Frequency domain design of the wheeled pendulum

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Introduction

Unlike four-wheel robot, wheeled inverted pendulum robot is easy to circle and it takes little space to move.

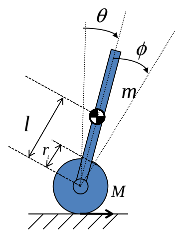
Fujita lab. uses zmp e-nuvo WHEEL. We learn control theorem and the usability of the control law through design and implementation of the control law using this.

zmp e-nuvo WHEEL

Outline

1. Introduction
2. Design of the servo system
   - Design of the servo system
   - Problems
3. Design of PID controller
   - Analysis of the wheeled pendulum
   - Design of PID controller
4. Summary

Derivation of the model



Parameters

- m : Mass of the pendulum
- M : Mass of the cart
- J_p : Moment of inertia of the body
- J_t : Moment of inertia of the cart
- J_m : Moment of inertia of the motor rotor
- l : Length between the wheel axle and the gravity center of the body
- r_t : Radius of the wheel
- c : Friction of the wheel axle
- K_t : Torque constant of the motor
- i : Reduction ratio of the gear
- u : Current references of the motor
- a : Gain from u to torque of the tire

Lagrange equation of motion

$$\begin{cases} \{(M+m)r_t^2+mlr_t\cos\theta+J_t+iJ_m\}\ddot{\theta}-mlr_t\sin\theta\cdot\dot{\theta}^2+\{(M+m)r_t^2+J_t+i^2J_m\}\ddot{\phi}+c\dot{\phi}=au \\ \{(M+m)r_t^2+2mlr_t\cos\theta+J_p+J_t+J_m\}\ddot{\theta}-mlr_t\sin\theta\cdot\dot{\theta}^2-mgl\sin\theta \\ +\{(M+m)r_t^2+mlr_t\cos\theta+J_t+iJ_m\}\ddot{\phi}=0 \end{cases}$$

Space state equation

Linearization around the equilibrium point $\theta_e=0$

$\cos\theta\approx 1,\sin\theta\approx\theta,\dot{\theta}^2\approx 0$

$$\alpha\begin{bmatrix}\ddot{\theta}\\ \ddot{\phi}\end{bmatrix}+\beta\begin{bmatrix}\dot{\theta}\\ \dot{\phi}\end{bmatrix}+\gamma\begin{bmatrix}\theta\\ \phi\end{bmatrix}=\delta u$$

where

$$\alpha=\begin{bmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{bmatrix}\quad \beta=\begin{bmatrix}0 & c\\ 0 & 0\end{bmatrix}\quad \gamma=\begin{bmatrix}0 & 0\\ -mgl & 0\end{bmatrix}\quad \delta=\begin{bmatrix}a\\ 0\end{bmatrix}$$

$\alpha_{11}=(M+m)r_t^2+mlr_t+J_t+iJ_m$ $\alpha_{12}=(M+m)r_t^2+J_t+i^2J_m$

$\alpha_{21}=(M+m)r_t^2+2mlr_t+ml^2+J_p+J_t+J_m$ $\alpha_{22}=(M+m)r_t^2+mlr_t+J_t+iJ_m$

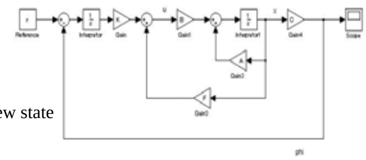
Determine the state

$$x=\begin{bmatrix}\theta\\ \phi\\ \dot{\theta}\\ \dot{\phi}\end{bmatrix}\quad \frac{dx}{dt}=Ax+Bu$$

where

$$A=\begin{bmatrix}0_{2\times2} & I_{2\times2}\\ -\alpha^{-1}\gamma & -\alpha^{-1}\beta\end{bmatrix},B=\begin{bmatrix}0_{2\times2}\\ \alpha^{-1}\delta\end{bmatrix}$$

Servo system



Space state

$$\begin{cases}\frac{dx}{dt}=Ax+Bu \quad y=Cx\\ C=[0\quad 1\quad 0\quad 0]\end{cases}$$

Description of the system with a new state

$$\frac{d\eta}{dt}=r-y=r-Cx$$

$$\frac{d}{dt}\begin{bmatrix}x\\ \eta\end{bmatrix}=\begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x\\ \eta\end{bmatrix}+\begin{bmatrix}B\\ 0\end{bmatrix}u+\begin{bmatrix}0\\ I\end{bmatrix}r$$

