma_2^2} \left\| \begin{bmatrix} \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{10}})} & 0 \\ 0 & \mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e10}})} \end{bmatrix} e_1 \right\|^2 + \frac{1}{2}\gamma_2^2 \|d_0\|^2 \\ &\leq -e_2^T N_2^T K_2 N_2 \nu_2 + \frac{1}{2\gamma_2^2}\|e_1\|^2 + \frac{1}{2}\gamma_2^2\|d_0\|^2 \quad \left(\because \mathrm{Ad}_{(e^{\hat{\xi}\theta_{ij}})} : \text{Unitary}\right) \end{aligned}$$

Proof of L_2 Gain Performance (3 Rigid Bodies)

Proof:

$$\dot{E}_2 \leq -e_2^T N_2^T K_2 N_2 \nu_2 + \frac{1}{2\gamma_2^2}\|e_1\|^2 + \frac{1}{2}\gamma_2^2\|d_0\|^2$$

$$= -e_2^T N_2^T K_2 N_2 \nu_2 - \frac{1}{2}\|e_2\|^2 + e_2^T \cdot \frac{1}{2}\begin{bmatrix} \left(1+\frac{1}{\gamma_2^2}\right) I_6 & 0 & 0 & 0 & 0 \\ 0 & I_6 & 0 & 0 & 0 \\ 0 & 0 & \left(1+\frac{1}{\gamma_2^2}\right) I_6 & 0 & 0 \\ 0 & 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & 0 & I_6 \end{bmatrix} e_2 + \frac{1}{2}\gamma_2^2\|d_0\|^2$$

$$= -e_2^T P_2 e_2 - \frac{1}{2}\|e_2\|^2 + \frac{1}{2}\gamma_2^2\|d_0\|^2$$

$$\leq -\frac{1}{2}\|e_2\|^2 + \frac{1}{2}\gamma_2^2\|d_0\|^2 \quad (\because P_2 > 0)$$

Integrating from 0 to T,

$$E_2(T) - E_2(0) \leq \frac{1}{2}\gamma_2^2 \int_0^T \|d_0\|^2 dt - \frac{1}{2}\int_0^T \|e_2\|^2 dt$$

➡ $$\|e_2\|_{\mathcal{L}_2} \leq \gamma_2 \|d_0\|_{\mathcal{L}_2} + \sqrt{2E_2(0)}$$

□

Condition of L_2 Gain Performance (3 Rigid Bodies)

Condition of K_2 for $P_2 > 0$ $\quad K_2 = \mathrm{diag}\{k_{c1}I_6, k_{c2}I_6, k_{e1}I_6, k_{e1}I_6, k_{e2}I_6\}$

$$P_2 := N_2^T K_2 N_2 - \frac{1}{2}\mathrm{diag}\left\{\left(1+\frac{1}{\gamma_2^2}\right)I_6, I_6, \left(1+\frac{1}{\gamma_2^2}\right)I_6, I_6, I_6\right\}$$

$$N_2 = \begin{bmatrix} I_6 & -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{21}})} & 0 & 0 & -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e21}})} \\ 0 & I_6 & 0 & -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e12}})} & 0 \\ 0 & 0 & I_6 & 0 & 0 \\ 0 & 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & 0 & I_6 \end{bmatrix}$$

$P_2 =$

$$\begin{bmatrix} \left(k_{c1} - \frac{\gamma_2^2+1}{2}\right)I_6 & -k_{c1}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{21}})} & 0 & 0 & -k_{c1}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e21}})} \\ -k_{c1}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{21}})} & \left(k_{c1}+k_{c2}-\frac{1}{2}\right)I_6 & 0 & -k_{c2}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e12}})} & k_{c1}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{21}})}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{-e21}})} \\ 0 & 0 & \left(k_{e1}-\frac{\gamma_2^2+1}{2}\right)I_6 & 0 & 0 \\ 0 & -k_{c2}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{e12}})} & 0 & \left(k_{c2}+k_{e1}-\frac{1}{2}\right)I_6 & 0 \\ -k_{c1}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{e21}})} & k_{c1}\mathrm{Ad}_{(e^{\hat{\xi}\theta_{e21}})}\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{21}})} & 0 & 0 & \left(k_{c1}+k_{e2}-\frac{1}{2}\right)I_6 \end{bmatrix}$$

P_2 : Symmetic $\quad P_2 > 0$: Linear Matrix Inequality (LMI) for k_{ci}, k_{ei}

➡ Convex Programming Problem

Outline

Pose Synchronization with VMO (n Rigid Bodies)

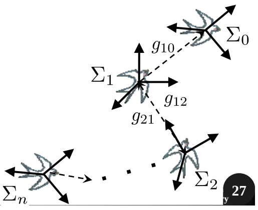
Relative Rigid Body Motion + Estimation Error System

$$\begin{bmatrix} V_{10}^b \\ \vdots \\ V_{n(n-1)}^b \\ V_{e10}^b \\ V_{e12}^b \\ \vdots \\ V_{en(n-1)}^b \end{bmatrix} = \begin{bmatrix} -\mathrm{Ad}_{(g_{10}^{-1})} & & & & & & 0 \\ & \ddots & & & & & \\ & & -\mathrm{Ad}_{(g_{n(n-1)}^{-1})} & & & & \\ & & & -\mathrm{Ad}_{(g_{e10}^{-1})} & & & \\ & & & & -\mathrm{Ad}_{(g_{e12}^{-1})} & & \\ & * & & & & \ddots & \\ & & & & & & -\mathrm{Ad}_{(g_{en(n-1)}^{-1})} \end{bmatrix} \begin{bmatrix} V_{w1}^b \\ \vdots \\ V_{wn}^b \\ u_{e10} \\ u_{e12} \\ \vdots \\ u_{en(n-1)} \end{bmatrix} + \begin{bmatrix} V_{w0}^b \\ 0 \\ \vdots \\ V_{w0}^b \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (11)$$

