Stabilization control of inverted pendulum

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Introduction

Invert pendulum
e-nuvo WHEEL(ZMP INC.)
had no controller with d-SPACE.

- Derive equation of motion.
- Control with d-SPACE.
- Apply some controller.

Outline

✓ Adapt to d-SPACE
  – Equation of motion
  – Calibration of potentiometer
  – LQ controller
  – Filter
- LQI Controller
- PID Controller
- Conclusion

Equation of motion(1)

Cart position

$$(x_c, y_c) = (l\sin\theta + r_t\,\phi,\ -l\cos\theta)$$

Pendulum position

$$(x_p, y_p) = (rt\phi,\ 0)$$

Lagrange equation of motion

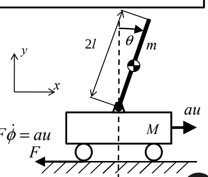
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} = 0$$

Result of Lagrange equation of motion

$$(ml^2 + Jp)\ddot\theta + c\dot\theta - gml\sin\theta + mlr_t\ddot\phi = 0$$

$$mlr_t\ddot\theta\cos\theta - mlr_t\dot\theta^2\sin\theta + \{(M+m)r_t^2 + Jt + i^2 Jm\}\ddot\phi + r_t^2 F\dot\phi = au$$

m : Mass of the pendulum
M : Mass of the cart
rat : Reduction ratio of the gear
Jp : Moment of inertia of the pendulum
Jt : Moment of inertia of the cart
Jm : Moment of inertia of the motor rotor
l : Length between the pendulum axle and the gravity center of the pendulum
r_t : Radius of the wheel
F : Viscous Friction of the system
c : Friction of the wheel axle
Kt : Torque constant of the motor
a :



Equation of motion(2)

Linearize the equation of motion with $\cos\theta \quad 1, \sin\theta \quad \theta$

$$\ddot\theta = \frac{\{(M+m) + Jt/r_t^2 + i^2 Jm/r_t^2\}(-c\dot\theta + mgl\theta) + mlr_t F\dot\phi - mlr_t au/r_t^2}{(ml^2+Jp)\{(M+m) + Jt/r_t^2 + i^2 Jm/r_t^2\} - (ml)^2}$$

$$\ddot\phi = \frac{ml/r_t(c\dot\theta - mgl\theta) - F(ml^2+Jp)\dot\phi + (ml^2+Jp)au/r_t^2}{(ml^2+Jp)\{(M+m) + Jt/r_t^2 + i^2 Jm/r_t^2\} - (ml)^2}$$

$\{(M+m) + Jt/r_t^2 + i^2 Jm/r_t^2\} = \alpha$, denominator $= \beta$

Give the state equation with state $x = (\theta,\ \phi,\ \dot\theta,\ \dot\phi)^{\mathrm T}$

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{\alpha mgl}{\beta} & 0 & -\frac{\alpha c}{\beta} & \frac{mlr_t}{\beta} \\ 0 & 0 & 0 & 1 \\ -\frac{m^2 l^2 g}{r_t\beta} & 0 & \frac{mlc}{r_t\beta} & -\frac{F(ml^2+Jp)}{\beta} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ -\frac{mlr_t a}{r_t^2\beta} \\ \frac{(ml^2+Jp)a}{r_t^2\beta} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x$$

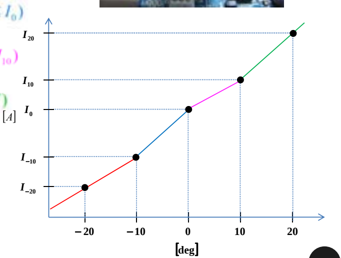
$$\Rightarrow \begin{cases} \frac{dx}{dt} = Ax + Bu \\ y = Cx + D \end{cases}$$

Calibration of potentiometer(1)

Interpolation line

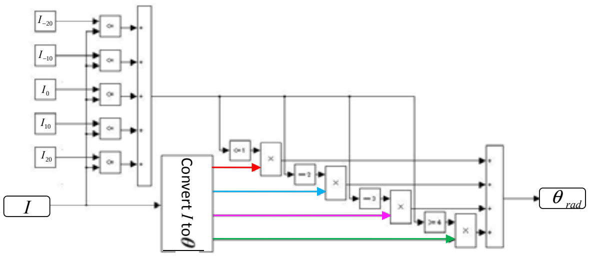
$$\theta_{\deg} = \begin{cases} \frac{I_{-10} - I_{-20}}{10} I - 20 + \frac{I_{-10} - I_{-20}}{10} I_{-20} & (I \le I_{-10}) \\ \frac{I_0 - I_{-10}}{10} I - 10 + \frac{I_0 - I_{-10}}{10} I_{-10} & (I_{-10} \le I \le I_0) \\ \frac{I_{10} - I_0}{10} I + 0 + \frac{I_{10} - I_0}{10} I_{10} & (I_0 \le I \le I_{10}) \\ \frac{I_{20} - I_{10}}{10} I + 10 + \frac{I_{20} - I_{10}}{10} I_{20} & (I_{10} \le I) \end{cases}$$

$$\theta_{rad} = \frac{2\pi}{360}\theta_{\deg}$$



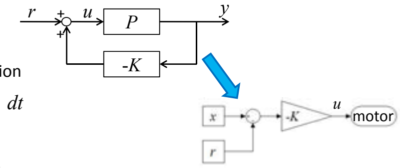
Calibration of potentiometer(2)