A_s X B_s

$$y=[C\quad 0]\begin{bmatrix}x\\ \eta\end{bmatrix}$$

Optimal feedback

$$u=-R^{-1}B_s^TP\begin{bmatrix}x\\ \eta\end{bmatrix}$$

This u minimize the evaluate function

P is the solution of the Riccatti equation

$$A_s^TP+PA_s-PB_sR^{-1}B_s^TP+Q=0$$

State Feedback

$$u=[F\quad K]\begin{bmatrix}x\\ \eta\end{bmatrix}$$

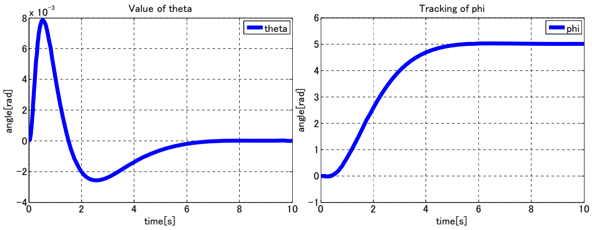
Evaluate function

$$J=\int_0^{\infty}(X^TQX+u^TRu)dt$$

Q : Weighting matrix $Q\in\Re^{5\times5}$

R : Weighting matrix $R\in\Re$

Simulation of servo system



θ ϕ

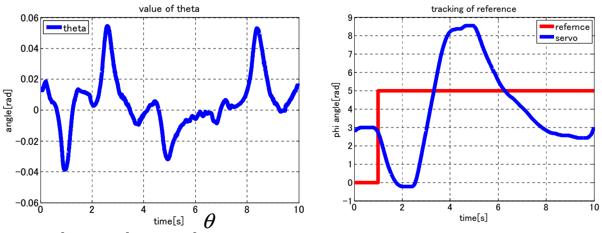
$$Q=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}, R=500 \qquad x_0=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} \quad r=5[rad]$$

Feedback gain

Wheeled pendulum moves about 0.3[m]

$F=[9.697 \quad 0.094 \quad 1.156 \quad 0.079] \quad K=-0.044$

Experiment of servo system



θ

There is a big overshoot.

To avoid this, we have to change the feedback and integral gain.

One of the method is changing the weight matrix.

But, it is difficult to understand straightforward how to change the weight and the cause of poor robustness.

→ Apply PID controller

We can change the gain directly and design the gain base on the frequency analysis.

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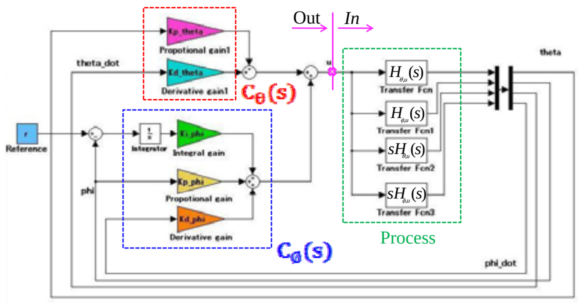
Block diagram of the PID system

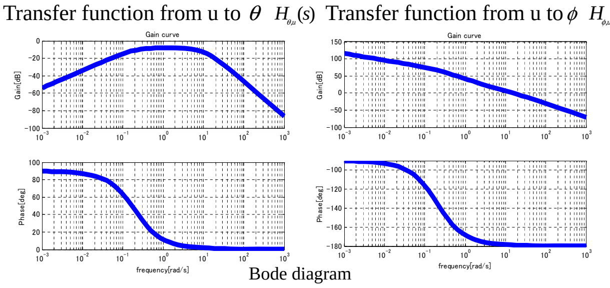


$$C_\theta(s)=K_{P,\theta}+K_{D,\theta}s$$

$$C_\phi(s)=K_{P,\phi}+\frac{K_{I,\phi}}{s}+K_{D,\phi}s$$

Analysis of the process

Transfer function from u to θ $H_{\theta u}(s)$ Transfer function from u to ϕ $H_{\phi u}(s)$



Bode diagram

Poles :	-11.01, -0.2, 0, 10.74
Zeros :	0

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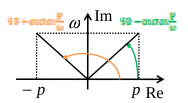
Zeros :	0

There is a pole in the right half plane.