$$* \begin{cases} (i+1,i), (n+2i,i+1), (n+2i+1,i) : I_6 \\ \text{otherwise } 0 \end{cases}$$

in 6×6 block structure

Input $u_n := \begin{bmatrix} V_{w1}^b \\ \vdots \\ V_{n(n-1)}^b \\ V_{e10}^b \\ \vdots \\ u_{en(n-1)} \end{bmatrix}$ $\quad$ Error $e_n := \begin{bmatrix} e_{c10} \\ \vdots \\ e_{cn(n-1)} \\ e_{e10} \\ \vdots \\ e_{en(n-1)} \end{bmatrix}$



Passivity (n Rigid Bodies)

Lemma 8 (Passivity)

If the leader is static $(V_{w0}^b = 0)$, then system (11) satisfies

$$\int_0^T u_n^T \nu_n dt \geq -\beta_n$$

where $\nu_n = -N_n e_n$ and β_n is a positive scalar.

$$N_n = \begin{bmatrix} I_6 & & ** \\ & \ddots & \\ 0 & & I_6 \end{bmatrix} \in \mathcal{R}^{(18n-6)\times(18n-6)} \quad ** \begin{cases} (i+1,i) : -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{(i+1)i}})} \\ (i+1,n+2i) : -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{ei(i+1)}})} \\ (i,n+2i+1) : -\mathrm{Ad}_{(e^{-\hat{\xi}\theta_{e(i+1)i}})} \end{cases}$$

in 6×6 block structure

N_n : Regular Matrix (Upper Triangular)

Proof:

Storage Function $\quad E(g_{ij}) = \frac{1}{2}\|p_{ij}\|^2 + \phi(e^{\hat{\xi}\theta_{ij}})$

$$E_n := \sum_{i=1}^{n}\left(E(g_{i(i-1)}) + E(g_{ei(i-1)})\right) + \sum_{i=1}^{n-1} E(g_{ei(i+1)})$$

Pose Synchronization with VMO (n Rigid Bodies)

Control Input for Pose Synchronization with VMO : Passivity Approach

$$u_n = -K_n \nu_n = K_n N_n e_n \quad (12) \quad K_n = \mathrm{diag}\{k_{c1}I_6 k_{en}I_6\} \quad k_{ci}, k_{ei} > 0$$

$$\begin{bmatrix} V_{w1}^b \\ V_{w2}^b \\ \vdots \end{bmatrix} = \begin{bmatrix} e_{c10} + e_{c12} + e_{e12} \\ e_{c21} + e_{c23} + e_{e21} + e_{e23} \\ \vdots \end{bmatrix}$$

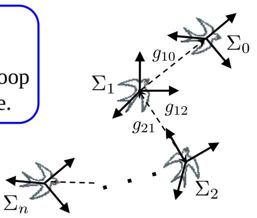
Estimation Error

Theorem 5

If the leader is static $(V_{w0}^b = 0)$, then the equilibrium point $e_n = 0$ for the closed-loop system (11) and (12) is asymptotic stable.

Proof: Refer to resume

Key Point: $N_n^T K_n N_n > 0$



Control Performance (3 Rigid Bodies)

Velocity of Leader V_{w0}^b : Disturbance d_0 ➡ e_2 $\quad L_2$ Gain Performance

Theorem 6

Given a positive scalar γ_n, assume $P_n > 0$ (defined below). Then the closed-loop system (11) and (12) has L_2-gain $\leq \gamma_n$.

$$N_n = \begin{bmatrix} I_6 & & ** \\ & \ddots & \\ 0 & & I_6 \end{bmatrix}$$

Upper Triangular

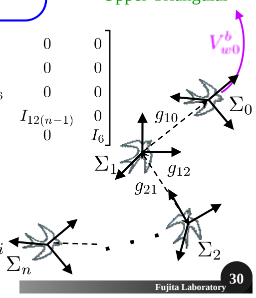
$$P_n := N_n^T K_n N_n - \frac{1}{2}\begin{bmatrix} \left(1+\frac{1}{\gamma_n^2}\right)I_6 & 0 & 0 & 0 & 0 \\ 0 & I_{6(n-1)} & 0 & 0 & 0 \\ 0 & 0 & \left(1+\frac{1}{\gamma_n^2}\right)I_6 & 0 & 0 \\ 0 & 0 & 0 & I_{12(n-1)} & 0 \\ 0 & 0 & 0 & 0 & I_6 \end{bmatrix}$$

Proof: Refer to resume

Condition of K_n for $P_n > 0$

P_n : Symmetic $\quad P_n > 0$: (LMI) for k_{ci}, k_{ei}

Convex Programming Problem



Speculation and Future Work

Speculation:

It is probably possible to deal in Arbitrary Communication Structure

Reason:

$$N_n = \begin{bmatrix} I_6 & & * \\ & \ddots & \\ 0 & & I_6 \end{bmatrix}$$

Relative Rigid Body Motion

Estimation Error System

: Upper Triangular Matrix

Future Work

- Simulation and Experiment (Omnidirectional Camera)
- Convergence to The Desired Relative Pose
- Unidirectional Information Flow (Changing Potential Function)
- Time Varying Desired Relative Pose
- Block Diagram
- (Solution to LMI)