Model of Calibration



LQ Controller (1)

LQ controller



Quadratic evaluation function

$$J=\int_0^\infty \left(x^T Q x + u^T R u\right)\, dt$$

When J is minimum

$$u=-R^{-1}B^T P x=-Kx$$

,where P is positive definite solution of Riccati equation

$$PA+A^T P-PBR^{-1}B^T P+Q=0$$

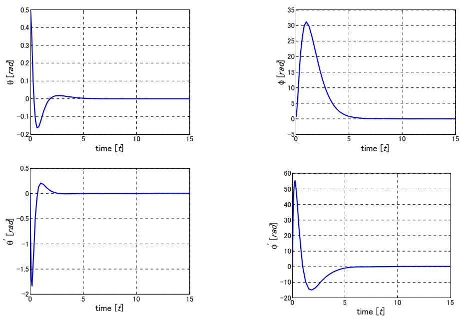
Q (weighting matrix) $\in \Re^{4\times 4}$

R (weighting matrix) $\in \Re$

Ex.

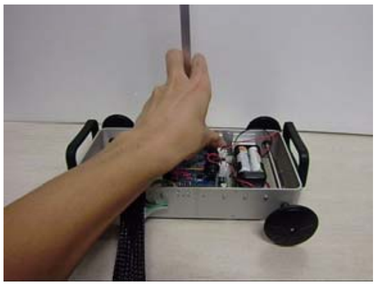
$$Q=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},\quad R=500 \dashrightarrow K=[-31.7\quad -5.7\quad -.0045\quad -0.086]$$

LQ controller (2)



$$x_0=[0.5\quad 0.5\quad 0\quad 0]^T$$

Movie

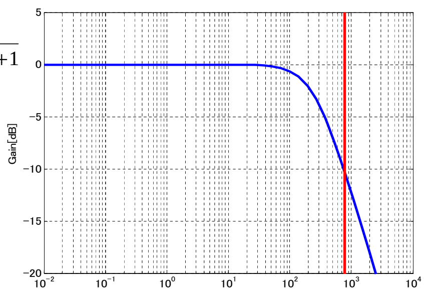


Filter (1)

$4\ [ms/puls] \Rightarrow \omega_{max}=785\ [rad/s]$

785 [rad/s]

$$G_f(s)=\frac{1}{0.004s+1}$$



Filter (2)

Transfer function of a filter

$$X(s)=\frac{1}{Ts+1}$$

Z Transformation

$$X(z)\equiv(1-z^{-1})\ Z\left[\frac{X(s)}{s}\right]$$

$$\therefore\ X(z)=\frac{1-\exp(-\tau/T)}{z-\exp(-\tau/T)}$$

Ex.) $\tau=0.001[s]$

① $T=0.004$

$$X(s)=\frac{1}{0.0025s+1}$$

$$\Rightarrow X(z)=\frac{0.2212}{z-0.7788}$$

② $T=0.01$

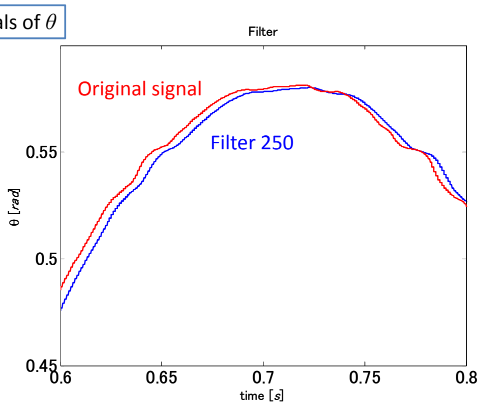
$$X(s)=\frac{1}{0.01s+1}$$

$$\Rightarrow X(z)=\frac{0.09516}{z-0.9048}$$

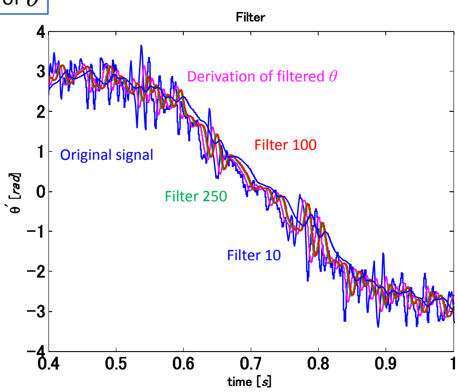
③ $T=0.1$

$$\Rightarrow X(z)=\frac{0.00995}{z-0.99}$$

Signals of θ



Signals of $\dot{\theta}$



250Hz

100Hz

10Hz

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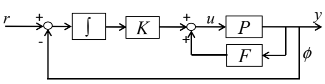
Servo integrator