Gain crossover frequency limitation

Gain crossover frequency inequality

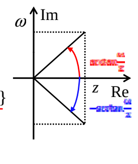
$$-\arg P_{ap}(j\omega_{gc})\le\pi-\varphi_m+n_{gc}\frac{\pi}{2}=:\varphi_l$$



When the process has a pole in the right half-plane

$$P_{ap}(s)=\frac{s+p}{s-p}, p>0 \quad -\arg P_{ap}(j\omega)=-\{\arg(p+j\omega)-\arg(-p+j\omega)\}$$

$$\omega_{gc}>\frac{p}{\tan(\varphi_l/2)} \qquad =2\arctan\frac{p}{\omega}$$



When the process has a zero in the right half-plane

$$P_{ap}(s)=\frac{z-s}{z+s}, z>0 \quad -\arg P_{ap}(j\omega)=-\{\arg(z-j\omega)-\arg(z+j\omega)\}$$

$$\omega_{gc}<z\tan(\varphi_l/2) \qquad =2\arctan\frac{\omega}{z}$$

Allowable phase lag of P_{ap} at ω_{gc} : φ_l

Required phase margin : $\varphi_m=60[\deg]$

Slope at ω_{gc} : $n_{gc}=-\frac{2}{3}$

⇒ $\varphi_l=60[\deg]$

So, gain crossover frequency must satisfy

$$\omega_{gc}>18.6[rad/s]$$

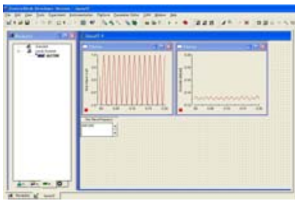
Properties of an actuator

Experiment

Give sine wave electrical signal to the motor.

Explore if the tire can reconstruct the motion.

We can know motion of the tire by the encoder.



Result

Over 500[rad/s],the encoder pulse can't reconstruct the sine wave.

Over 100[rad/s],we can see the tire can't turn around enough to move the cart.

Properties of sensors

Gyro sensor

Zmp original gyro board

ENC-03RC
- Murata Manufacturing Company
- Cycle 10[ms]
- Bandwidth 50Hz LPF

⇒ $\omega < 300[rad/s]$

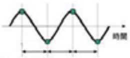
Rotary encoder

KE203
- Kodenshi Coporation
- Cycle 10[ms]
- Resolution 100[puls/rev]

*Resolution of the axis
12000[puls/rev]
0.000523[rad/puls]

Consider the cycle

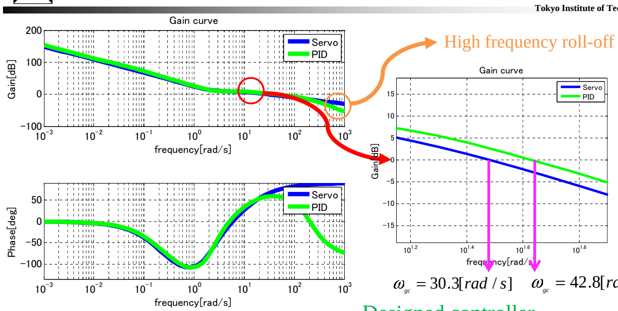
$\omega < 1200$ [rad/s] ∴For sampling theorem



⇒ The sensor can't detect over 1200[rad/s] signals

To take into account properties of the actuator and sensors, we want to cut down the gain about over 150[rad/s].

Open loop transfer function



$\omega_{gc} = 30.3[rad/s]$ $\omega_{gc} = 42.8[rad/s]$

Theta	Gain	Phi	Gain
Kp_theta	10.697	Kp_phi	0.112
Kd_theta	1.383	Ki_phi	-0.070
		Kd_phi	0.075

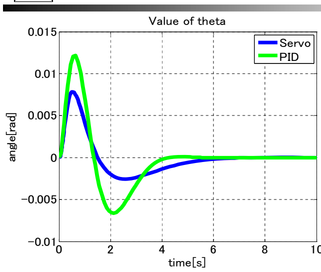
Designed controller

Add quadratic filter to PID controller

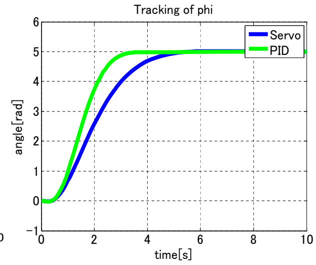
$$C_f(s) = \frac{1}{\frac{1}{45000}s^2 + \frac{1}{150}s + 1}$$

Cut off frequency 150[rad/s]

Simulation of PID system



θ



ϕ

$$x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad r = 5$$

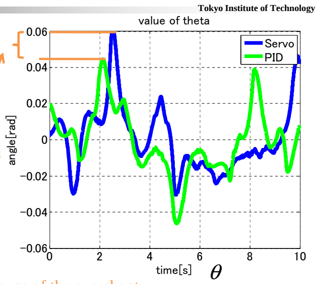
Because of the increasing of the integral gain, the angle of the tire converge at the reference angle faster. But, the body leans bigger.