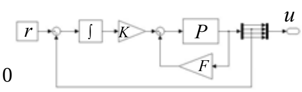
$$\frac{d\eta}{dt} = r - \phi = r - \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} x = r - C_\phi x$$



Augmented system

$$\frac{d}{dt}\begin{bmatrix} x \\ \eta \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ -C_\phi & 0 \end{bmatrix}}_{A_d} \begin{bmatrix} x \\ \eta \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_d} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r$$

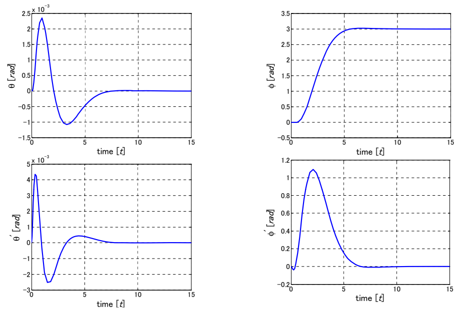
$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix}$$



Riccati Equation

$$PA_d + A_d^T P - PB_d R^{-1} B_d^T P + Q = 0$$

$$\begin{bmatrix} F & K \end{bmatrix} = -R^{-1} B_d^T P$$



$$Q = I \qquad x_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$R = 500 \qquad r = \begin{bmatrix} 0 & 3 & 0 & 0 \end{bmatrix}^T$$

Movie

$r = const$

Outline

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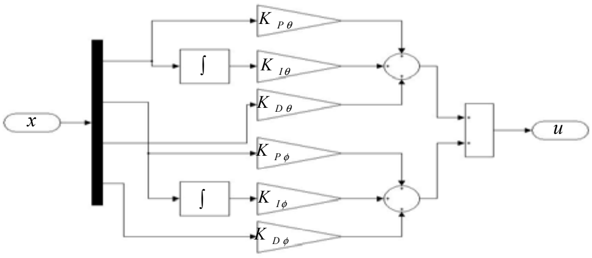
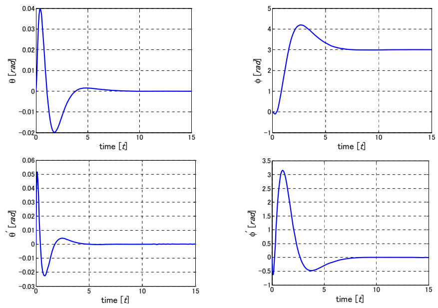
PID controller



Simulation



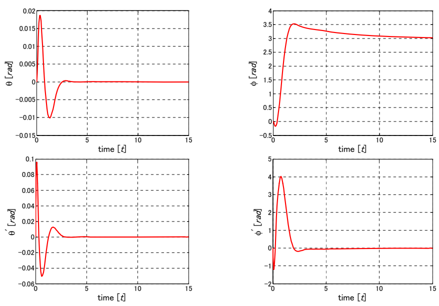
$x_0 = [0 \quad 0 \quad 0 \quad 0]^T \qquad [K_{P\theta} \quad K_{I\theta} \quad K_{D\theta}] = [-36.1 \quad 0 \quad -6.47]$

$r = [0 \quad 3 \quad 0 \quad 0]^T \qquad [K_{P\phi} \quad K_{I\phi} \quad K_{D\phi}] = [-0.11 \quad -0.045 \quad -0.13]$

Movie

$r = const$

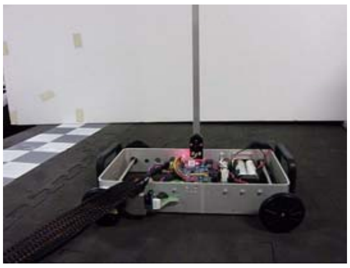
Tuned PID



$x_0 = [0 \quad 0 \quad 0 \quad 0]^T \qquad [K_{P\theta} \quad K_{I\theta} \quad K_{D\theta}] = [-44.3 \quad 0 \quad -8.0]$

$r = [0 \quad 3 \quad 0 \quad 0]^T \qquad [K_{P\phi} \quad K_{I\phi} \quad K_{D\phi}] = [-0.24 \quad -0.045 \quad -0.21]$

Movie



$$r = 3\sin \omega t$$

Outline

- Adapt to d-SPACE
- LQI Controller
- PID Controller
- ✓ Conclusion

Conclusion

Conclusion

- Controlled inverted pendulum with d-SPACE
- Applied some basic controllers

Future works

- Analyze with transfer function
- Apply another controller