Experiment of the PID system (1)

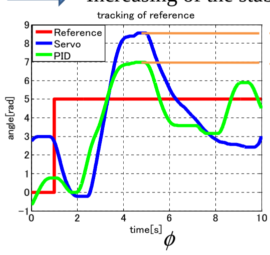
When the wheeled pendulum starts to move forward, the perturbation of the body angle becomes smaller.

Difference of the perturbation

⇒ Increasing of the stability



θ



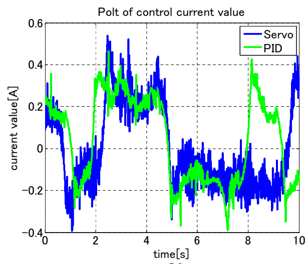
ϕ

Difference of the overshoot

The tracking speed is almost same as the servo system. The overshoot becomes smaller.

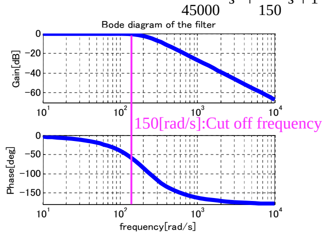
⇒ Increasing of the tracking ability.

Experiment of the PID system (2)



u

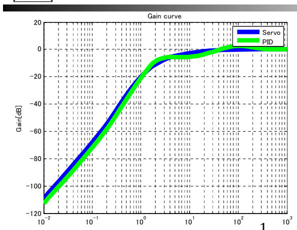
Applied filter $C_f(s) = \frac{1}{\frac{1}{45000}s^2 + \frac{1}{150}s + 1}$

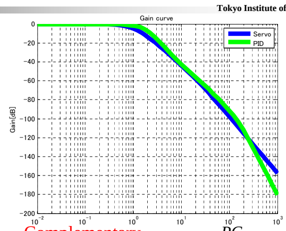


There are oscillations in the wheeled pendulum because the measurement noise affect the control input. From the graph, you can see the measurement noise attenuates if applied the filter.

As a result, the oscillation disappeared when the wheeled pendulum moves forward.

Gang of four (1)





Sensitivity function $S = \frac{1}{1+PC}$

Complementary sensitivity function $T = \frac{PC}{1+PC}$

Large peaks of the sensitivity can result unless there are closed loop poles close to the fast process poles.

T increases for frequencies close to the process zeros unless there is a closed loop pole in the neighborhood.

Closed loop poles Fast process pole : -11.01 Process zero : 0.00

Servo		
-18.1049	-8.6762	-4.2193
-0.8699 + 0.5083i	-0.8699 - 0.5083i	

PID	-53.83	-5.85	-1.32
	-118.49 + 124.04i	-118.49 – 124.04i	
	-1.25 + 1.52i	-1.25 - 1.52i	

Gang of four (2)

Load sensitivity function

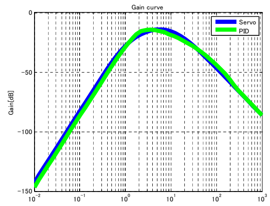
$PS = \frac{P}{1+PC}$

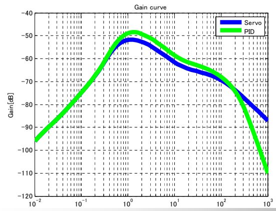
From u to theta

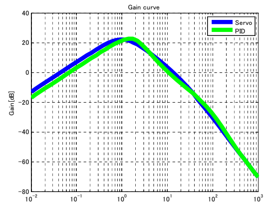
From u to phi

Noise sensitivity function

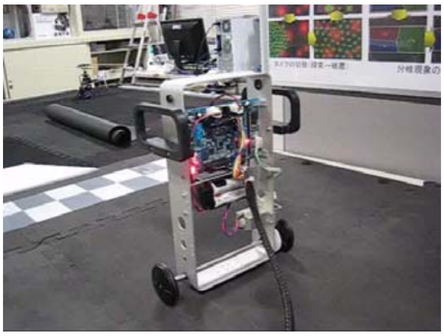
$CS = \frac{C}{1+PC}$







Experiment movie



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Summary

- Introduced what I did for two months
  - Servo system is considered based on only the time domain. It is necessary to design a controller based on the frequency domain.
  - It is not good to design a controller base on the open loop transfer function. We have to consider the Gang of four in addition when we design a controller.
- Future works
  - ➢ Design more robust controller
    - Find more robust gain
    - Apply another controller